

## KEYWORDS

Endogenous  
economic growth

Net energy

EROI

Energy  
transition

## ENDOGENOUS ECONOMIC GROWTH, EROI, AND TRANSITION TOWARDS RENEWABLE ENERGY

Victor COURT, Pierre-André JOUVET and  
Frédéric LANTZ

Due to their initial lack of emphasis on energy and natural resources, exogenous and endogenous growth models have suffered the same critic regarding the limits to economic growth imposed by finite Earth resources. Thus, various optimal control models that incorporate energy or natural resources have been developed during the last decades. However, in all these models the importance of the Energy Return On Energy Investment (EROI) has never been raised. The EROI is the ratio of the quantity of energy delivered by a given process to the quantity of energy consumed in this same process. Hence, the EROI is a measure of the accessibility of a resource, meaning that the higher the EROI the greater the amount of net energy delivered to society in order to support economic growth. The present article build a bridge upon the vacuum lying between the different literatures related to endogenous economic growth, the EROI and the necessary transition from nonrenewable to renewable energy. We provide an endogenous economic growth model subject to the physical limits of the real world (i.e. fossil and renewable energy production costs have functional forms that respect physical constraints). The model is able to reproduce (based on world data) an increasing reliance on fossil fuels from an early renewable era and the subsequent inevitable transition towards complete renewable energy that human will have to deal with in a not-too-far future. Through simulation we define the conditions for having a smooth transition from fossil to renewable energy and we study in which circumstances this transition can have negative impacts on economic growth (peak followed by a degrowth phase). In such cases, the implementation of a carbon tax can partially smooth this unfortunate dynamics depending on the ways of use of the carbon tax income.

Chaire Economie  
du Climat  
Palais Brongniart,  
4<sup>ie</sup> étage  
28 place de la  
bourse  
75002 PARIS

**Victor COURT** is a phd student at EconomiX, UMR 7235, Université Paris Ouest Nanterre - La Défense, 200 av. de la République, 92001 Nanterre, France.

**Pierre-André JOUVET** works at the Climate Economics Chair, Palais Brongniart, 28 place de la Bourse, 75002 Paris, France.

**Frédéric LANTZ** works at IFP Énergies Nouvelles, IFP School, 1-4 av. du Bois Préau, 92852 Rueil-Malmaison, France.



# Endogenous economic growth, EROI, and transition towards renewable energy

Victor COURT<sup>a,b,c,\*</sup>, Pierre-André JOUVET<sup>a,b</sup>, and Frédéric LANTZ<sup>c</sup>

<sup>a</sup>*EconomiX, UMR 7235, Université Paris Ouest Nanterre - La Défense, 200 av. de la République, 92001 Nanterre, France.*

<sup>b</sup>*Chaire Economie du Climat, Palais Brongniart, 28 place de la Bourse, 75002 Paris, France.*

<sup>c</sup>*IFP Énergies Nouvelles, IFP School, 1-4 av. du Bois Préau, 92852 Rueil-Malmaison, France.*

---

## Abstract

Due to their initial lack of emphasis on energy and natural resources, exogenous and endogenous growth models have suffered the same critic regarding the limits to economic growth imposed by finite Earth resources. Thus, various optimal control models that incorporate energy or natural resources have been developed during the last decades. However, in all these models the importance of the Energy Return On Energy Investment (EROI) has never been raised. The EROI is the ratio of the quantity of energy delivered by a given process to the quantity of energy consumed in this same process. Hence, the EROI is a measure of the accessibility of a resource, meaning that the higher the EROI the greater the amount of net energy delivered to society in order to support economic growth. The present article build a bridge upon the vacuum lying between the different literatures related to endogenous economic growth, the EROI and the necessary transition from nonrenewable to renewable energy. We provide an endogenous economic growth model subject to the physical limits of the real world (i.e. fossil and renewable energy production costs have functional forms that respect physical constraints). The model is able to reproduce (based on world data) an increasing reliance on fossil fuels from an early renewable era and the subsequent inevitable transition towards complete renewable energy that human will have to deal with in a not-too-far future. Through simulation we define the conditions for having a smooth transition from fossil to renewable energy and we study in which circumstances this transition can have negative impacts on economic growth (peak followed by a degrowth phase). In such cases, the implementation of a carbon tax can partially smooth this unfortunate dynamics depending on the ways of use of the carbon tax income.

**Key words:** Endogenous economic growth, net energy, EROI, energy transition.

**JEL classification:** C6, O4, Q3, Q4.

---

\*Corresponding author. E-mail: victor.court@chaireeconomieduclimat.org

# 1. Introduction

Compared to previous millennia of existence, human societies have experienced tremendous increases of development during the last one hundred and fifty years (Maddison, 2001). This pattern is commonly measured by the growth of the Gross World Product (GWP) at global scale, or Gross Domestic Product (GDP) at national level. As imperfect as these indicators are, they remain the mere focus of attention of economists, policy makers and media. Since Solow (1956), an important literature on endogenous growth models has been developed proposing different mechanisms for the origin of growth: human capital accumulation (Lucas, 1988; Rebelo, 1991), physical capital accumulation (Romer, 1986, 1990, Aghion and Howitt, 1992), public investment in education or research and development (Grossman and Helpman, 1991, Barro and Sala-i-Martin, 1995, Barro, 1996). But all these approaches ultimately raise the questions of the constitutive elements of the production function, the possibilities of taking into account their effects on growth, and the ways of deploying them in order to encourage their accumulation to support economic growth. Furthermore, these models have received two main critics. First, they are essentially qualitative since their main variables (physical capital, human capital, and innovation) are not always readily quantifiable (Warr and Ayres, 2012). Second, they do not take into account the limits to economic growth imposed by finite Earth resources formally addressed by Meadows et al. (1972). Responding to these critics in economic growth theory has implied to integrate a natural resource input in models and to postulate that human-made capital would be a perfect substitute for this natural resource input, or that technological progress would have to be infinite (Solow, 1974; Stiglitz, 1974; Dasgupta and Heal, 1974). Endogenous growth models that took into account the finiteness of natural resources as Smulders (1995) concluded that sustainable growth is possible as long as new knowledge is steadily accumulated thanks to constant or positive returns to scale (i.e. spillovers from human capital accumulation). In such models, it is acknowledged that material growth is finite, but because value added is supposed to become increasingly dematerialized and based on knowledge, economic growth can continue forever. In addition, it is worth noting that most of these conceptual frameworks must be understood as short-term explanation for contemporary growth, but they are completely unable to explain how humanity was able to leave a state of Malthusian stagnation characterized by a very low rate of growth for population, output and technological progress, to the state of sustained growth that countries affected by the Industrial Revolution have been experiencing for the last two hundred years or so.

It is also important to understand that the work of Meadows et al. (1972) took place in a broader critic of neoclassical economics because of its incompatibility with the physical reality governed by the laws of the thermodynamics. This view of the human society as a thermodynamic system was particularly supported by the 'bioeconomics' approach of Georgescu-Roegen (1971, 1979), the 'ecological system' approach of Odum (1971, 1973) and their peers (Daly, 1977; Hall et al., 1986). Despite some pure conceptual papers related to entropy and sustainability (Perrings, 1987; O'Connor, 1991; Ayres, 1998; Krysiak, 2006), or to the need for a broader paradigm shift in economics (Faber, 1985; Hall et al., 2001; Hall and Klitgaard, 2006), what we prefer to call 'biophysical economics' has been more practically represented by the energy science literature (input/output analysis, energy and mass flows accounting, etc.) that started at the same time. In particular, the Energy Return On (Energy) Investment (EROI or EROEI) has attracted many attention since any organism or system needs to procure at least as much energy as it consumes in order to pursue its existence. The EROI is the ratio of the quantity of energy delivered by a given process to the quantity of

energy consumed in this same process. Hence, the EROI is a measure of the accessibility of a resource, meaning that the higher the EROI the greater the amount of net energy delivered to society in order to support growth (Hall et al., 2014). It is a well-spread idea (despite the lack of emphasis of mainstream economics on this subject) that the development of industrial societies has been largely dependent on fossil fuels and in particular on their high EROI and consequent capacity to deliver large amounts of net energy to society. Since nonrenewable energy sources have high EROI compared to renewable ones (Murphy and Hall, 2010; Hall et al., 2014), a complete transition towards renewable energy would imply a decrease of the EROI at societal level. One can easily postulate that without an adequate increase in energy efficiency<sup>1</sup>, the energy transition towards complete renewable energy could force the economy to devote higher levels of investments to the energy sector at the expense of the rest of the economy. Despite an important literature, people working on the EROI concept have never developed aggregated models able to assess the impact of a complete energy transition on the EROI and economic growth. An exception to this fact is the GEMBA model of Dale et al. (2012), which incorporate a dynamic EROI function into an aggregated model but without any specification on the agents' behaviour and thus completely differs from neoclassical optimal growth models.

On the other hand, some researchers have focused their attention on the transition between a nonrenewable and a renewable natural resource. The optimal growth model of Tahvonen and Salo (2001) is able to represent a first phase of human development that only rely on renewable energy, a second phase where the simultaneous exploitation of nonrenewable and renewable energy is possible thanks to previous capital accumulation and increasing energy demand, and a third phase where the share of nonrenewable energy is decreasing because of increasing extraction costs, thus leading to a society that rely on renewable energy only. Here again, the impact of the complete transition towards renewable energy on economic growth is not specifically studied, and technological progress takes the form of learning by doing process. In Tsur and Zemel (2005) the attention is more focused on the R&D investments that allow a reduction in the cost of use of a backstop technology, but the broader effect of the energy transition on economic growth is not studied. Finally, Acemoglu et al. (2012) have studied in a more recent work the importance of the substitutability level between nonrenewable and renewable inputs in directing endogenous technical change, and influencing the optimal mix of environmental policies between carbon tax and R&D subsidy. It is worth noting that in these different energy transition models, the parallel with the EROI concept and its interaction with economic growth is never made. Hence, the consequences of a complete energy transition towards renewable energy and its impact on economic growth due to a potential EROI decrease have never been studied in the mainstream economics framework.

In light of what has been presented so far, the reader can now understand that the purpose of the present article is to build a bridge upon the vacuum lying between the different literatures related to endogenous economic growth, the EROI and the necessary transition from nonrenewable to renewable energy. In fact, Fagnart and Germain (2014) have started to bridge this gap in a recent working paper in which the possibility of a smooth transition between nonrenewable and renewable energy and its impact on the EROI and economic growth is studied. Despite its novelty, this study presents some features that we would like to address in the present paper: the nonrenewable energy is extracted without any capital requirement and consequently presents an infinite EROI, the backstop technology has a

---

<sup>1</sup> To be more precise, we should here speak about “energy efficiency net of any rebound effect”.

constant capital requirement per unit of energy produced<sup>2</sup>, technological progress is bounded but completely exogenous, and the production function in the final good sector is of Leontief type. Furthermore in this model, it is impossible to represent a three phases development of the energy use as in Tahvonen and Salo (2001) since the simulations need to be started with an initial economy just before “peak oil” and no production of renewable energy. In order to address these particular settings and other problems mentioned earlier in this introduction we will seek to provide an endogenous economic growth model subject to the physical limits of the real world, (i.e. fossil and renewable energy production costs have functional forms that respect physical constraints). Our model will also be able to reproduce an increasing reliance on fossil fuels from an early renewable era and the subsequent inevitable complete transition toward renewable energy that human will have to deal with in a not-too-far future. In order to do so the structure of our paper is as follows. Section 2 presents the model of a decentralized economy subject to knowledge accumulation to improve technological progress in the production of a final output good which primordially rely on the accessibility of a nonrenewable and a renewable energy resource. In section 3 we precise our simulation approach and calibration procedure. We run some simulations of the model in section 4 in order to study its dynamics, in particular the necessary conditions to have a smooth transition towards complete renewable energy. In section 5 we analyze the interest of the implementation of a tax on fossil energy production to smooth the transition towards complete renewable energy from an original simulation setting in which the energy transition has negative impacts on economic growth. We conclude our work and discuss some of our hypotheses for further research development in section 6.

## 2. The model<sup>3</sup>

The economy under consideration has three competitive markets for: final output good, capital and energy. The production of the final good that is then used for consumption or investment requires capital and energy. The accumulated capital is detained by households and rented to the: nonrenewable energy, renewable energy and final good sectors. Nonrenewable and renewable energy (hereafter NRE and RE respectively) are considered to be perfect substitutes and are consequently sold at the same price. It is important to highlight, especially regarding the EROI definition in section 2.6, that the NRE and RE supplies  $F_t$  are both productions net of the intrinsic energetic consumptions of their producers. A representative firm operates the NRE stock (aggregation of coal, oil, gas and uranium resources) with an increasing unitary cost of extraction that is however attenuated by the technological progress level. In the same way, another representative firm exploits a free primary RE flow (say radiant energy from the sun) considered to be constant and so large that its availability cannot constraint the economy. This RE flow is operated with a decreasing unitary cost of production and under decreasing returns to scale. Technological progress increases the overall productivity (i.e. both energy and capital efficiencies) of the final good

---

<sup>2</sup> To be exact, Fagnart and Germain (2014) postulated a theoretical decreasing (strictly positive) capital requirement per unit of energy produced for the backstop technology exploiting the renewable energy flow but when performing the simulations of their model they used instead a constant parameter.

