Energy and Capital in a New-Keynesian Framework

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Outline

Goals

Model

Household

Firms
- The Final Good Firm
- Intermediate Good Firms

Government
- GDP and GDP Deflator

Estimation
- Setting
- Estimation Results

Impulse Response Functions
Outline

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Impulse Response Functions
• This paper constructs a New-Keynesian model with oil in the production function and in consumption.

• The model’s parameters are estimated using Bayesian techniques.

• We observe the impact of the oil shock in this economy.
Outline

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Impulse Response Functions

Goals

Model

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Impulse Response Functions
Model Structure

Domestic Economy
Model Structure

- Domestic Economy
  - Household
  - Final Good Firm

- Firms
  - Invests
  - Works
  - Consumes
  - Stays taxes
  - Capital
  - Bonds

- Final Goods
  - Energy

- Intermediate Firms
  - Labor
  - Capital
  - Energy
  - Exo Pro.
  - Profits
  - Foreign exo Pro.
  - Exogenous price

- Government
  - Taxes
  - Taylor
Model Structure

Domestic Economy → Final Good Firm

Household:
- works
- consumes
- pays taxes
- invests

Final Goods:
- Energy
- Intermediate Firms

Firms:
- Labor
- Capital
- Exogenous Price
- Profits

Foreign:
- Exogenous Price

Government:
- Taylor

Goals:
- Model
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- Estimation
- Impulse Response Functions
Model Structure

- **Household**
  - works
  - consumes
  - pays taxes
  - invests
  - capital
  - bonds

- **Final Good Firm**
  - Domestic Economy

- **Domestic Economy**
  - Energy
  - Intermediate Firms
  - Energy
  - Labor
  - Capital
  - Exogenous price

- **Government**
  - Taylor

- **Firms**
  - produces
Model Structure

Domestic Economy
- Household
  - consumption
  - works
  - capital
  - invests
- Final Goods
  - Energy

Final Good Firm
Model Structure

Domestic Economy → Final Good Firm

Household
- consumes
- works
- invests
- l.s taxes
- capital

Final Goods

Energy

Foreign exo p.

Taylor

Intermediate Firms: Energy, Labor, Capital

Exogenous price

Government

Taylor

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Domestic Economy

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Final Good Firm

Intermediate Firms

Final Goods

Energy

l.s taxes

bonds

capital

consumes

invests

works

produces

Foreign exo p.

Government

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Model Structure

- **Household**
  - consumes
  - works
  - invests
  - pays taxes

- **Domestic Economy**
  - produces Final Goods

- **Final Good Firm**

- **Intermediate Firms**
  - produces Energy, Labor, Capital

- **Foreign Firms**
  - exogenous price

- **Government**

- **Goals**
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  - Government
  - Estimation
  - Impulse Response Functions
Model Structure

- Domestic Economy
  - Household
    - consumes
    - works
    - invests
  - final goods
    - Energy
    - Labor
    - Capital
    - exogenous price
  - exogenous price
    - foreign

- Final Good Firm
  - Intermediate Firms
    - profits
    - exo p.
  - exo p.
Model Structure

Government

Domestic Economy

Final Good Firm

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Intermediate Firms

Foreign

Final Goods

Energy

Labor

Capital

Invests

Works

Consumes

Taylor

l.s taxes

bonds

capital

profits

produces

exogenous price

Energy

exo p.

Profit
Outline

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Household

Problem

\[
\max \mathbb{E}_0 \left[ \sum_{t=0}^{\infty} \beta^t U(C_t, L_t) \right], \quad 0 < \beta < 1
\]

s. t

\[
P_{e,t} C_{e,t} + P_{q,t} C_{q,t} + P_{k,t} l_t + B_t + T_t \\
\leq (1 + i_{t-1}) B_{t-1} + W_t L_t + D_t + r^k_t P_{k,t} K_t
\]
Household

Problem

$max \mathbb{E}_0 \left[ \sum_{t=0}^{\infty} \beta^t U(C_t, L_t) \right], \quad 0 < \beta < 1$

s. t

$P_e, t C_{e, t} + P_q, t C_{q, t} + P_k, t I_t + B_t + T_t \leq (1 + i_{t-1}) B_{t-1} + W_t L_t + D_t + r^k_t P_{k, t} K_t$

$\Theta_x := x^{-x}(1 - x)^{-(1-x)}$

$C_t := \Theta_x C_{e, t}^x C_{q, t}^{1-x}$
Household

**Problem**

\[
\max E_0 \left[ \sum_{t=0}^{\infty} \beta^t U(C_t, L_t) \right], \quad 0 < \beta < 1
\]

**s. t.**

\[
P_{e,t} C_{e,t} + P_{q,t} C_{q,t} + P_{k,t} I_t + B_t + T_t \\
\leq (1 + i_{t-1}) B_{t-1} + W_t L_t + D_t + r^k_t P_{k,t} K_t
\]

**θ_x :=**

\[
x^{-x}(1 - x)^{-(1-x)}
\]

**C_t :=**

\[
\Theta_x C_{e,t}^x C_{q,t}^{1-x}
\]

**U(C_t, L_t) =**

\[
\log(C_t) - \frac{L_t^{1+\phi}}{1+\phi}
\]
Household

Problem

\[ \max \mathbb{E}_0 \left[ \sum_{t=0}^{\infty} \beta^t U(C_t, L_t) \right], \quad 0 < \beta < 1 \]

s. t

\[ P_{e,t} C_{e,t} + P_{q,t} C_{q,t} + P_{k,t} I_t + B_t + T_t \leq (1 + i_{t-1})B_{t-1} + W_t L_t + D_t + r_k^k P_{k,t} K_t \]

\[ C_{q,t} := \left( \int_0^1 C_{q,t}(i)^{1-\frac{1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}} \]

\[ \Theta_x := x^{-x}(1 - x)^{-(1-x)} \]

\[ C_t := \Theta_x C_{e,t}^x C_{q,t}^{1-x} \]

\[ U(C_t, L_t) = \log(C_t) - \frac{L_t^{1+\phi}}{1+\phi} \]
Household

\[ \text{Problem} \]

\[ \max \mathbb{E}_0 \left[ \sum_{t=0}^{\infty} \beta^t U(C_t, L_t) \right], \quad 0 < \beta < 1 \]

s. t

\[ P_{e,t} C_{e,t} + P_{q,t} C_{q,t} + P_{k,t} I_t + B_t + T_t \leq (1 + i_{t-1}) B_{t-1} + W_t L_t + D_t + r^k P_{k,t} K_t \]

\[ C_{q,t} := \left( \int_0^1 C_{q,t}(i)^{1 - \frac{1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon - 1}} \]

\[ U(C_t, L_t) = \log(C_t) - \frac{L_t^{1+\phi}}{1+\phi} \]

\[ I_t := K_{t+1} - (1 - \delta) K_t \]

\[ \Theta_x := x^{-x}(1 - x)^{-(1-x)} \]

\[ C_t := \Theta_x C_{e,t}^x C_{q,t}^{1-x} \]
Optimization

Household’s Optimal Expenditure Allocation
Optimization

Household’s Optimal Expenditure Allocation

\[
\max_{C_{q,t}, C_{e,t}} P_{c,t} C_t
\]

s. t

\[
P_{c,t} C_t = P_{e,t} C_{e,t} + P_{q,t} C_{q,t}
\]

\[
C_t = \Theta x C_{e,t}^{x} C_{q,t}^{1-x}
\]
Optimization

Household's Optimal Expenditure Allocation

\[
\max_{C_{q,t}, C_{e,t}} P_{c,t} C_t
\]

subject to

\[
P_{c,t} C_t = P_{e,t} C_{e,t} + P_{q,t} C_{q,t}
\]

\[
C_t = \Theta x C_{e,t} C_{q,t}^{1-x}
\]

\[
P_{q,t} C_{q,t} = (1-x) P_{c,t} C_t
\]

\[
P_{e,t} C_{e,t} = x P_{c,t} C_t
\]

\[
P_{c,t} = P_{e,t}^x P_{q,t}^{(1-x)}
\]
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Estimation