<sup>3</sup> In parts of our model that are common with the one of Fagnart and Germain (2014) we have tried, as much as possible, to keep the same denomination for variables. Doing so allows the reader to easily spot similarities and discrepancies between both models and to consequently understand the sources of differences in results.

sector and also affects the capital intensiveness of both NRE and RE sectors. This technological progress evolved endogenously but it is formally bounded implying that the energy and capital requirements for industrial or energy production cannot be nil and tend asymptotically towards positive values. Finally, it is worth adding that in this model the possibilities of common or rare metal shortages that are incorporated in the different forms of capital are out of scope<sup>4</sup>.

## 2.1 Optimization behaviors of the consumer, the two energy producers, and the final good producer

### Intertemporal utility maximization of the household

We consider a representative household that consumes the final good and accumulates the physical capital that is rented to the different productive sectors. At each period  $t$ , the household receives the entire macroeconomic income from the rented capital and the different profits  $\Pi_t, \Omega_t, \Psi_t$  of the respective NRE, RE and final sectors. This income is spent in consumption and capital investment. Taking the final good as the numeraire, we can write the household's budget constraint as follows:

$$C_t + I_t = v_t K_t + \Pi_t + \Omega_t + \Psi_t \quad (1)$$

And the dynamics of the capital investment level is:

$$I_t = \frac{K_{t+1}}{\rho} \quad (2)$$

Here  $K_t$  is the total capital stock of the economy accumulated in  $t - 1$ ,  $v_t$  is the rental price of capital.  $\rho > 0$  represents the productivity of the transformation of investments goods into productive capital. For analytical simplicity we assume a unitary depreciation rate, implying that the time period length  $t$  corresponds to the average capital lifetime set to 15 years.

Preferences of the household are represented by an isoelastic utility function<sup>5</sup>  $U_t(C_t)$  that depends on the consumption level  $C_t$ . Parameter  $\sigma$  is the constant relative risk aversion coefficient of Arrow-Pratt, and  $0 < \beta < 1$  is the discount factor. Thus, the intertemporal decision of the household is to maximize its lifetime utility over the time horizon  $T$  under constraints (1) and (2).

$$\max_{\{C_t, K_{t+1}\}_{0 < t < T}} U = \sum_{t=0}^T \beta^t \frac{C_t^{1-\sigma} - 1}{1-\sigma}, \sigma > 0 \quad (3)$$

The first order condition with respect to  $K_t$  leads to:

$$\left[ \frac{C_{t+1}}{C_t} \right]^\sigma = \beta \rho v_{t+1}, \quad \forall t \in \{0, \dots, T\} \quad (4)$$

<sup>4</sup> In doing so we do not take into account the sensitivity of the EROI of the energy sectors to the increasing energy cost associated with ores grades degradation of metals incorporated in the physical capital of energy sectors as highlighted by Fizaine and Court (2015).

<sup>5</sup> This functional form respects the usual properties required since it is twice continuously differentiable and that  $U'(C) > 0$ ,  $U''(C) < 0 \forall C$ . We assume  $\lim_{C \rightarrow 0} U'(C) = +\infty$ .

This equation describes the well-known smoothing behavior of the agent where  $\rho v_{t+1} \equiv 1 + \mu$ , where  $\mu$  corresponds to the real interest rate of the economy.

Profit maximization of the NRE producer

The NRE resource, endowed with the initial stock  $\mathcal{S}$  (also called the Ultimately Recoverable Resource, or URR), is a common resource exploited during the time length  $T_e$  (observed during simulations) by a representative firm<sup>6</sup>. Extracting this resource implies to consume some physical capital  $Z_t$  that represents the only source of production cost. It is assumed that the representative firm knows  $\mathcal{S}$  and that the NRE stock will be operated until it is completely depleted or before that if the NRE operator cannot earn any profit because its production cost is too high compared to the RE sector where another representative firm is present. Thus, in each period  $t$  the NRE producer maximizes its profit  $\Pi_t$  and consequently chooses a capital stock  $Z_t$  in order to extract the amount  $X_t$  sold at the unitary price  $p_t$  by solving:

$$\max_{X_t, Z_t} \Pi_t = p_t X_t - v_t Z_t, \quad \forall t \in \{0, \dots, T_e\} \quad (5)$$

Under the constraint:

$$\sum_{t=1}^{T_e} X_t \leq \mathcal{S}, \quad \forall t \in \{0, \dots, T_e\} \quad (6)$$

And,

$$Z_t = (X_t * D_t)^{\frac{1}{\theta}}, \quad \text{with } \theta < 1 \quad \forall t \in \{0, \dots, T_e\} \quad (7)$$

Where  $\theta < 1$  means that returns to scale are decreasing in the NRE sector.  $D_t$  represents the capital intensiveness of the extraction process (i.e. the capital requirement per unit of fossil energy produced), which detailed definition is given in section 2.3.

After inserting (7) into the objective (5), the first order condition with respect to  $X_t$  gives:

$$X_t = \left[ \frac{p_t \theta}{D_t^{\frac{1}{\theta}} v_t} \right]^{\frac{\theta}{1-\theta}}, \quad \forall t \in \{0, \dots, T_e\} \quad (8)$$

Thus, considering (7) and (8),  $Z_t$  is defined by:

$$Z_t = \left[ \frac{p_t \theta}{D_t v_t} \right]^{\frac{1}{1-\theta}}, \quad \forall t \in \{0, \dots, T_e\} \quad (9)$$

---

<sup>6</sup> The authors would like to highlight that a competitive setting of the NRE or the RE sector is unnecessary since we do not seek to observe the labor allocation between sectors. In Fagnart and Germain (2014), such competitive framework is set for the RE sector (but not for the NRE resource since it is operated by the household) but it proves to be superfluous since after resolution the sector behave exactly as if it was represented by a unique firm.



Profit maximization of the RE producer

We suppose that a very large solar flow is accessible to the economy and that a representative firm is in charge of exploiting this RE flow. The firm chooses the net amount  $F_t$  of RE that is produced. In order to deliver the RE flow  $F_t$  some capital is necessary for the capture and transformation of a part of the primary RE flow. Thus in each period  $t$ , the RE producer maximizes its profit  $\Omega_t$  and consequently chooses a capital stock  $G_t$  in order to supply a flow  $F_t$  sold at the unitary price  $p_t$  by solving:

$$\max_{F_t, G_t} \Omega_t = p_t F_t - v_t G_t, \quad \forall t \in \{0, \dots, T\} \quad (10)$$

Under the constraint,

$$G_t = (F_t * B_t)^{\frac{1}{\gamma}}, \quad \text{with } 0 < \gamma < 1 \quad \forall t \in \{0, \dots, T\} \quad (11)$$

Where the variable  $B_t$  represents the capital intensiveness of the RE producer (i.e. the capital requirement per unit of renewable energy produced), which detailed definition is given in section 2.3.

Once  $G_t$  is substituted by its expression (11) into (10), the first order condition with respect to  $F_t$  leads to the following definition of the RE sector capital requirement:

$$F_t = \left[ \frac{\gamma p_t}{B_t^{\frac{1}{\gamma}} v_t} \right]^{1-\gamma}, \quad \forall t \in \{0, \dots, T\} \quad (12)$$

Hence considering (11) and (12),  $G_t$  is given by:

$$G_t = \left[ \frac{\gamma p_t}{B_t v_t} \right]^{\frac{1}{1-\gamma}}, \quad \forall t \in \{0, \dots, T\} \quad (13)$$

Profit maximization of the final good producer

In order to produce the final good  $Y_t$ , capital  $H_t$  and energy  $E_t$  are combined in a production function of Cobb-Douglas type with constant returns to scale.

$$Y_t = A_t E_t^\alpha H_t^{1-\alpha}, \quad \forall t \in \{0, \dots, T\} \quad (14)$$

Where  $E_t$  is the total energy sum of NRE ( $X_t$ ) and RE ( $F_t$ ) productions consumed by the final good sector and  $H_t$  is the capital allocated to this same sector. The output elasticities of energy and capital inputs are respectively represented by  $\alpha$  and  $1 - \alpha$ .  $A_t > 0$  represents the technological progress level of the economy (sometimes called Total Productivity Factor in other models) that increases through time thanks to knowledge accumulation. We suppose that in the final good sector, the technological progress affects both energy and capital uses and is thus . Considering the final good price as the numeraire, the representative firm in the final good sector seeks to solve:

$$\max_{E_t, H_t} \Psi_t = Y_t - p_t E_t - v_t H_t, \quad \forall t \in \{0, \dots, T\} \quad (15)$$

Under the constraint (14). The resolution of this problem leads to the following.

$$H_t = \frac{1 - \alpha p_t}{\alpha} \frac{p_t}{v_t} E_t, \quad \forall t \in \{0, \dots, T\} \quad (16)$$

$$Y_t = A_t E_t \left[ \frac{1 - \alpha p_t}{\alpha} \frac{p_t}{v_t} \right]^{1-\alpha}, \quad \forall t \in \{0, \dots, T\} \quad (17)$$

$$p_t = \left[ A_t \alpha \right]^{\frac{1}{1-\alpha}} \frac{(1 - \alpha)^{\frac{1-\alpha}{\alpha}}}{\alpha v_t}, \quad \forall t \in \{0, \dots, T\} \quad (18)$$

For the clarity of the following of the presentation, let us define now  $s_t$  as the saving rate of the economy.

$$s_t = \frac{K_{t+1}}{\rho Y_t} \quad (19)$$

## 2.2 Endogenous technological progress

The technological progress level  $A_t$  is necessarily bounded from above by a strictly positive constant  $\bar{A}$  since physical laws imply to always use some (even small) quantity of energy and capital to produce the industrial output.  $\bar{A}$  should be considered as the maximum technological progress level that humans will eventually achieve in the future. This uncertain parameter (for which we test several values in the following of the paper) is logically exogenous and in fact, given the increasing sigmoid functional form given to  $A_t$ , its maximum value  $\bar{A}$  represents an asymptotic limit that is never formally reached. This implies that the technological progress level continuously and infinitely increases over time, but at some point (when the maximum limit  $\bar{A}$  is close) the incremental gains are so small that the dynamic system describing the economy is in a quasi-steady state. Furthermore, we suppose that the speed of convergence between the initial technological progress level  $\underline{A}$  and its asymptotic value  $\bar{A}$  (verifying  $0 < \underline{A} < \bar{A}$ ) directly depends on the variation of the knowledge stock of the economy. This knowledge stock depends on the effort deployed in the R&D sector that itself follows the level of investment  $I_{t-1}$  in the economy compared to the level of production  $Y_{t-1}$  of this same previous period. In other words, we define the speed of convergence of the technological progress as the saving rate of the economy at the previous period,  $s_{t-1}$ . Hence, we choose the following law of motion for  $A_t$ :

$$A_t = \underline{A} + \frac{\bar{A} - \underline{A}}{1 + \exp(-s_{t-1}(t - t_{A_t \max}))}, \quad \forall t \in \{0, \dots, T\} \quad (20)$$

Where  $t_{A_t \max}$  is the time at which the technological progress growth rate is maximum. Readers familiar with diffusion processes will have recognized in  $A_t$  an increasing sigmoid function. This formulation of the technological progress insures that in our model both technological progress and economic growth are endogenous.

## 2.3 Unitary capital requirements in the NRE and RE sectors

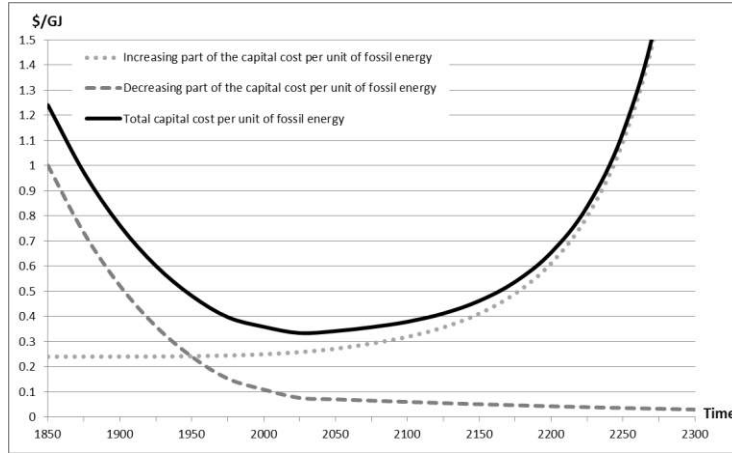
### Unitary capital requirement in the NRE sector

The unitary capital requirement per fossil energy unit produced  $D_t$  is composed of two parts as defined in equation (20) and shown in Figure 1. A first one that increases through the extraction process because of the quality depletion of the NRE resource, thus this term depends on the ratio of exploited resource  $\phi_t$ , varying between 0 when the fossil energy resource is still virgin and 1 when it is fully depleted. The second part is decreasing thanks to the impact of the current, initial, and asymptotic level of technological progress levels, respectively represented by  $A_t$ ,  $\underline{A}$ , and  $\bar{A}$ . With  $d_1 + d_2$  as the initial unitary capital cost and  $\delta$  as a constant parameter representing the rate of quality degradation of the NRE resource we can define  $D_t$  as:

$$D_t(\phi_t, A_t) = d_1 \exp^{\delta \phi_t^\varphi} + d_2 \left( \frac{\bar{A} - A_t}{\bar{A} - \underline{A}} \right)^\vartheta, \quad \forall t \in \{0, \dots, T_e\} \quad (21)$$

Where  $\varphi$  and  $\vartheta$  are positive integer which values (and the ones of other constants too) will be determined when calibrating the model on historical world data in section 3. The variable  $\phi_t$  representing the ratio of exploited energy resource is defined as follows in (22).

$$\phi_t = \frac{\sum_{i=0}^{t-1} X_i}{S}, \quad \forall t \in \{0, \dots, T_e\} \quad (22)$$



**Figure 1.** Example of a possible form for the capital requirement per unit of fossil energy produced between 1850 and 2300 with decomposition of the increasing and decreasing parts.