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Final Good Producers

Final Good Firm
Final Good Producers

Intermediate Good $i \in [0, 1]$

Final Good Firm

$Q_t = \left( \int_0^1 Q_t(i) \epsilon \epsilon - 1 \epsilon d_i \right) \epsilon \epsilon - 1 \epsilon$: the elasticity of substitution among intermediate goods
Final Good Producers

Intermediate Good \( i \in [0, 1] \)

Final Good Firm

\[ Q_t = \left( \int_0^1 Q_t(i) \frac{e^{-1}}{\epsilon} \, di \right)^{-\frac{\epsilon}{\epsilon-1}} \]
Final Good Producers

Intermediate Good $i \in [0, 1]$

Final Good Firm

$Q_t = \left( \int_0^1 Q_t(i) \frac{e-1}{\epsilon} \, di \right)^{\frac{\epsilon}{\epsilon-1}}$

$\epsilon$: the elasticity of substitution among intermediate goods
Final Good Producer Problem

Final Good Firm Profit Optimization

\[
\max_{Q_t(i)} P_{q,t} Q_t - \int_0^1 P_{q,t}(i) Q_t(i) \, di
\]

s. t

\[
Q_t = \left( \int_0^1 Q_t(i) \frac{\epsilon - 1}{\epsilon} \, di \right)^{\frac{\epsilon}{\epsilon - 1}}
\]

\[
Q_t(i) = \left( \frac{P_{q,t}(i)}{P_{q,t}} \right)^{-\epsilon} Q_t
\]

\[
P_{q,t} = \left( \int_0^1 P_{q,t}(i)^{1-\epsilon} \, di \right)^{\frac{1}{1-\epsilon}}
\]
Intermediate Good Firms

Intermediate Firms
Intermediate Good Firms

\[ Q_t(i) = A_t E_t(i)^{\alpha_e} L_t(i)^{\alpha_\ell} K_t(i)^{\alpha_k} \]

\[ \alpha_e, \alpha_\ell, \alpha_k \geq 0, \quad \alpha_e + \alpha_\ell + \alpha_k \leq 1 \]
Intermediate Good Firms

\[ Q_t(i) = A_t E_t(i)^{\alpha_e} L_t(i)^{\alpha_\ell} K_t(i)^{\alpha_k} \]

\[ \alpha_e, \alpha_\ell, \alpha_k \geq 0, \quad \alpha_e + \alpha_\ell + \alpha_k \leq 1 \]

strategy of firm \( i \): Marginal cost pricing behavior

Given: \( P_{e,t}, P_{k,t}, W_t \) and \( Q_t(i) \)

Choses: \( E_t(i), L_t(i) \) and \( K_t(i) \)
Intermediate Good Firms

\[ Q_t(i) = A_t E_t(i)^{\alpha_e} L_t(i)^{\alpha_{\ell}} K_t(i)^{\alpha_k} \]

\[ \alpha_e, \alpha_{\ell}, \alpha_k \geq 0, \quad \alpha_e + \alpha_{\ell} + \alpha_k \leq 1 \]

strategy of firm \( i \): Marginal cost pricing behavior

Given: \( P_{e,t}, P_{k,t}, W_t \) and \( Q_t(i) \)

Choses: \( E_t(i), L_t(i) \) and \( K_t(i) \)

Given: prices and quantities

Choses: \( P_{q,t} \)
Price Optimization

Price Maximization (at each date $t$) (Calvo Price Setting)

$$P_{q,t}(i) = P_{q,t-1}(i)$$

$$P_{q,t}(i) = P_{q,t}^o(i)$$

$\theta$ cannot change

$1 - \theta$ can change
Outline

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GDP and GDP Deflator

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GDP and GDP Deflator Definition

\[ P_{y,t} Y_t = P_{q,t} Q_t - P_{e,t} E_t \]
GDP and GDP Deflator Definition

\[ P_y, t \ Y_t = P_q, t \ Q_t − P_e, t \ E_t \]

GDP (in value added)

GDP Deflator

\[ P_y, t = P_c, t \]
<table>
<thead>
<tr>
<th>Goals</th>
<th>Model</th>
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<th>Impulse Response Functions</th>
</tr>
</thead>
</table>

**Government**

\[
\begin{align*}
\Pi_q, t &= \beta(\Pi_q, t) \phi_\pi(Y_t, Y_t) \phi_y \varepsilon_{i, t},
\end{align*}
\]

\[
\begin{align*}
\Pi_q, t &= P_q, t - 1 \ln(\varepsilon_{i, t}) = \rho_i \ln(\varepsilon_{i, t-1}) + \varepsilon_{i, t}(1+i_{t-1}B_{t-1} + G_{t}) = B_t + T_t
\end{align*}
\]

\[
\begin{align*}
\ln(G_r, t) &= (1 - \rho_g)\ln(\omega Q) + \rho_g \ln(G_r, t-1) + \rho_{alk, ge} e_{alk, t} + \rho_{ae, ge} e_{ae, t} + e_{ge, t}
\end{align*}
\]

**budget constraint**
Government

Central Bank → Government

\[ \Pi_{q,t} := \frac{P_{q,t}}{P_{q,t-1}} \ln \left( \varepsilon_{i,t} \right) = \rho_i \ln \left( \varepsilon_{i,t-1} \right) + \varepsilon_{i,t} \left( 1 + i_{t-1} B_{t-1} + G_{t} \right) \]

\[ \ln \left( G_r,t \right) = \left( 1 - \rho_g \right) \left( \ln \left( \omega Q \right) \right) + \rho_g \ln \left( G_r,t-1 \right) + \rho_{alg} e_{alg},t + \rho_{ae} e_{ae},t + e_{g},t \]

budget constraint
Government

Central Bank

Government

\[ 1 + i_t = \frac{1}{\beta} (\Pi_{q,t})^{\phi_\pi} \left( \frac{Y_t}{Y} \right)^{\phi_y} \varepsilon_{i,t} \]
Government

$1 + i_t = \frac{1}{\beta} (\Pi_{q,t})^{\phi_\pi} \left( \frac{Y_t}{Y} \right)^{\phi_y} \varepsilon_{i,t}$

$\Pi_{q,t} := \frac{P_{q,t}}{P_{q,t-1}}$

$\ln(\varepsilon_{i,t}) = \rho_i \ln(\varepsilon_{i,t-1}) + e_{i,t}$
Government

Central Bank

\[ 1 + i_t = \frac{1}{\beta} (\Pi_{q,t})^{\phi_\pi} \left( \frac{Y_t}{Y} \right)^{\phi_y} \varepsilon_{i,t} \]

Government

\[ (1 + i_{t-1})B_{t-1} + G_t = B_t + T_t \]

\[ \Pi_{q,t} := \frac{P_{q,t}}{P_{q,t-1}} \]

\[ \ln(\varepsilon_{i,t}) = \rho_i \ln(\varepsilon_{i,t-1}) + e_{i,t} \]
Government

\[ \ln(G_r,t) = (1 - \rho_g)\ln(\omega Q) + \rho_g \ln(G_r,t-1) + \rho_{alk,g} e_{alk,t} + \rho_{ae,g} e_{ae,t} + e_{g,t} \]

Central Bank

\[ 1 + i_t = \frac{1}{\beta}(\Pi_q,t)^{\phi_{\pi}} \left( \frac{Y_t}{Y} \right)^{\phi_y} \varepsilon_{i,t} \]

Government

\[ (1 + i_{t-1})B_{t-1} + G_t = B_t + T_t \]

\[ \ln(\varepsilon_{i,t}) = \rho_i \ln(\varepsilon_{i,t-1}) + e_{i,t} \]

\[ \Pi_{q,t} := \frac{P_{q,t}}{P_{q,t-1}} \]
Other Shocks

\[ S_{e,t} := \frac{P_{e,t}}{P_{q,t}} \]

\[ \log(S_{e,t}) = \rho_s \log(S_{e,t-1}) + e_{se,t} \]

\[ \text{Oil Price} \]

\[ \text{AR}(1) \]
Other Shocks

Oil Price

\[ S_{e,t} := \frac{P_{e,t}}{P_{q,t}} \]

\[ \log(S_{e,t}) = \rho_s e \log(S_{e,t-1}) + e_{se,t} \]