### Unitary capital requirement in the RE sector

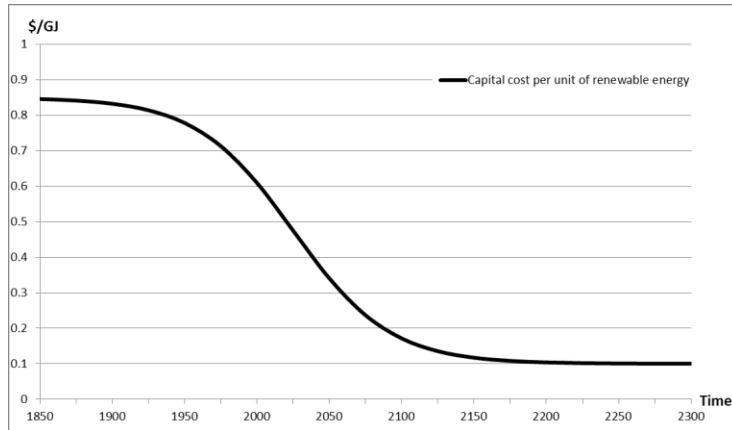
To be accurate, the unitary capital requirement per renewable energy unit produced  $B_t$  should be represented by a decreasing function since over time less capital is necessary to capture the same amount of RE thanks to technological progress  $A_t$ . This function starts to a point  $\bar{b}$  and decrease at constant speed  $\tau > 1$  to a strictly positive bound  $\underline{b}$  since the production of any RE flow would always require a minimum quantity of capital (Figure 2).

$$B_t(A_t) = \underline{b} + (\bar{b} - \underline{b}) \left( 1 - \frac{1}{1 + \exp(-\tau(A_t - A_{inflexion}))} \right), \quad \forall t \in \{0, \dots, T\} \quad (23)$$

The fact that  $0 < \gamma < 1$  means that returns to scale are decreasing and that consequently the capital intensiveness of the RE firm is increasing in the production level<sup>7</sup>. The evolution of this decreasing sigmoid function depends on the general technological progress level of the economy  $A_t$ , and on the particular technological progress level  $A_{inflexion}$  at which the function  $B_t$  presents an inflexion point. In other words  $A_{inflexion}$  is the technological progress level of the economy at which the rate of degrowth of  $B_t$  is maximum. We define  $A_{inflexion}$  as follow:

$$A_{inflexion} = \eta \bar{A}, \text{ with } 0 < \eta < 1 \quad (24)$$

Where  $0 < \eta < 1$  is a constant parameter.



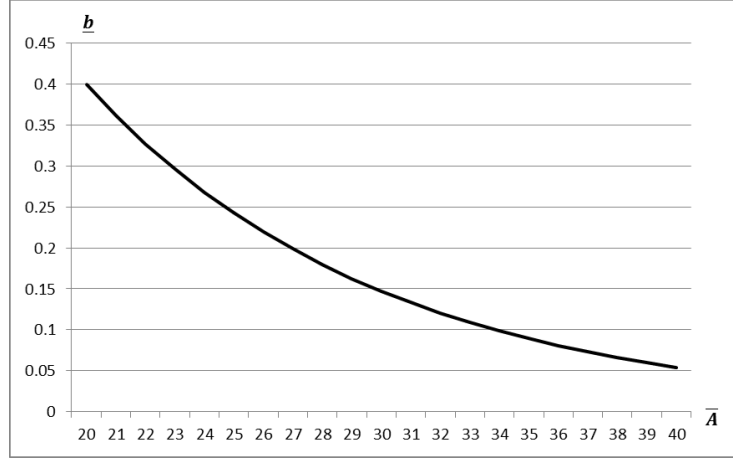
**Figure 2.** Example of a possible form for the capital requirement per unit of renewable energy produced between 1850 and 2300.

Furthermore, since  $\underline{b}$  is the final unitary cost of RE production, it necessarily depends on the final technological progress level of the whole economy  $\bar{A}$ . The idea behind this relation is that the higher the final technological progress level  $\bar{A}$ , the lower the final unitary cost of RE production  $\underline{b}$ .

$$\underline{b} = \underline{\underline{b}} * \exp\left(-\xi(\bar{A} - \bar{\bar{A}})\right) \quad (25)$$

<sup>7</sup> The assumption of declining returns to scale in the RE sector is supported by Dale et al. (2011) where it is argued that it represents the likelihood of the most optimal sites being used earlier. For example, deployment of wind turbines presently occurs only in sites where the average wind speed is above some lower threshold and that are close to large demand centers to avoid the construction of large distribution networks. Over time, the availability of such optimal sites will decrease, pushing deployment into sites offering lower energy returns. Dale et al. (2011) used two databases of the National Renewable Energy Laboratory (NREL, 2010a, 2010b) to demonstrate that plotting the potential of wind and solar resources in the USA as a function of the sites frequency present clear declining trends. In the same view and in two separate studies, Honnery and Moriarty (2009) and Hoogwijk et al. (2004) have shown that as wind energy production increases, the marginal capacity factor of wind turbines decreases.

Where  $0 < \xi$  is a constant parameter,  $\bar{A}$  is the minimum level that the final technological progress  $\bar{A}$  can have (i.e. the final technological progress level  $\bar{A}$  is at least equal to  $\bar{A}$ ) and  $\underline{b}$  is the final unitary cost of RE production when  $\bar{A}$  equals  $\bar{A}$ .



**Figure 3.** Relation between the final unitary cost of RE production  $\underline{b}$  and the final technological progress level  $\bar{A}$  with  $\underline{b} = 0.4$ ,  $\bar{A} = 20$ , and  $\xi = 0.1$ .

### 2.3 Markets equilibrium conditions

In order to close the model we need to define three equilibrium conditions. First, the economic output produced by the final good sector is allocated either to consumption or to investment.

$$Y_t = C_t + \frac{K_{t+1}}{\rho}, \quad \forall t \in \{0, \dots, T\} \quad (26)$$

On the energy market, the supplies of NRE and RE comply with the demand generated by the final good sector.

$$E_t = \begin{cases} X_t + F_t, & \forall t \in \{0, \dots, T_e\} \\ F_t, & \forall t \in \{T_e + 1, \dots, T\} \end{cases} \quad (27)$$

The total capital stock of the economy is either allocated to the NRE, RE or to the final good sector.

$$K_t = \begin{cases} Z_t + G_t + H_t, & \forall t \in \{0, \dots, T_e\} \\ G_t + H_t, & \forall t \in \{T_e + 1, \dots, T\} \end{cases} \quad (28)$$

## 2.4 EROI of NRE, RE and entire energy sector

In order to define the different EROI of our energy sectors, we need to decompose the saving rate  $s_t$  in three parts  $s_t^z$ ,  $s_t^g$  and  $s_t^h$ .

$$s_t = s_t^z + s_t^g + s_t^h, \text{ with } s_t^z = \frac{Z_{t+1}}{\rho Y_t}, \quad s_t^g = \frac{G_{t+1}}{\rho Y_t}, \quad \text{and } s_t^h = \frac{H_{t+1}}{\rho Y_t} \quad (29)$$

$s_t^h$  (respectively  $s_t^z$ ,  $s_t^g$ ) is the fraction of period  $t$  output invested in the final good sector (respectively NRE, RE sector) in period  $t + 1$ . According to Hall et al. (2014), the Energy Return On (Energy) Investment (EROI) is “the ratio between the energy delivered by a particular fuel to society and the energy invested in the capture and delivery of this energy”. This invested energy takes usually two forms: direct (actual energy carrier like electricity or liquid fuel) and indirect (embodied in capital) input energy. Since our energy productions are considered net of any direct energy consumption, the energy investment (denominator of the EROI) will only consist in the energy embodied in the capital produced by the final sector and later used in the energy sector. Hence, the NRE production  $X_t$  requires the capital stock level  $Z_t$  that comes from the investment  $Z_t/\rho$  in period  $t - 1$ . This investment corresponds to a fraction  $Z_t/K_t$  of the total investment  $s_{t-1}Y_{t-1} = \frac{K_t}{\rho Y_{t-1}}Y_{t-1}$ , hence the quantity of economic output from the final sector that is invested (as capital) in the NRE sector to fulfill its production in period  $t$  is  $\frac{Z_t}{K_t} \frac{K_t}{\rho Y_{t-1}} Y_{t-1} = s_{t-1}^z Y_{t-1}$ . Since the production of  $Y_t$  has required the consumption of energy  $E_t$ , it follows that the EROI,  $\varepsilon_t^{NRE}$ , of the NRE production in period  $t$  is:

$$\varepsilon_t^{NRE} = \frac{X_t}{s_{t-1}^z E_{t-1}} \quad (30)$$

Similarly, the EROI,  $\varepsilon_t^{RE}$ , of the RE production in period  $t$  is:

$$\varepsilon_t^{RE} = \frac{F_t}{s_{t-1}^g E_{t-1}} \quad (31)$$

At the macroeconomic level, it is possible to define the EROI of the whole economy,  $\varepsilon_t$ , since producing the total energy  $E_t = X_t + F_t$  has indirectly required the embodied energy  $s_{t-1}^z E_{t-1} + s_{t-1}^g E_{t-1}$ . Thus globally, the EROI of the entire energy sector in period  $t$  is equal to:

$$\varepsilon_t = \frac{E_t}{(s_{t-1}^z + s_{t-1}^g) E_{t-1}} \quad (32)$$

Due to its highly nonlinear formulation, studying the analytical solution of the model that we have designed would prove to be rather difficult if not impossible. Thus, it is preferable to study the dynamics of the model through simulation.

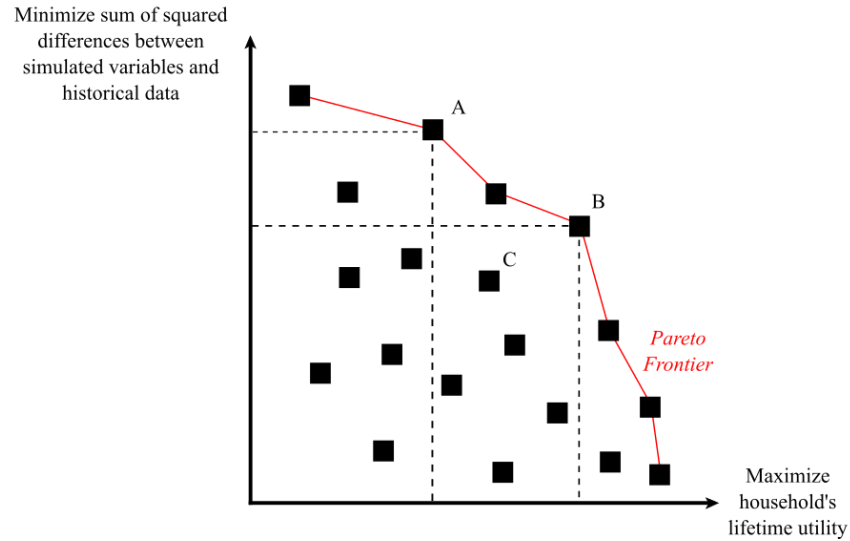
### 3. Simulation approach to study the dynamics

The purpose of our simulation approach is to see if our model is able to reproduce what has been happening at the global scale in terms of energy and Gross World Production (GWP) since the industrial revolution and simulate what might be ahead of us in a close future. Thus, the model shall be able to represent an economy relying increasingly on fossil fuels (coal, oil, gas and uranium) from an early renewable era (based on biomass) and the subsequent transition towards complete renewable energy (based on solar, wind and biomass) that is inevitable because of the finiteness of fossil fuels. In the same time, our model shall reproduce the global world GWP pattern. The simulation of the model has been operated with the Vensim software, which is very convenient for simulating highly nonlinear dynamic systems, but is not equipped to handle optimization of objective functions under constraints. Nevertheless, we follow a metaheuristic procedure describe below to estimate and then simulate the dynamic problem using the Vensim software.