Capital Price

\[ S_{k,t} := \frac{P_{k,t}}{P_{q,t}} \]

\[ \log(S_{k,t}) = \rho_s k \log(S_{k,t-1}) + e_{sk,t} \]
Other Shocks

\[ \ln(A_t) = \rho_a \ln(A_{t-1}) + e_{a,t} \]
Other Shocks

TFP

\[ \ln(A_t) = \rho_a \ln(A_{t-1}) + e_{a,t} \]

Price Markup

\[ \varepsilon_{p,t} = \rho_p \varepsilon_{p,t-1} + e_{p,t} - \nu_p e_{p,t-1} \]
Definition of Equilibrium
Definition of Equilibrium

agents maximize its problems

all markets clear  Equilibrium  Government budget const. fulfilled
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## Data

<table>
<thead>
<tr>
<th>Observed Variable</th>
<th>Transformation</th>
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</thead>
<tbody>
<tr>
<td>invobs</td>
<td>detrend ((\ln(\frac{PFI}{GDPDEF}) \times 100))</td>
</tr>
<tr>
<td>yobs</td>
<td>detrend ((\ln(\frac{GDPC09}{LNSIndex}) \times 100))</td>
</tr>
<tr>
<td>labobs</td>
<td>(\ln\left(\frac{\text{Averagehours} \times CE16OVIndex}{LNSIndex}\right) \times 100 - \text{mean}\left(\ln\left(\frac{\text{Averagehours} \times CE16OVIndex}{LNSIndex}\right) \times 100\right))</td>
</tr>
<tr>
<td>infobs</td>
<td>(\ln\left(\frac{\text{GDPDEF}}{\text{GDPDEF}(-1)}\right) \times 100 - \text{mean}\left(\ln\left(\frac{\text{GDPDEF}}{\text{GDPDEF}(-1)}\right) \times 100\right))</td>
</tr>
<tr>
<td>iobs</td>
<td>((\ln\left(1 + \frac{\text{FEDFUND}}{400}\right) - \text{mean}\left(\ln\left(1 + \frac{\text{FEDFUND}}{400}\right)\right)) \times 100)</td>
</tr>
<tr>
<td>eobs</td>
<td>(\ln\left(\frac{\text{TotalSAOil}}{\text{LNSIndex}}\right) \times 100 - \text{mean}\left(\ln\left(\frac{\text{TotalSAOil}}{\text{LNSIndex}}\right) \times 100\right))</td>
</tr>
</tbody>
</table>
Calibrated Parameters

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>$\delta$</th>
<th>$\omega$</th>
<th>$x$</th>
<th>$\epsilon$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.99</td>
<td>0.025</td>
<td>0.18</td>
<td>0.023</td>
<td>8</td>
</tr>
</tbody>
</table>

Table: Calibrated Parameters
## Estimation Results - $\theta$ estimated

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Prior distribution</th>
<th>Posterior distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Mode</td>
</tr>
<tr>
<td>$\theta$ estimated</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Capital elasticity</td>
<td>$\alpha_k$ IGamma(0.1,2)</td>
<td>0.3728</td>
</tr>
<tr>
<td>Labor elasticity</td>
<td>$\alpha_\ell$ IGamma(0.4,2)</td>
<td>0.6424</td>
</tr>
<tr>
<td>Oil elasticity</td>
<td>$\alpha_e$ IGamma(0.6,2)</td>
<td>0.1234</td>
</tr>
<tr>
<td>Inverse Frisch elasticity</td>
<td>$\phi$ IGamma(1.17,0.5)</td>
<td>0.6209</td>
</tr>
<tr>
<td>Taylor rule response to inflation</td>
<td>$\phi_\pi$ Normal(1.2,0.1)</td>
<td>1.2235</td>
</tr>
<tr>
<td>Taylor rule response to output</td>
<td>$\phi_y$ Normal(0.5,0.1)</td>
<td>0.8020</td>
</tr>
<tr>
<td>Calvo price parameter</td>
<td>$\theta$ Beta(0.5,0.1)</td>
<td>0.9812</td>
</tr>
</tbody>
</table>

Table: Prior and Posterior Distribution of Structural Parameters
Table: Prior and Posterior Distribution of Shock Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Prior distribution</th>
<th>Mode</th>
<th>Mean</th>
<th>10%</th>
<th>90%</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Autoregressive parameters</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Technology</td>
<td>$\rho_a$ Beta(0.5,0.2)</td>
<td>0.8619</td>
<td>0.8481</td>
<td>0.7960</td>
<td>0.8999</td>
</tr>
<tr>
<td>Real oil price</td>
<td>$\rho_{se}$ Beta(0.5,0.2)</td>
<td>0.5761</td>
<td>0.5611</td>
<td>0.4629</td>
<td>0.6669</td>
</tr>
<tr>
<td>Real capital price</td>
<td>$\rho_{sk}$ Beta(0.5,0.2)</td>
<td>0.7210</td>
<td>0.7080</td>
<td>0.6647</td>
<td>0.7524</td>
</tr>
<tr>
<td>Price markup1</td>
<td>$\rho_p$ Beta(0.5,0.2)</td>
<td>0.9418</td>
<td>0.9283</td>
<td>0.8955</td>
<td>0.9640</td>
</tr>
<tr>
<td>Price markup2</td>
<td>$\nu_p$ Beta(0.5,0.2)</td>
<td>0.9796</td>
<td>0.9760</td>
<td>0.9610</td>
<td>0.9913</td>
</tr>
<tr>
<td>Government</td>
<td>$\rho_g$ Beta(0.5,0.2)</td>
<td>0.9058</td>
<td>0.8995</td>
<td>0.8712</td>
<td>0.9258</td>
</tr>
<tr>
<td>Tech. in Gov.</td>
<td>$\rho_{ag}$ Beta(0.5,0.2)</td>
<td>0.6904</td>
<td>0.6127</td>
<td>0.3549</td>
<td>0.9472</td>
</tr>
<tr>
<td>Monetary</td>
<td>$\rho_i$ Beta(0.5,0.2)</td>
<td>0.9399</td>
<td>0.9308</td>
<td>0.9035</td>
<td>0.9581</td>
</tr>
<tr>
<td><strong>Standard deviations</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Technology</td>
<td>$\sigma_a$ IGamma(1,2)</td>
<td>0.4361</td>
<td>0.4435</td>
<td>0.3901</td>
<td>0.4942</td>
</tr>
<tr>
<td>Real oil price</td>
<td>$\sigma_{se}$ IGamma(1,2)</td>
<td>2.0000</td>
<td>1.9373</td>
<td>1.8652</td>
<td>2.000</td>
</tr>
<tr>
<td>Real capital price</td>
<td>$\sigma_{sk}$ IGamma(1,2)</td>
<td>0.7740</td>
<td>0.7675</td>
<td>0.6379</td>
<td>0.8781</td>
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<tr>
<td>Price markup</td>
<td>$\sigma_p$ IGamma(1,2)</td>
<td>0.1814</td>
<td>0.1854</td>
<td>0.1615</td>
<td>0.2094</td>
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<tr>
<td>Government</td>
<td>$\sigma_g$ IGamma(1,2)</td>
<td>2.0000</td>
<td>1.7921</td>
<td>1.5508</td>
<td>1.9998</td>
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<tr>
<td>Monetary</td>
<td>$\sigma_i$ IGamma(1,2)</td>
<td>0.5410</td>
<td>0.4566</td>
<td>0.3859</td>
<td>0.5205</td>
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</tbody>
</table>
## Estimation Results - $\theta$ calibrated