#### 3.1 From the theoretical model to the practical simulation on real data

From a theoretical point of view our model is a dynamic system of equations with only one intertemporal maximization under constraint (i.e. household's lifetime utility maximization under budget constraint). But from a practical point of view, another objective appears from the fact that we want our model to represent as best as possible what has been happening in the past. So we want our simulated variables for global NRE production, RE production and GWP to fit as much as possible with historical data. In other words, we have to calibrate the model's parameters according to a criterion that is the minimization of the sum of squared errors between the simulated and the historical values (for the three variables NRE and RE production and GWP). As a consequence, the simulation is a multi-purpose constrained optimization were possible solutions are defined by a set of two vectors: a first one embedding all the capital cost value across time (as the result of the lifetime utility maximization), a second one containing the values of the constant parameters of the model (as the result of the minimization of the sum of the squared differences between our simulated variables and historical data). In such case, we have to use a metaheuristic in order to define a solution among the set of strictly non-dominated solutions that represent the *Pareto Frontier* of our problem. For readers that are not familiar with multi-purpose optimization under constraints, Figure 4 should help understanding that there is no unique global optimum to our problem but rather a set of strictly non-dominated solutions among which it is impossible to decide if one solution is better than another.

Since we are looking for a unique solution per scenario to make a straightforward cross-comparison between scenarios, we have to use a metaheuristic to help us define a final solution (among the set of solution of the Pareto Frontier) for each scenario.



**Figure 4. Example of a Pareto Frontier in our practical simulation problem.** The boxed points represent feasible solutions, and larger values are preferred to smaller ones. Solution C is not on the Pareto Frontier because it is dominated by both solution A and solution B. Solutions A and B are not strictly dominated by any other, and hence do lie on the frontier.

### 3.2 Calibration procedure through metaheuristic

The manual metaheuristic used to define the optimal solution of a given scenario can be explained as follows:

1. An initializing iteration of the model is operated with a constant capital cost.
2. The constant parameters (of the model) of this first run are calibrated in order to minimize the sum of the squared differences between simulated variables  $Y_t$  (GWP),  $X_t$  (NRE production),  $F_t$  (RE production), and historical data of these same variables.
3. A set of non-constant capital costs values that satisfy the smoothing behavior of equation (4) is calculated.
4. A new iteration is performed using the time dependent set of capital costs calculated at step 3 as an input (instead of a constant capital cost).
5. Constant parameters of the model are recalibrated to optimize the fit between simulated variables  $Y_t$  (GWP),  $X_t$  (NRE production),  $F_t$  (RE production), and historical data of these same variables.
6. Another set of time dependent capital costs values that satisfy the smoothing behavior of equation (4) is calculated for the current iteration.
7. Step 4 is repeated until the stopping criterion is reached.

The stopping criterion is defined as the sum of the absolute value of the differences between the capital cost of the current iteration and the capital cost of the preceding iteration. Once this criterion reached  $1.0E^{-1}$ , the manual metaheuristic is stopped and the last iteration of the model is considered to be the final run representing the scenario under study.



### 3.3 Historical data, fossil Ultimately Recoverable Resource (URR) and common parameters presentation

#### *Historical data*

Let us first presents the historical data that are used for the calibration of the model in all scenarios. Three time series were used to calibrate the different scenarios on historical data: historical nonrenewable energy production, historical renewable energy production and historical GWP. As explained at the beginning of section 2, the time period length  $t$  of the model corresponds to the average capital lifetime, which is set to 15 years. Thus, by starting our simulations with year 1850 as the initial time period ( $t=0$ ), our historical data time series consist in 10 discrete points up to year 2000 ( $t=10$ ). To suit our single nonrenewable resource model, the different historical data for global primary fossil fuels production of coal, oil, gas and nuclear have been aggregated in a single NRE production expressed in exajoule (EJ). In the same way, historical global primary production of biomass (including noncommercial wood) was aggregated with historical global renewable energy production from hydro, solar and wind in a single RE production expressed in exajoule (EJ). For historical fossil fuels productions, data from Etemad and Luciani (1991) was used for coal, oil, gas and nuclear production up to 1985, and completed with the BP statistical review (2014) for the last data point in 2000. For biomass, data was retrieved from Grüebler et al. (1996) and completed with Smil (2010). The BP statistical review (2014) was used for historical production of hydro, wind and solar energy. Regarding the historical GWP (expressed in billions of 1990 International Geary-Khamis dollar<sup>8</sup>), we used data from The Maddison Project (2013). All historical data values used in our simulations are summarized in Table 1 below.

**Table 1. Historical data used for the calibration of global GWP, NRE production and RE production.**

Time period	NRE production (EJ/Year)	RE production (EJ/Year)	Gross World Production (Billion 1990 Int. GK\$, B\$)
0 (1850)	2.10	17.12	649.10
1 (1865)	4.59	18.10	1049.25
2 (1880)	9.18	18.98	1438.52
3 (1895)	17.00	19.54	1841.73
4 (1910)	32.73	20.72	2441.56
5 (1925)	43.84	22.34	3429.24
6 (1940)	51.78	24.26	4945.08
7 (1955)	89.44	25.91	7027.80
8 (1970)	198.62	30.04	13805.29
9 (1985)	289.46	44.45	22845.98
10 (2000)	379.60	57.19	38170.35

#### *Determination of the fossil energy URR*

As explain before, some values of the constant parameters of the model differ from one scenario to another. Among them, one important parameter of the model requires a specific attention: the Ultimately Recoverable Resource (URR) represented by parameter  $\mathcal{S}$  in the model. This parameter represents the total amount of recoverable NRE resource<sup>9</sup>. To

<sup>8</sup> The 1990 International Geary–Khamis dollar, more commonly known as the international dollar, is a standardized and fictive unit of currency that has the same purchasing power parity that the U.S. dollar had in the United States in 1990.

<sup>9</sup> According to BP (2015): “URR is an estimate of the total amount of a given resource that will ever be recovered and produced. It is a subjective estimate in the face of only partial information. Whilst some consider URR to be fixed by geology and the laws of physics, in practice estimates of URR continue to

obtain the value of this parameter it is needed to aggregate the values found in the literature for the URR of the different kind of nonrenewable energy forms we use, namely coal, conventional and unconventional oil, conventional and unconventional gas, and uranium. These values, presented in Table 2, were retrieved from the recent work of McGlade and Ekins (2015) for oil (Gb: Giga barrel), gas (Tcm: terra cubic meters), coal (Gt: Giga tons), and from Dale (2012) for uranium (EJ: Exajoule). After conversion and aggregation, the total nonrenewable URR value retained for our simulations is 175 000 EJ.

**Table 2. Data used for the calculation of the global NRE URR. Sources: McGlade and Ekins, 2015; Dale, 2012.**

Energy resource	URR (diverse units)	Conversion factors (diverse units)	URR (EJ)
Conventional oil	2615 (Gb)	6.1E-9 EJ/barrel	16 000
Unconventional oil	2455 (Gb)	6.1E-9 EJ/barrel	15 000
Conventional gas	375 (Tcm)	4.0E-11 EJ/cm	15 000
Unconventional gas	300 (Tcm)	4.0E-11 EJ/cm	12 000
Hard coal	2565 (Gt)	32.5E-9 EJ/ton	85 000
Lignite coal	1520 (Gt)	14.0E-9 EJ/ton	22 000
Uranium	-	-	10 000
<b>Total NRE URR (EJ)</b>			<b>175 000</b>

Off course, other values could be found in the literature for the URR of these different resources due to the great sensitivity of this parameter to geologic assumptions<sup>10</sup>. It is not our purpose to discuss the discrepancies between URR values found in the literature and their underlying calculation assumptions.

#### Calibration of parameters that are common to all scenarios

When simulating the model, we have found that it can exhibit four kinds of scenarios that we have entitled *Plateau*, *Smooth*, *Recovery* and *Degrowth*. More precisely we have found that it is possible to obtain a representative example of each distinct scenario type by only changing four parameters values and keeping all the sixteen other parameters, and the three initial capital stocks constant between the four scenario examples. The following of this paragraph will present a representative set of these common parameters. In our model the length of a time period was set to be 15 years since we have assumed a unitary depreciation rate for capital. Thus, in order to have an annual discount rate of approximately 2%, we have chosen a discount factor value of  $\beta = 0.75$  ( $((0.75)^{-1/15}) - 1 = 1.9\%$ ). The output elasticity to energy input in the final sector was set to  $\alpha = 0.6$ . With a productivity of the transformation of investments goods into productive capital assumed to be  $\rho = 3$ , the constant capital cost for the initializing iteration of the simulations was set to  $v_{initializing} = 0.343$  in order to represent a real interest rate for the economy  $\mu$  equals to 3% ( $v_{initializing} =$

---

be increased as knowledge grows, technology advances and economics change. The ultimately recoverable resource is typically broken down into three main categories: cumulative production, discovered reserves and undiscovered resource<sup>7</sup>.

<sup>10</sup> Contrary to the idea advanced by BP and presented in the footnote 7 that URR are re-estimated due to technological progress, Sorrell et al., (2010) highlight that unlike reserves, URR estimates are not dependent on technology assumptions and thus should only be determined by geologic hypotheses. Unfortunately, this apparent contradiction on the URR definition is only a tiny example of the fuzziness of point of views that one could find in the literature regarding the different notions of resources and reserves.

$\frac{1+\mu}{\rho} = \frac{1+0.03}{3} = 0.343$  ). Other parameters values remaining constant among the three scenarios were found by calibration to historical data. For the sake of clarity, there are all presented in Table 3.

**Table 3. Example of a set of parameters values remaining constant between the three scenarios.**

Parameter	Definition (unit)	Value
$\rho$	Transformation productivity of investment goods (dmnl)	3.0
$\beta$	Discount factor (dmnl)	0.75
$\sigma$	Arrow-Pratt Constant Relative Risk Aversion coefficient (dmnl)	0.5
$URR$	Ultimately Recoverable Resource (EJ)	175 000
$d_1$	First part of the initial unitary NRE extraction cost (B\$/EJ)	0.24
$\delta$	Rate of quality degradation of the NRE resource (dmnl)	20
$\varphi$	Power exponent of the ratio of exploited resource $\phi_t$	4
$\vartheta$	Power exponent of the normalized technological progress used in equation (8)	4
$\bar{b}$	Initial unitary RE production cost (B\$/EJ)	0.85
$\underline{b}$	Final unitary cost of RE production when $\bar{A}$ equals $\bar{\bar{A}}$ (B\$/EJ)	0.4
$\xi$	Degrowth rate of $\underline{b}$ from $\bar{b}$ (dmnl)	0.1
$\tau$	Growth rate of $\bar{b}$ towards $\underline{b}$ (dmnl)	0.5
$\theta$	Returns to scale in the NRE sector (dmnl)	0.5
$\gamma$	Returns to scale in the RE sector (dmnl)	0.5
$\underline{A}$	Initial technological progress level (dmnl)	10
$\bar{\bar{A}}$	Minimum level that the final technological progress $\bar{A}$ can have (dmnl)	20
$\alpha$	Output elasticity of energy input in final production (dmnl)	0.6
$Z_0$	Initial capital in the NRE sector (B\$)	10
$G_0$	Initial capital in the RE sector (B\$)	200
$H_0$	Initial capital in the final sector (B\$)	350
$v_{initializing}$	Constant capital cost for initializing iteration (dmnl)	0.343
$T$	Time horizon of the model (dmnl)	30

We are now ready to present the four different kinds of scenarios that we have identified when performing the simulations of our model. The scenarios are mainly represented by a change in the final technological progress  $\bar{A}$ , the remaining distinctive parameters values of each scenario are then determined through the calibration procedure.

## 4. Results of simulations and scenarios analyses

### 4.1 Detailed analysis of the baseline run a.k.a *Plateau* scenario ( $\bar{A} = 30$ )

The baseline run that we have called *Plateau* scenario is obtained thanks to a final technological progress value  $\bar{A} = 30$  that also implies other specific parameters values presented in the Table 4. In the *Plateau* scenario the GWP pursue its steady growth during 90 years but encounters an abrupt plateau at the end of the XXIth century. Figure 5 which shows the main output results of the *Plateau* scenario helps understand that the emergence of the plateau in GWP correspond to the beginning of the declining in NRE production (fossil energy peak occurs around 2060).

**Table 4. Specific parameters values of the *Plateau* scenario.**

Parameter	Definition (unit)	Value
$\bar{A}$	Final technological progress level (dmnl)	30
$t_{A_t \max}$	Time of maximum technological progress growth rate (time period)	8.5
$d_2$	Second part of the initial unitary NRE extraction cost (B\$/EJ)	1.0
$\eta$	Share of final technological progress level of the economy that helps define the technological progress level $A_{inflexion}$ at which the rate of degrowth of $B_t$ is maximum (dmnl)	0.85

After a 30 years (2090-2120) period of stagnation, the GWP increases again (yet with a lower growth rate compared to the past century) when renewable energy production becomes sufficient to compensate the fossil energy continuous decline. However, even if GWP increases again after the plateau phase, its growth rate starts to decrease slowly from 2150 up to the end of the simulation where finally a value of 235,000 B\$ (compared to 38,170 B\$ in 2010) is reached. By extending further the simulation time horizon (not shown here), we saw that up to 2300 the GWP remains broadly constant and the whole system is almost in a steady state where renewable energy is used at an annual level of approximately 1300 EJ/year (compared to 60 EJ/year in 2010). Nonrenewable energy production reaches a peak of 665 EJ/year in 2060 (compared to 390 EJ/year in 2010) and then constantly decreases up to a level of 35 EJ/year in 2300. At the end of the simulation, the saving rate of the economy is stabilized to a value of 0.275, while a maximum of 0.38 is attained in 1985. Technological progress reaches its maximum at the very end of the simulation in 2300. We can see that the EROI of the NRE sector is maximal at the second period of extraction (1865) with a value of 145 and then sharply decline to 8 in the third period (1880). After that, the EROI of the NRE sector fluctuates around 15 during 150 years and starts to decline again in 2030 when the rate of growth of the NRE production starts to decline (i.e. 30 years before the fossil energy peak). The EROI of the NRE sector steadily decline during 70 years and stabilized in 2100 to a value of approximately 5.5. Around 2130 when the rate of degrowth of the NRE production decreases, the EROI logically raises slightly to a value of 7. On the RE side, we can see that the EROI of the RE sector constantly increases up to 2060, time of the fossil energy peak and consequent need for increasing RE capital investment. If we remember that the RE sector is subject to declining returns to scale and that between 2060 and 2180 the rate of growth of the RE production is maximal, it is logical to see that the EROI of the RE sector decreases a lot during this same period and less once the RE production starts to slow down. All of this also translates in the net energy supply to society as shown in Figure 2. From 1850 to 2000, total and net energy supplies are virtually identical, but then the net energy supply to society starts to be lower than the total energy production. This can be easily explained because when fossil production approaches its peak, the energy requirement to extract energy increases through increasing capital requirements. In the same way, as renewable production progresses, increasing amount of energy must be dedicated just to produce the capital required to harness the renewable energy flow (solar flow). Hence, around 2060 the difference between gross and net flows to society of renewable energy starts to be significant. For the record, these results are in accordance with the GEMBA model of Dale et al. (2012).

## 4.2 Comparison of the baseline *Plateau* scenario ( $\bar{A} = 30$ ) with the *Smooth* scenario ( $\bar{A} = 32$ ), *Recovery* scenario ( $\bar{A} = 28$ ) and *Degrowth* scenario ( $\bar{A} = 26$ )

The *Smooth*, *Recovery* and *Degrowth* scenarios are obtained thanks to the respective final technological progress values of 32, 28 and 26 that also imply to adjust parameters  $t_{lag}$ ,  $d_2$ , and  $\eta$  as presented in the Table 5, 6 and 7 below.

**Table 5. Specific parameters values of the *Smooth* scenario.**

Parameter	Definition (unit)	Value
$\bar{A}$	Final technological progress level (dmnl)	32
$t_{A_t max}$	Time of maximum technological progress growth rate (time period)	8.8
$d_2$	Second part of the initial unitary NRE extraction cost (B\$/EJ)	0.9
$\eta$	Share of final technological progress level of the economy that helps define the technological progress level $A_{inflexion}$ at which the rate of degrowth of $B_t$ is maximum (dmnl)	0.83

**Table 6. Specific parameters values of the *Recovery* scenario.**

Parameter	Definition (unit)	Value
$\bar{A}$	Final technological progress level (dmnl)	28
$t_{A_t max}$	Time of maximum technological progress growth rate (time period)	8.3
$d_2$	Second part of the initial unitary NRE extraction cost (B\$/EJ)	1.1
$\eta$	Share of final technological progress level of the economy that helps define the technological progress level $A_{inflexion}$ at which the rate of degrowth of $B_t$ is maximum (dmnl)	0.86

**Table 7. Specific parameters values of the *Degrowth* scenario.**

Parameter	Definition (unit)	Value
$\bar{A}$	Final technological progress level (dmnl)	26
$t_{A_t max}$	Time of maximum technological progress growth rate (time period)	7.8
$d_2$	Second part of the initial unitary NRE extraction cost (B\$/EJ)	1.4
$\eta$	Share of final technological progress level of the economy that helps define the technological progress level $A_{inflexion}$ at which the rate of degrowth of $B_t$ is maximum (dmnl)	0.88

As can be seen in Figure 6, (where only the most interesting variables of the four scenarios are presented For the sake of clarity) in the *Smooth* scenario it is possible to avoid the plateau phase of the GWP that occurs in the *Plateau* scenario when fossil energy peaks because in the *Smooth* scenario renewable energy production is already sufficiently important thanks to a larger final technological progress. Indeed, having a larger final technological progress level  $A_*$  implies to have a lower final unitary cost of renewable energy production as stipulated by equation (24), shown in Figure 3 below and also visible in Figure 4. Having a final unitary cost of RE production of 0.12 \$/GJ in the *Smooth* scenario allows a larger RE production of 2450 EJ/year at the end of the simulation which explains that a very larger global GWP of 480,000 B\$ is reached (235,000 B\$ in the baseline *Plateau* scenario).

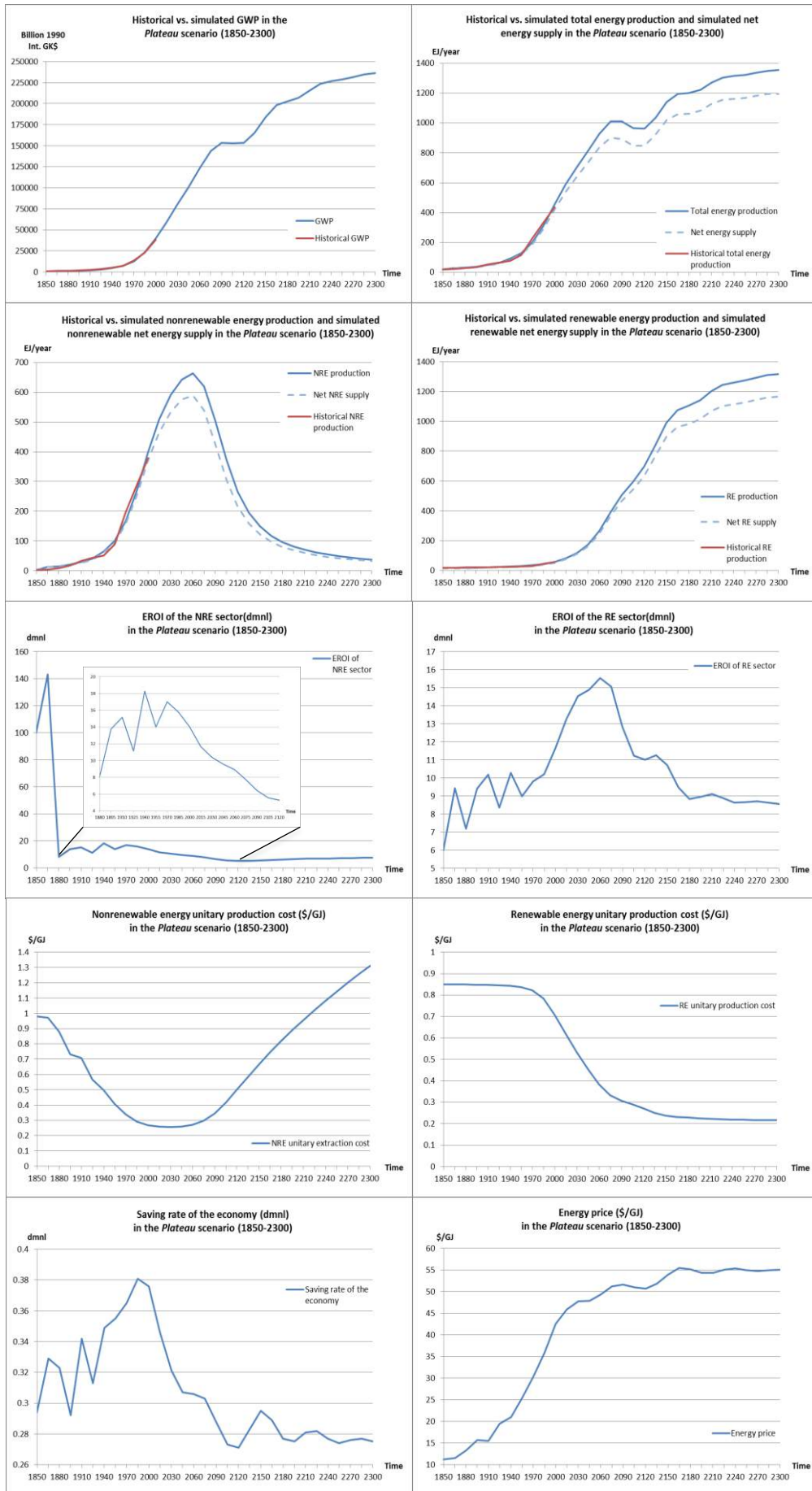


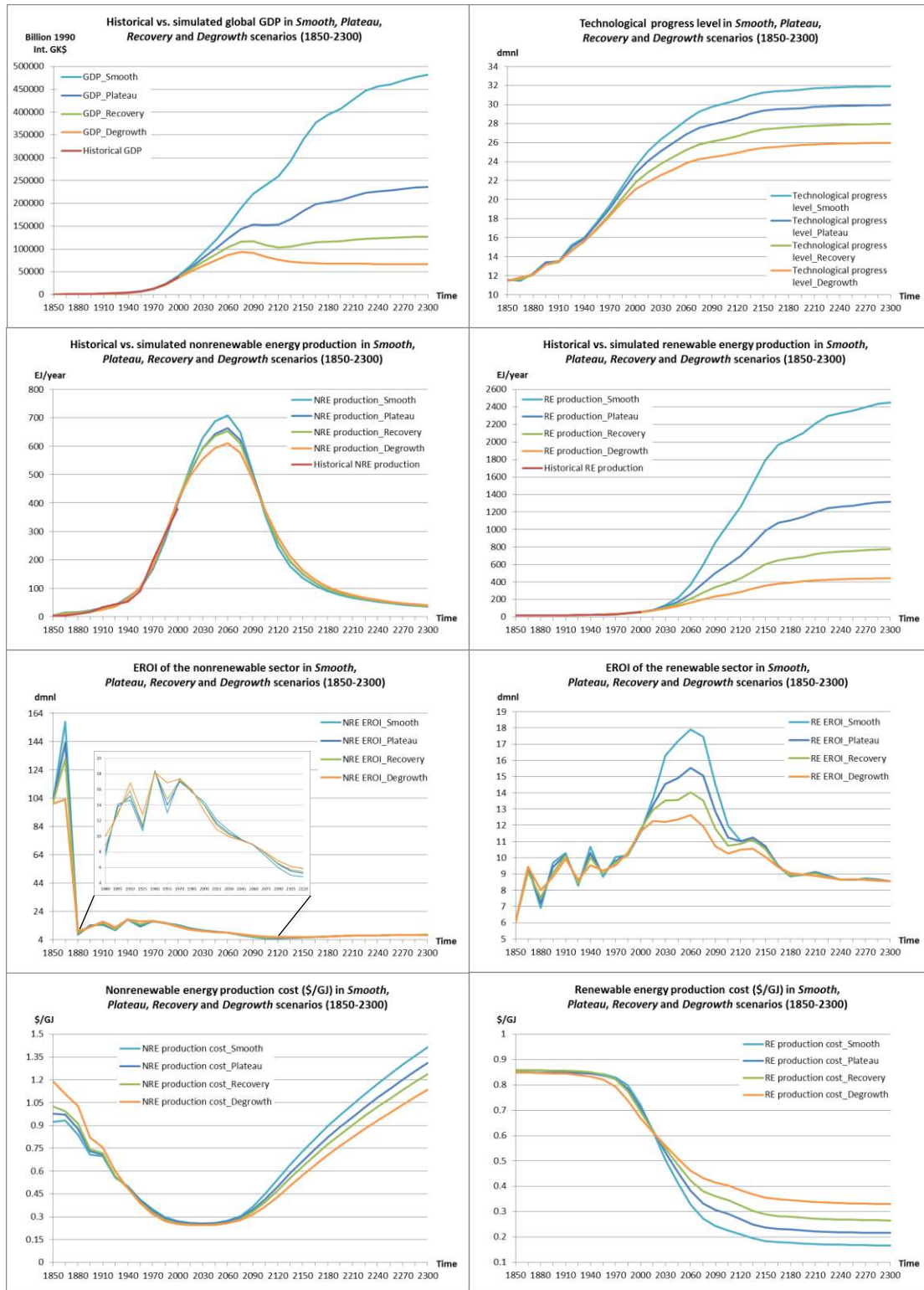
Figure 5. Main output results of the *Plateau* scenario (and comparison to historical data when available).

On the other hand, simulation results presented in Figure 6 show that if the final technological progress level is lower, GWP peaks and then decrease just after the fossil energy peak because the renewable energy production is not sufficient to sustain past rate of economic growth. In the case of the *Recovery* scenario, since the final technological progress level is only lowered to 28, the GWP decreases for a while but then recovers and stabilized to a level of 125,000 B\$ approximately similar to its level when fossil energy peak occurred. In the *Degrowth* scenario however, since the final technological progress level is even lower ( $\bar{A} = 26$ ), GWP peaks at a lower level because fossil energy peak (610 EJ/year) is also lower than in other scenarios. After its peak, GWP starts a slow decrease to eventually stabilize and finish in a steady state of 66,000 B\$ which is slightly higher than current level (40,000 B\$). Furthermore, in the *Degrowth* scenario the final renewable energy production is only about 450 EJ/year because of a higher final unitary production cost of 0.22 \$/GJ. In such scenario, the GWP overshoot comes from the fact that the maximum level of renewable energy production (450 EJ/year) is lower than fossil energy peak (610 EJ/year). Thus, compare to other scenarios, in the *Degrowth* scenario the economy remains even more accustomed and dependent to fossil energy which prevents a correct anticipation of the necessary transition towards renewable energy.