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Prior distribution</th>
<th>Posterior distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Mode</td>
</tr>
<tr>
<td>$\theta$ calibrated</td>
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</tr>
<tr>
<td>Capital elasticity $\alpha_k$</td>
<td>IGamma(0.2,2)</td>
<td>0.3918</td>
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<tr>
<td>Labor elasticity $\alpha_\ell$</td>
<td>IGamma(0.4,2)</td>
<td>0.5947</td>
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<tr>
<td>Oil elasticity $\alpha_e$</td>
<td>IGamma(0.5,2)</td>
<td>0.1132</td>
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<tr>
<td>Inverse Frisch elasticity $\phi$</td>
<td>IGamma(1.17,0.5)</td>
<td>1.2562</td>
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<tr>
<td>Taylor rule response to inflation $\phi_\pi$</td>
<td>Normal(1.2,0.1)</td>
<td>1.5236</td>
</tr>
<tr>
<td>Taylor rule response to output $\phi_y$</td>
<td>Normal(0.5,0.1)</td>
<td>0.0265</td>
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</table>

**Table**: Prior and Posterior Distribution of Structural Parameters
### Estimation Results - $\theta$ calibrated

#### Table: Prior and Posterior Distribution of Shock Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Prior distribution</th>
<th>Posterior distribution</th>
<th>Mode</th>
<th>Mean</th>
<th>10%</th>
<th>90%</th>
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</thead>
<tbody>
<tr>
<td><strong>Autoregressive parameters</strong></td>
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<tr>
<td>Technology</td>
<td>$\rho_a$ Beta(0.5,0.2)</td>
<td>0.9605 0.9401 0.9033 0.9774</td>
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<tr>
<td>Real oil price</td>
<td>$\rho_{se}$ Beta(0.5,0.2)</td>
<td>0.9934 0.9872 0.9754 0.9977</td>
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<tr>
<td>Real capital price</td>
<td>$\rho_{sk}$ Beta(0.5,0.2)</td>
<td>0.8940 0.8924 0.8483 0.9314</td>
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<tr>
<td>Price markup1</td>
<td>$\rho_p$ Beta(0.5,0.2)</td>
<td>0.9839 0.9621 0.9299 0.9971</td>
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<tr>
<td>Price markup2</td>
<td>$\nu_p$ Beta(0.5,0.2)</td>
<td>0.1652 0.1711 0.0593 0.2758</td>
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<tr>
<td>Government</td>
<td>$\rho_g$ Beta(0.5,0.2)</td>
<td>0.9373 0.9312 0.9061 0.9560</td>
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<tr>
<td>Tech. in Gov.</td>
<td>$\rho_{ag}$ Beta(0.5,0.2)</td>
<td>0.7129 0.6589 0.3808 0.9541</td>
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<tr>
<td>Monetary</td>
<td>$\rho_i$ Beta(0.5,0.2)</td>
<td>0.1914 0.2104 0.1249 0.2856</td>
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<tr>
<td><strong>Standard deviations</strong></td>
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<tr>
<td>Technology</td>
<td>$\sigma_a$ IGamma(1,2)</td>
<td>0.4538 0.4542 0.3981 0.5078</td>
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</tr>
<tr>
<td>Real oil price</td>
<td>$\sigma_{se}$ IGamma(1,2)</td>
<td>2.0000 1.9475 1.8842 2.000</td>
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<tr>
<td>Real capital price</td>
<td>$\sigma_{sk}$ IGamma(1,2)</td>
<td>0.5459 0.5750 0.4722 0.6714</td>
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<tr>
<td>Price markup</td>
<td>$\sigma_p$ IGamma(1,2)</td>
<td>0.4235 0.4645 0.2868 0.6602</td>
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<tr>
<td>Government</td>
<td>$\sigma_g$ IGamma(1,2)</td>
<td>2.0000 1.8359 1.6425 2.000</td>
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<tr>
<td>Monetary</td>
<td>$\sigma_i$ IGamma(1,2)</td>
<td>0.4778 0.4769 0.4062 0.5455</td>
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</table>
Outline

Goals
Model
Household
Firms
Government
Estimation
Impulse Response Functions
IRF to a Real Oil Price Shock. Case: $\theta$ Estimated
IRF to a Real Oil Price Shock. Case: $\theta$ Calibrated
Optimization

\[1 = \beta E_t \left[ (1 + i_t) \frac{C_t}{C_{t+1}} \frac{P_{c,t}}{P_{c,t+1}} \right]\]