Our model clearly shows that in order to have a smooth transition between fossil and renewable energy that does not negatively impact the GWP (either through a plateau or a straight decrease), the final unitary production cost of RE production must be sufficiently low (below 0.2 \$/GJ in practice in our model). This requires having a sufficiently high final technological progress level  $A_t$  for the whole economy (superior to 30 in practice in our model). In the following section we discuss different strategies that can help smoothing the GWP dynamic in scenarios that initially present the settings of the *Degrowth* scenario.

## 5. Discussion on the implementation of a carbon tax and the way to use its revenue

The intuition we want to test is to consider that the final technological progress level  $\bar{A}$  of an economy, which is the most important parameter determining the dynamics of our model, is not something that can be changed endogenously by a given policy action. This asymptotic value  $\bar{A}$  cannot be known a priori but only a posteriori once it is reached. Even if as seen in the previous section 4, this parameter is primordial for determining the ultimate state of the economic system, there must be ways to change the trajectory that leads to this deterministic end, especially if this path is considered to generate welfare losses. In other words, the policy actions that must be investigated are the ones that help avoiding as much as possible the lock-in phenomenon described previously that is characteristic of both the real world and our model: the tendency of the economic system to stay accustomed to fossil fuels without anticipating its inevitable supply peak and decline. Starting from a *Degrowth* scenario setting, the strategy we propose to avoid its adverse outcome (GWP peak followed by a straight degrowth phase) is to implement a tax on the nonrenewable energy production and to use the income revenue of this tax to direct the energy transition dynamics and smooth its negative impact on GWP. Such a tax could be indexed on the polluting potential of the fossil energy and more precisely on its greenhouse gases (GHG) content. Even if our model does not include a climate module able to formalize the impacts of the GHG emissions resulting from the fossil fuels use, it makes no doubt that such impacts exist in reality.



**Figure 6.** Comparison of the main variables of the *Smooth*, *Plateau*, *Recovery* and *Degrowth* scenarios (and historical data when possible).

Hence in our model, the tax that we are going to exogenously implement on the NRE production could perfectly be seen as a *carbon tax*. It is important to see that the income from the annual carbon tax can be used in three different ways that can be combined in various proportions to generate many different policy mixes. The annual income from the carbon tax could be used to: (1) subsidize the general R&D sector of the economy; (2) subsidize the



R&D specific to the RE sector; (3) subsidize direct capital investment in the RE sector. Actually, there is a fourth way of using the income from the carbon tax, it is to compensate the consumer for its loss due to higher fossil energy prices. This option will not be incorporated in the following of the discussion because preliminary simulations that are not presented here showed that it was simply the worse way of using the income carbon tax. Indeed, all other options necessarily imply some positive spillovers on the potential of renewable energy production which subsequently affect positively the GWP dynamics. Providing a free compensation to the consumer cannot generate such positive dynamics in our model, hence this option cannot be considered as an interesting policy choice in our setting.

In the following of this section we will first present the different equation changes resulting from the implementation of the carbon tax. Then, the specific mathematical formalization of the use of the carbon tax income will be successively presented. Finally, we will propose four policy mixes scenarios and compare simultaneously the results of their simulations.

### 5.1 Common equation changes in the model due to the carbon tax implementation

Let us define  $q_t$  as the unitary carbon tax at period  $t$  (i.e. the carbon tax per unit of fossil energy pollution content, hence expressed in  $\$/\text{tCO}_2\text{eq}$ , or  $\text{B}\$/\text{GtCO}_2\text{eq}$  in order to be consistent with the previous sections). It is nil prior to the time period  $t_{q_t \text{ start}}$  at which the carbon tax is implemented and will evolve towards the maximum unitary carbon tax value  $\bar{q}$  at exogenous speed  $\lambda$  following a sigmoid increasing form (Figure 7). The maximum growth rate of the unitary carbon tax occurs when  $t = t_{q_t \text{ start}} + t_{q_t \text{ lag}}$ .

$$q_t = \frac{\bar{q}}{1 + \exp\left(-\lambda(t - t_{q_t \text{ start}} - t_{q_t \text{ lag}})\right)}, \quad \forall t \geq t_{q_t \text{ start}} \quad (33)$$

Since the NRE producer has to pay the tax  $q_t$  for every unit of pollution ( $\text{B}\$/\text{GtCO}_2\text{eq}$ ), he has to pay the amount  $q_t \kappa_t$  per unit of fossil energy produced ( $\text{B}\$/\text{EJ}$ ), with  $\kappa_t$  representing the GHG emission factor of fossil energy (expressed in  $\text{GtCO}_2\text{eq}/\text{EJ}$ ). Hence, we deduce that the carbon tax income  $Q_t$  of the period  $t$  is defined by:

$$Q_t = X_t * q_t \kappa_t, \quad \forall t \geq t_{q_t \text{ start}} \quad (34)$$

Implementing the carbon tax also logically change the equations relative to the NRE producer behavior. More precisely, the implementation of the carbon tax implies to respectively replace equations (5), (8) and (9) by the following (35), (36) and (37).

$$\max_{X_t, Z_t} \Pi_t = (p_t - q_t \kappa_t) X_t - v_t Z_t, \quad \forall t \in \{0, \dots, T_e\} \quad (35)$$

$$X_t = \left[ \frac{(p_t - q_t \kappa_t) \theta}{D_t^{\frac{1}{\theta}} v_t} \right]^{\frac{\theta}{1-\theta}}, \quad \forall t \in \{0, \dots, T_e\} \quad (36)$$

$$Z_t = \left[ \frac{(p_t - q_t \kappa_t) \theta}{D_t v_t} \right]^{\frac{1}{1-\theta}}, \quad \forall t \in \{0, \dots, T_e\} \quad (37)$$

In addition to the equation changes previously presented that concern the NRE producer, some equation changes will also be specific to each way of using the annual carbon tax income  $Q_{t-1}$  of the previous period  $t-1$ .

## 5.2 Specific equation changes in the model due to the particular use of the carbon tax income

Let us note here that since we have three ways of using the annual income carbon tax, each option will use a share  $\omega_i$ ,  $i \in [1,2,3]$  of the total annual carbon tax income  $Q_{t-1}$ , with  $\omega_1 + \omega_2 + \omega_3 = 1$ .

### Option (1): carbon tax income used to subsidize the general R&D sector

A first option to use the carbon tax income is to allocate it to the general R&D sector in order to increase the growth rate of the technological progress level  $A_t$ . Doing so should also increase the rate of degrowth of the capital cost of renewable energy  $B_t$  through equation (23). However, since  $A_t$  is also present in other equations, such as (14), (18), and (21), increasing the growth rate of  $A_t$  will also have other impacts that we will analyze through simulation. Hence, in order to mathematically formalize the use of the annual income from the carbon tax to subsidize the general R&D sector, we have to replace equation (20) defining the law of motion of the technological progress by the following equation (38).

$$A_t = \underline{A} + \frac{\bar{A} - \underline{A}}{1 + \exp(-(s_{t-1} + \omega_1 \varepsilon_1 Q_{t-1}) * (t - t_{A_t \max}))}, \quad \forall t \in \{0, \dots, T\} \quad (38)$$

Where parameter  $\omega_1$  is the share of the carbon tax income of the previous period that is recycled as a subsidy to the general R&D sector. The constant  $\varepsilon_1$  represents the efficiency of the transformation of the carbon tax income into technological progress gains through general R&D. In other words it measures the efficiency of the general R&D sector to use the carbon tax income to produce innovations that materialized in the form of increases in the speed of convergence of  $A_t$  towards  $\bar{A}$ . The functional form given in (38) insures that the higher the carbon tax income of period  $t-1$  and the higher the share  $\omega_1$  of this income dedicated to the general R&D sector, the faster the technological progress will converge towards its asymptotic limit  $\bar{A}$ .

### Option (2): carbon tax income used to subsidize the specific R&D of the RE sector

A second way to use the income from the carbon tax is to allocate it to the R&D that is specifically dedicated to the renewable energy sector. Doing so will affect the rate of degrowth of the unitary capital cost of RE production  $B_t$  towards its asymptotic limit  $\underline{b}$ . An appropriate way to formalize this is to replace (23) by the following (39).

$$B_t(A_t) = \underline{b} + (\bar{b} - \underline{b}) \left( 1 - \frac{1}{1 + \exp(-(\tau + \omega_2 \varepsilon_2 Q_{t-1}) * (A_t - A_{inflection}))} \right), \quad \forall t \in \{0, \dots, T\} \quad (39)$$

Where parameter  $\omega_2$  is the share of the annual income carbon tax of the previous period that is recycled as a subsidy to the specific R&D of the RE sector. The constant  $\varepsilon_2$  represents the efficiency of the transformation of the carbon tax income into a decrease of the RE production cost through specific R&D in renewable technology. In other words it measures the efficiency of the specific R&D of the RE sector to use the carbon tax income to produce innovations that materialized in the form of RE production cost decreases. The functional form given in (39) insures that the higher the carbon tax income of period  $t-1$  and the higher the share  $\omega_2$  of this income dedicated to the specific R&D of the RE sector, the faster the unitary capital cost of RE production will converge towards its asymptotic limit  $\underline{b}$ .

#### Option (3): carbon tax income used as a direct capital investment in the RE sector

The third option that we are going to explore for using the income from the carbon tax consists in a direct subsidy to the RE sector in order to increase the amount of installed capital. This should be seen as the capacity of the RE producer to install an additional amount of capital at period  $t$  thanks to a subsidy that equals the carbon tax income of the previous period. Hence, to formalize this effect, we propose to replace (13) by the following (40).

$$G_t = \left[ \frac{\gamma p_t}{B_t v_t} \right]^{\frac{1}{1-\gamma}} + \omega_3 \varepsilon_3 Q_{t-1}, \quad \forall t \in \{0, \dots, T\} \quad (40)$$

Where parameter  $\omega_3$  is the share of the annual income carbon tax of the previous period that is recycled as a direct capital investment in the RE sector. The constant  $\varepsilon_3$  represents the efficiency of the transformation of the carbon tax income given in the form of a subsidy at the previous period into an additional installation of capital in the RE sector. In other words it measures the efficiency of the RE sector to use the subsidy that is received at the previous period (equaling the amount of the carbon tax income) to build new capital in the RE sector. The functional form given in (40) insures that the higher the carbon tax income of period  $t-1$  and the higher the share  $\omega_3$ , the higher the additional capital installation in the RE sector in period  $t$ .

### **5.3 Comparison of the simulation results with the common carbon tax implementation and different ways to use its revenue**

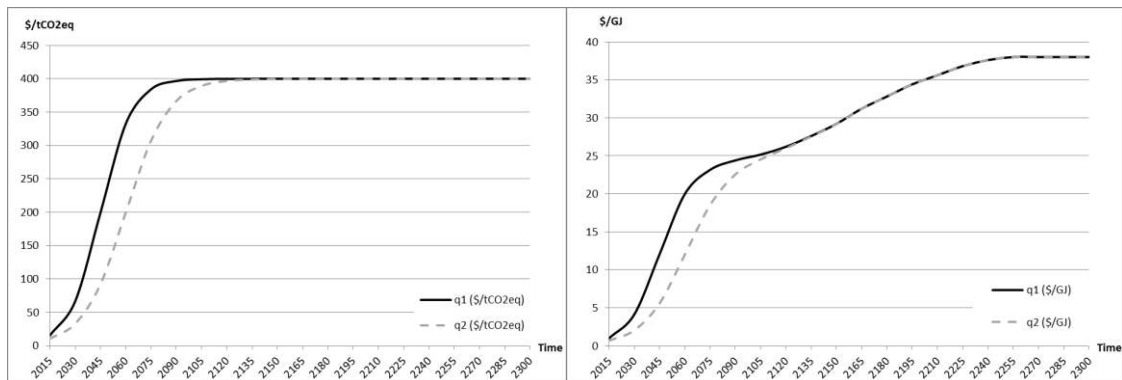
#### Defining the carbon tax profile, the policy mixes scenarios and their specific parameters

As previously mentioned, we make the hypothesis that in all the new scenarios in which we implement the carbon tax, we start with the settings of the *Degrowth* scenario summarized in Table 7. Then, we must defined the exogenous carbon tax variable (mutual to all scenarios) and the values of the three parameters  $\varepsilon_1$ ,  $\varepsilon_2$  and  $\varepsilon_3$ . Regarding  $\varepsilon_3$  the reasoning we have taken is to consider that this parameter should have the same value as  $\rho$  since both parameters represents productivities of the transformation of an investments goods into productive capital, and that there is no apparent reason to think that this transformation productivity in the RE sector (represent by  $\varepsilon_3$ ) should be different than the one of the broader economy (represented by  $\rho$ ). Hence, we define  $\varepsilon_3 = \rho = 3$ . On the other hand, since we have no clear way to estimate parameters  $\varepsilon_1$  and  $\varepsilon_2$ , we have taken the decision to arbitrarily choose the same value of 0.00002 for both parameters and to define two sets of values for parameters  $\bar{q}$ ,  $\lambda$ ,  $t_{q_t \text{ start}}$  and  $t_{q_t \text{ lag}}$  summarized in Table 8 in order to test two distinct exogenous carbon tax profiles  $q1$  and  $q2$  presented on Figure 7. On this point, it is worth

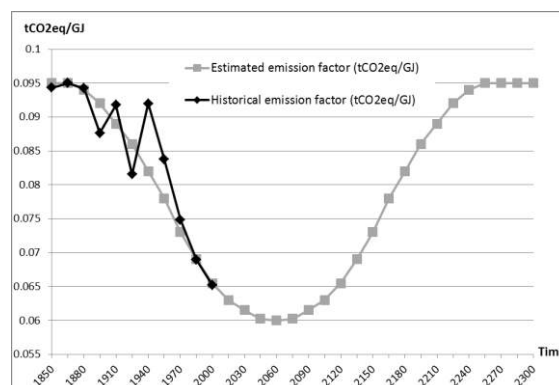
noting that the tax profiles were first expressed in dollar per unit of pollution (B\$/GtCO<sub>2</sub>eq). Then, in order to run this carbon tax in our simulations, we had to express these carbon taxes per unit of fossil energy (B\$/EJ). For that, we took the historical global GHG emissions from fossil fuels estimated by Boden et al. (2012) and divide them by the historical nonrenewable energy production presented in Table 1 to obtain an emission factor in GtCO<sub>2</sub>eq/EJ for each of our ten time periods between 1850 and 2000. As shown on Figure 8, this average emission factor is decreasing over time since the share of the dirtiest energy forms (e.g. coal) in the total mix have been decreasing so far. Considering that the emission factor profile will be symmetric (with the date of the fossil peak as the symmetric center), we can compute exogenously the rest of the profile of the fossil energy emission factor up to the end of our simulation horizon (2300). This is necessary for understanding that on Figure 7 the time profile of the carbon tax is not sigmoid when expressed in \$/GJ (or B\$/EJ).

**Table 8. Values for parameters defining the two possible carbon taxes  $q1$  and  $q2$ .**

Parameter	Definition (unit)	Value for	Value for
		carbon tax	carbon tax
		$q1$	$q2$
$\bar{q}$	Maximum level of the carbon tax (\$/tCO <sub>2</sub> eq)	400	400
$\lambda$	Exogenous rate of growth of the carbon tax (dmnl)	1.6	1.2
$t_{q_t \text{ start}}$	Time period for implementing the carbon tax (time period)	10	11
$t_{q_t \text{ lag}}$	Time lag to obtain the maximum rate of growth of the carbon tax after its implementation time (time period)	3	5



**Figure 7. Profiles of the two carbon taxes  $q1$  and  $q2$  in \$/tCO<sub>2</sub>eq (or B\$/GtCO<sub>2</sub>eq) (left) and \$/GJ (or B\$/EJ) (right) to be used in all scenarios with carbon tax implementation.**



**Figure 8. Historical and estimated average GHG emission factor per unit of fossil energy in tCO<sub>2</sub>eq/GJ (or GtCO<sub>2</sub>eq/EJ) between 1850 and 2300.**

We can now define the different policy mixes scenarios characterized by their relative parameters  $\omega_i$ ,  $i \in [1,2,3]$ . We choose to simulate four scenarios:

- *General R&D* scenario: the integrality of the annual carbon tax income is allocated to the general R&D sector, so  $\omega_1 = 1$ ,  $\omega_2 = 0$ , and  $\omega_3 = 0$ .
- *One third for each* scenario: the annual income from the carbon tax is split equally between the three ways of revenue recycling, so  $\omega_1 = 1/3$ ,  $\omega_2 = 1/3$ , and  $\omega_3 = 1/3$ .
- *50/50 RE R&D/Investments* scenario: the annual carbon tax income is split equally between the specific R&D of the RE sector and the direct capital investment in the RE sector, there is no additional subsidy to the general R&D sector, so  $\omega_1 = 0$ ,  $\omega_2 = 0.5$ , and  $\omega_3 = 0.5$ .
- *30/70 RE R&D/Investments* scenario: 30% of the annual income from the carbon tax goes to the specific R&D of the RE sector, whereas 70% is used as a direct capital investment in the RE sector. In this scenario again there is no additional subsidy to the general R&D sector, so  $\omega_1 = 0$ ,  $\omega_2 = 0.3$ , and  $\omega_3 = 0.7$ .

Simulation results of *General R&D*, *One third for each*, *50/50 RE R&D/Investments*, and *30/70 RE R&D/Investments* scenarios with carbon taxes  $q1$  and  $q2$

In the Figure 9, we compare the GWP, the nonrenewable and the renewable energy productions of the four scenarios that include the carbon tax (*General R&D*, *One third for each*, *50/50 RE R&D/Investments*, and *30/70 RE R&D/Investments*) with their original scenario *Degrowth*. What can be observed when comparing the left (carbon tax  $q1$ ) and right (carbon tax  $q2$ ) sides of Figure 9 is that as could be expected, the more ‘initially stringent’ carbon tax  $q1$  induces larger reductions of nonrenewable energy production levels and higher reductions of the GWP peak. From this Figure 9 it is also clear that introducing the carbon tax induces the expected main outcome: a smoothing of the GWP dynamics during the energy transition. Indeed in all the scenarios that include the carbon tax, whether  $q1$  or  $q2$ , the peak and subsequent degrowth phase of the GWP is lower compared to the original *Degrowth* scenario that does not incorporate the carbon pricing. In all carbon tax scenarios (whether  $q1$  or  $q2$ ), the smoother dynamics is displayed by scenarios of type *General R&D* where the totality of the carbon tax income is allocated to the general R&D sector of the economy. However, *General R&D* scenarios lead eventually to a lower GWP level at the end of the simulation compared to the original *Degrowth* scenario and to all other scenarios that include the carbon tax. A price on carbon and the recycling of its revenue in general R&D does not necessarily lead to the optimal economic growth. This result supports the critics made by Weyant (2011) about the “price fundamentalism” advanced by Nordhaus (2011), implying that additional incentives directed specifically to the renewable sector are needed to overcome its market failures. Such propositions are modeled in the other three scenarios from which simulation results show that the *50/50 RE R&D/Investments*, and the *30/70 RE R&D/Investments* scenarios present more volatile dynamics compared to the *One third for each* scenario in which the carbon tax income is split equally between the three ways of revenue recycling. At first sight, this *One third for each* scenario seems to be the ‘best’ policy option since it implies a smoother transition compared to the *50/50 RE R&D/Investments*, and *30/70 RE R&D/Investments* scenarios and a higher final GWP level compared to the *General R&D* scenario. Of course, further refinements of the model would be needed to correctly

define the ‘best policy option’ for which we do not have an optimization criterion in the current modeling state. Moreover we have only tested scenarios in which the relative allocation shares  $\omega_i$  of the carbon tax revenue remain constant during the entire simulation time but there is a lot of reason to think that this is different in reality. Nevertheless, implementing the carbon tax in our model was interesting to see that it seems to represent an adequate strategy (among others surely) to attenuate, at least partially, the unfortunate future outcomes featured by the *Degrowth* scenario. Implementing the same smoothing strategy in a *Recovery* or *Plateau* type setting lead to the same conclusions.

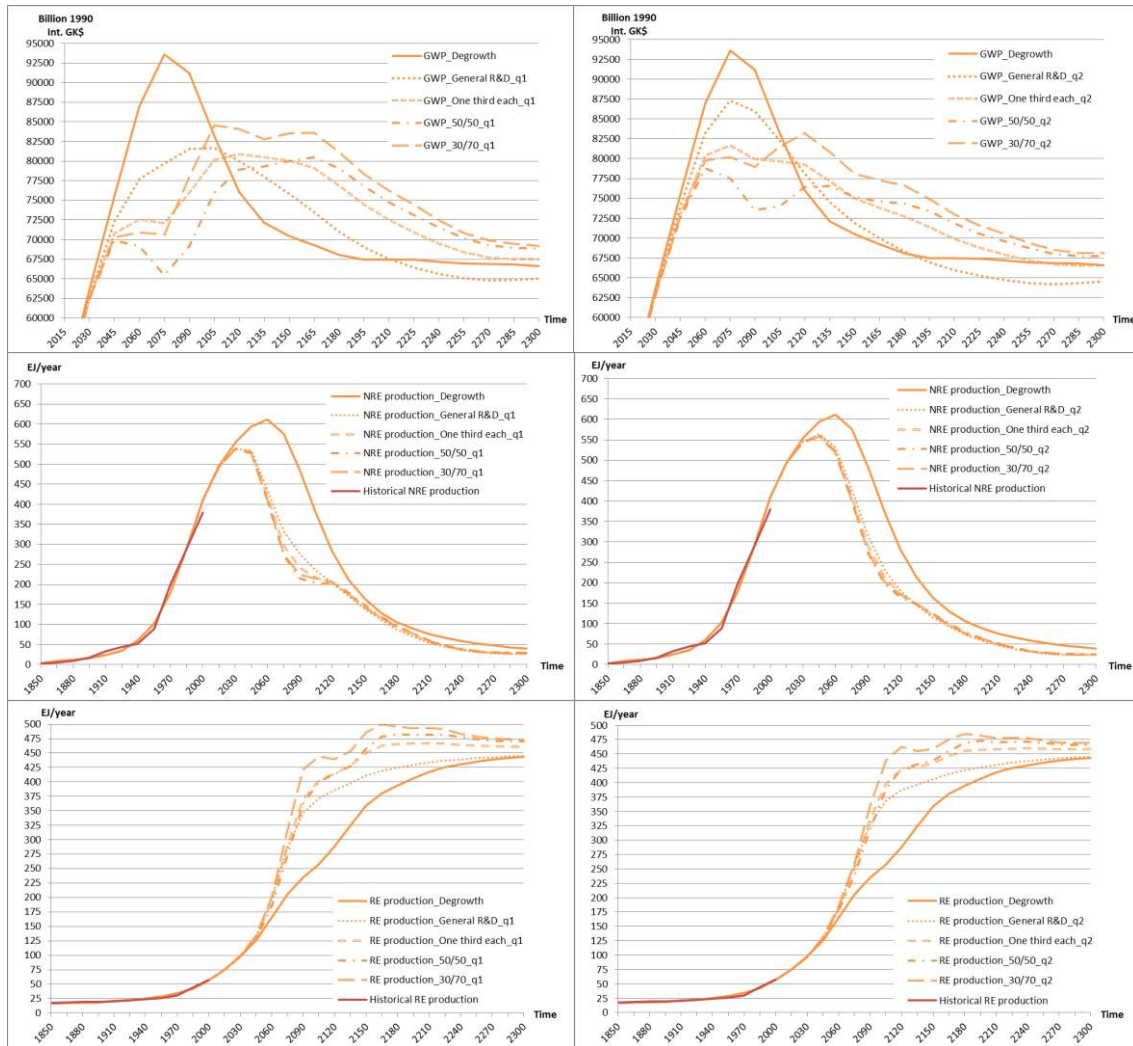


Figure 9. Comparison of the GWP, the nonrenewable and the renewable energy productions of the *Degrowth*, *General R&D*, *One third for each*, *50/50 RE R&D/Investment*, and *30/70 RE R&D/Investment* scenarios with carbon tax  $q_1$  (left) and  $q_2$  (right).

## 6. Conclusions

Through the present article we have developed a theoretical model of endogenous economic growth that, to our mind, accurately represents the fact that energy consumption is one of the main production factors allowing economic growth. Our model is able to closely reproduce the historical production of nonrenewable and renewable energy, and the way they have influenced past economic growth. To our knowledge, we are the first to develop a simple theoretical model that can be simulated on real data and indeed correctly reproduce global historical trends. This is mainly because contrary to other theoretical models we have ensured that our model respects some fundamental physical limits of the real world. Those are formalized in the functional forms that we have established for the fossil and the renewable energy production capital costs. As a consequence, our model is able to reproduce an increasing reliance on fossil fuels from an early renewable era and the subsequent inevitable transition towards complete renewable energy that human will have to deal with in a not-too-far future. To our knowledge, the only other theoretical model able to do so is the one of Tahvonen and Salo (2001), except that contrary to the present work, their simulations were operated with purely hypothetical values for parameters and could not be calibrated on real data to reproduce historical trends of fossil and renewable energy production, and economic production.