Euler

First Order Conditions

\[1 = \beta E_t \left[ \frac{C_t}{C_{t+1}} \frac{P_{c,t}}{P_{c,t+1}} \frac{P_{k,t+1}}{P_{k,t}} (r_{t+1}^k + 1 - \delta) \right]\]

Fisher

\[\frac{W_t}{P_{c,t}} = C_t L_t^\phi\]
No Ponzi Scheme

Transversality condition (no Ponzi Scheme)

$$\lim_{k \to \infty} E_t \left( \frac{B_{t+k}}{t+k-1} \prod_{s=0}^{t+k-1} (1 + i_{s-1}) \right) \geq 0, \quad \forall t.$$
Stochastic Discount Factor

1. from date $t$ to date $t + 1$

$$d_{t,t+1} := \frac{\beta U_C(C_{t+1}, L_{t+1})}{U_C(C_t, L_t)} \frac{P_{c,t}}{P_{c,t+1}}, \text{ i.e., } \frac{1}{1 + i_t} = \mathbb{E}_t (d_{t,t+1}).$$

2. from date $t$ to date $t + k$

$$d_{t,t+k} := \prod_{s=t}^{t+k-1} \Delta_s^{s+1}, \text{ then, } d_{t,t+k} := \frac{\beta^k U_C(C_{t+k}, L_{t+k})}{U_C(C_t, L_t)} \frac{P_{c,t}}{P_{c,t+k}}.$$
Cost Minimization

\[ mc_t(i) := \frac{W_t}{Q_t(i)} = \frac{r^k P_{i,t}}{Q_t(i)} = \frac{P_{e,t}}{Q_t(i)} \]

\[ cost(Q_t(i)) = (\alpha_e + \alpha_\ell + \alpha_k) F_t Q_t(i) \]

\[ F_t := \left( \frac{A \alpha_e}{r^k P_{i,t}} \right)^{-1} \]

\[ mc_t(i) = F_t Q_t(i)^{\frac{1}{\alpha_e + \alpha_\ell + \alpha_k}} - 1 \]
Price Optimization

**Price Maximization (at each date $t$)**

**Flexible Price Setting**

**Calvo Price Setting**

\[
\max_{P_{q,t}(i)} P_{q,t}(i)Q_{t}(i) - \text{cost}(Q_{t}(i))
\]

s.t

\[
\mu^p = \frac{\epsilon}{\epsilon - 1}
\]

\[
P_{q,t} = \mu^p mc_t
\]

\[
Q_{t}(i) = \left(\frac{P_{q,t}(i)}{P_{q,t}}\right)^{-\epsilon} Q_t
\]
Calvo Price Setting

\[ P_{q,t}(i) = P_{q,t-1}(i) \]

\[ P_{q,t}(i) = P_{q,t}^o(i) \]

\[ P_{q,t} = (\theta P_{q,t-1}^{1-\epsilon} + (1 - \theta)(P_{q,t}^o)^{1-\epsilon})^{\frac{1}{1-\epsilon}} \]
Calvo Price Setting

Calvo Price Setting Problem

\[
\max_{P_{q,t}(i)} \mathbb{E}_t \left[ \sum_{k=0}^{\infty} \theta^k d_{t,t+k} \left[ P_{q,t}(i) Q_{t,t+k}(i) - \text{cost}(Q_{t,t+k}(i)) \right] \right]
\]

s.t

\[
Q_{t,t+k}(i) = \left( \frac{P_{q,t}(i)}{P_{q,t+k}} \right)^{-\epsilon} Q_{t+k}, \quad \forall k \geq 0
\]
Calvo Price Setting

Calvo Price Setting Solution

\[ E_t \left[ \sum_{k=0}^{\infty} \theta^k d_{t,t+k} Q_o^{t,t+k} \left( P_o^{q,t} - \mu P m c_o^{t,t+k} \right) \right] = 0 \]

\[ m c_o^{t,t+k} := F_{t+k} \left( Q_o^{t,t+k} \right)^{\frac{1}{\alpha e + \alpha \ell + \alpha_k} - 1} \]

\[ Q_o^{t,t+k} = \left( \frac{P_o^{q,t}}{P_{q,t+k}} \right)^{-\epsilon} Q_{t+k} \]