By considering several values for the parameter characterizing the level of final technological progress, sensitivity analyses have underlined the various possible trajectories for the nonrenewable and renewable energy production and their impacts on economic growth. Thus, these sensitivity analyses can be interpreted as a way to take into account uncertainties of the real economy in the stylized model. These simulations have highlighted that the use of the current price system (energy price and capital cost in our model) does not necessarily lead to the best path of development because of the uncertainties existing on the final technological level of the economy and the consequent level of investments that are needed to reach this uncertain level of final technological progress. The main conclusion of this paper is rather clear: in order to have a smooth transition between nonrenewable and renewable energy that does not negatively impact the economic growth, the final technological progress level of the economy must be sufficiently high in order for the final renewable production cost to be sufficiently low and ensure an adequate development of renewable energy. Having a final technological progress that is too low can have harmful consequences on economic production, from which partial recovery is however possible. Indeed, in the simulations of our model presented in section 4 we clearly have a threshold under which a *Recovery* scenario is no longer possible and only *Degrowth* type scenarios occur. Of course, no one is able to predict what will be the value of the final technological progress level. That is why we have proposed that in such uncertain context, implementing a carbon tax on the nonrenewable energy production and recycling its revenue could help in the choice of the best development path that consists in a smooth energy transition that does not negatively impact the economic production. In particular, our simulations show that in our model when starting from a *Degrowth* type setting, splitting the income from the carbon tax between the different possibilities of revenue recycling (general R&D, specific R&D of the RE sector, direct support to capital investment in the RE sector) seems to be the best option. Doing so generates the double-winning situation of having both an increase of the growth rate of the general level of the technological progress and a decrease of the degrowth rate of the renewable energy capital requirement which helps smoothing the GWP dynamics.

In conclusion, our model supports the idea that the economic production of our society and its combined growth is highly dependent on the energy supply, and more precisely the net energy that the energy sector supplies to the society. We have shown that future transition towards complete renewable energy could occur with potential negative impact on economic growth, in particular if fossil energy peak is not adequately anticipated. Avoiding such lock-in behavior of our economic system can be (at least partially) done through the implementation of a carbon price. This would not only decrease GHG emissions from fossil fuels use, but also allow reaching more rapidly the final technological progress level of the economy (which is set by thermodynamic constraints). Of course, the model we have presented in this paper relies on important hypotheses. Despite critics that regards the neoclassical approach in itself (unique representative agents for households and firms, pure rational behavior, etc.), our model especially presents the following strong assumptions: the aggregation of the diverse fossil resources into a unique nonrenewable resource, the formalization of a unique renewable energy flow, the perfect substitutability of the fossil and renewable energy forms, the omission of the direct energy consumed by the energy sector itself (energy productions are net of any direct energy requirement since we have only considered the indirect energy embodied in the capital allocated to the two energy sectors). Moreover, our model does not correctly take into account the GHG emissions dynamics and the associated climate change issue. More precisely, in its current formulation our model cannot be used to define endogenously the optimal time path of the carbon price, nor the optimal time path allocation of the carbon tax revenue between the different recycling uses (general R&D subsidy, renewable energy R&D subsidy, renewable energy market support, etc.). These features could be corrected in more complex models (Integrated Assessment Models for example) for which our model would prove to be an adequate basis.

## **Acknowledgments**

The authors would like to thank Clément Bonnet and Simon Quemin for their review of an earlier version of this article, particularly for their respective helpful comments regarding the carbon taxes scenarios and the simulation approach. Many thanks go to Dr. Michael Dale for providing excels sheets containing easily usable data of historical nonrenewable and renewable energy productions.



## References

Acemoglu, D., P., Aghion, L., Bursztyn, D., Hemous, 2012. The environment and directed technical change. *American Economic Review*, 102(1), pp. 131-166.

Aghion, P. and P., Howitt, 1992. A model of growth through creative destruction, *Econometrica*, 60, pp. 323-351.

Ayres, R.U., 1998. Eco-thermodynamics: economics and the second law. *Ecological Economics*, 26, pp. 189-209.

Barro, R., Sala-i-Martin, X., 1995. *Economic Growth*. McGraw-Hill, New York.

Barro, R., 1996. Democracy and growth. *Journal of Economic Growth*, vol.1, issue 1, pp.1-27.

Boden, T.A., G., Marland, and R.J. Andres, 2012. Global, Regional, and National Fossil-Fuel CO<sub>2</sub> Emissions. Carbon Dioxide Information Analysis Center, Oak Ridge National Laboratory, U.S. Department of Energy, Oak Ridge, Tennessee, USA.

BP, 2014. *Statistical Review of World Energy*: British Petroleum.

BP, 2015. Official website of BP accessed 21/05/15: <http://www.bp.com/en/global/corporate/about-bp/energy-economics/statistical-review-of-world-energy/review-by-energy-type/oil/oil-reserves/oil-reserve-definitions.html>

Dale M., S. Krumdieck and P. Bodger, 2011. Net energy yield from production of conventional oil. *Energy policy*, 39, pp. 7095-7102.

Dale M., S. Krumdieck and P. Bodger, 2012. Global energy modelling - A biophysical approach (GEMBA) Part 2: Methodology. *Ecological Economics*, 73, pp. 158-167.

Dale, 2012. Meta-analysis of non-renewable energy resource estimates. *Energy Policy* 43,102–122.

Daly, H.E., 1977. *Steady-State Economics*. Freeman, San Francisco, CA, 185 pp.

Dasgupta, P.S., and G.M., Heal. 1974. The optimal depletion of exhaustible resources. *The Review of Economic Studies*, 41, pp. 85-104.

Etemad, B., and Luciani, J. *World energy production, 1800-1985*. Production mondiale d'énergie, 1800-1985. 1991, Genève: Librairie Droz.

Faber, M., 1985. *A biophysical approach to the economy: Entropy, environment and resources*. In: W. van Gool and J. Bruggink (Editors), *Energy and Time in Economic and Physical Sciences*. North-Holland, Amsterdam, pp. 315-337.

Fagnart, J.F., Germain, M., 2014. Can the energy transition be smooth? CEREC working paper, Université Saint-Louis, Brussels, Belgium.

Fizaine, F., Court, V., 2015. Renewable electricity producing technologies and metal depletion: A sensitivity analysis using the EROI. *Ecological Economics* 110, pp. 106-118.

Georgescu-Roegen, N., 1971. *The Entropy Law and the Economic Process*. Harvard

University Press, Cambridge, Massachusetts.

Georgescu-Roegen, N. 1979. Energy analysis and economic valuation. *Southern Economic Journal*, 45, pp. 1023-1058.

Grossman, M. G. and E., Helpman. 1991. *Innovation and growth in the global economy*. The MIT Press: Cambridge.

Grübler, A., Jefferson, M., and Nakicenovic, N., 1996. Global energy perspectives: A summary of the joint study by the international institute for applied systems analysis and world energy council. *Technological Forecasting and Social Change*, 51(3), pp. 237-264.

Hall C. A. S., C. J. Cleveland, and R. K. Kaufmann. 1986. *Energy and Resource Quality: The Ecology of the Economic Process*, Wiley Interscience, New York.

Hall, C.A.S., D. Lindenberger, R. Kümmel, T. Kroeger, and W. Eichhorn, 2001. The need to reintegrate the natural sciences with economics. *BioScience*, 51(8), pp. 663-673.

Hall, C.A.S, and K.A. Klitgaard, 2006. The need for a new, biophysical-based paradigm in economics for the second half of the age of oil. *International Journal of Transdisciplinary Research*, Vol. 1, No. 1, 4-22.

Hall, C.A.S., Lambert, J.G., Balogh, S., 2014. EROI of different fuels and the implications for society, *Energy Policy*, 64, 141-152.

Honnery, D, Moriarty, P., 2009. Estimating global hydrogen production from wind. *International Journal of Hydrogen Energy*, 34, pp. 727-736.

Hoogwijk M, de Vries B, Turkenburg W. Assessment of the global and regional geographical, technical and economic potential of onshore wind energy. *Energy Economics*, 26 (5), pp. 889-919.

Lucas, R. E., 1988. On the mechanics of economic development. *Journal of Monetary Economics*, 22, pp. 3-42.

Maddison, A. (2001), *The World Economy: A Millennia Perspective* (OECD, Paris).

McGlade and Ekins, 2015. The geographical distribution of fossil fuels unused when limiting global warming to 2°C. *Nature*, 57, 187-190.

Meadows, D.H., D.L., Meadows, J., Randers and W.W., Behrens. 1972. *The Limits to Growth: A Report for the Club of Rome's Project on the Predicament of Mankind*. Universe Books, New York, 1972.

Murphy, D.J. and Hall, C.A.S., 2010. Year in review - EROI or energy return on (energy) invested. *Annals of the New York Academy of Sciences*, 1185, pp. 102-118.

Nordhaus, W., 2011. Designing a friendly space for technological change to slow global warming. *Energy Economics*, 33, pp. 665-673.

NREL (2010a) Western Wind Dataset, National Renewable Energy Laboratory.

NREL (2010b) National Solar Radiation Database, National Renewable Energy Laboratory.

- O'Connor, M., 1991. Entropy, structure, and organisational change. *Ecological Economics*, 3, pp. 95-122.
- Odum, H.T., 1971. *Environment, Power, and Society*. Wiley-Interscience, New York, 331 pp.
- Odum, H.T., 1973. Energy, ecology and economics. *AMBIO*, Vol.2, No.6, In Energy in Society: A Special Issue, pp. 220-227.
- Perrings, C., 1987. *Economy and Environment: A Theoretical Essay on the Interdependence of Economic and Environmental Systems*. Cambridge University Press, Cambridge.
- Rebelo, S., 1991, Long-run policy analysis and long-run growth. *Journal of Political Economy*, 1991, vol. 99, 3, p. 500-521.
- Romer, P. M., 1986. Increasing returns and long run growth. *Journal of Political Economy*, 94, pp. 1002-1037.
- Romer, P. M., 1990. Endogenous technical change. *Journal of Political Economy*, 98, pp. 71-102.
- Smil, V. *Energy Transitions: History, Requirements, Prospects*. Praeger Publishers Inc, 2010.
- Smulders, S., 1995. Entropy, Environment, and Endogenous Economic Growth. *International Tax and Public Finance*, 2, pp. 319-340.
- Solow, R.M., 1956. A Contribution to the Theory of Economic Growth. *Quarterly Journal of Economics*, 70, pp. 65-94.
- Solow, R. M., 1974. The Economics of Resources or the Resources of Economics. *The American Economic Review*, 64(2), pp. 1-14.
- Sorrell, S., Spiers, J., Bentley, R., Brandt, A., Miller, R., 2010. Global oil depletion: a review of the evidence. *Energy Policy* 38 (9), pp. 5290–5295.
- Stiglitz, J.E., 1974. Growth with exhaustible natural resources, efficient and optimal growth paths. *Review of Economic Studies* 41, pp. 123-137.
- Tahvonen O. and S. Salo, 2001. Economic growth and transitions between renewable and nonrenewable energy resources. *European Economic Review*, 45, pp. 1379-1398.
- The Maddison-Project, <http://www.ggd.net/maddison/maddison-project/home.htm>, 2013 version.
- Tsur Y. and Zemel A., 2005. Scarcity, growth and R&D. *Journal of Environmental Economics and Management*, 49, pp. 484-499.
- Warr, B., and R.U., Ayres, 2012. Useful work and information as drivers of economic growth. *Ecological Economics*, 73, pp. 93-102.
- Weyant, J.P., 2011. Accelerating the development and diffusion of new energy technologies: Beyond the “valley of death”. *Energy Economics*, 33, pp. 674-682.

# WORKING PAPER

n° 2015-07 • Septembre 2015



## LAST ISSUES

- |   |            |
|---|------------|
| <b>Endogenous economic growth, EROI, and transition towards renewable energy</b><br>Victor COURT, Pierre-André JOUVET and Frédéric LANTZ  | n°2015-07  |
| <b>Accessibility to work by public transit and its social distribution in Lille, France</b><br>Claire PAPAIX, Ariane DUPONT-KIEFFER   | n°2015-06  |
| <b>Deforestation, Leakage and avoided deforestation policies: A Spatial Analysis</b><br>Philippe DELACOTE, Elizabeth J.Z. ROBINSON, Sébastien ROUSSEL                             | n°2015-05  |
| <b>Growth, Green Capital and Public Policies</b><br>Pierre-André JOUVET, Julien WOLFERSBERGER   | n°2015-04  |
| <b>Simulating the use of biomass in electricity with the Green Electricity Simulate model: An application to the French power generation</b><br>Vincent Bertrand, Elodie Le Cadre | n°2015-03  |
| <b>Mitigation costs Through alternative crop rotations in agriculture : An Assessment for 5 European regions</b><br>Benjamin Dequiedt, Vera Eory, Juliette Maire et al.           | n°2015-02  |
| <b>Policy packages for modal shift and CO<sub>2</sub> reduction in Lille, France</b><br>Hakim Hammadou, Claire Papaix   | n° 2015-01 |
| <b>The dynamics of deforestation and reforestation in a developing economy</b><br>Julien Wolfersberger, Gregory Amacher, Philippe Delacote, Arnaud Dragicevic                     | n° 2014-14 |

---

Working Paper Publication Director: Philippe Delacote

Les opinions exposées ici n'engagent que les auteurs. Ceux-ci assument la responsabilité de toute erreur ou omission

La Chaire Economie du Climat est une initiative de CDC Climat et de l'Université Paris-Dauphine sous l'égide de la Fondation Institut Europlace de Finance

[contact@chaireeconomieduclimat.org](mailto:contact@chaireeconomieduclimat.org)