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Preservation and Endogenous Uncertain Future Preferences

Alain Ayong Le Kama[1]

We extend the Beltratti, Chichilnisky and Heal's (1993) and (1998) continuous-time stochastic dynamic framework to analyze the optimal depletion of an environmental asset whose consumption is irreversible, in the face of an exogenous uncertainty about future preferences. We introduce an endogenous uncertainty about future preferences. The idea is that the ability of the future generations to change their preferences will depend on the state of the asset. More precisely, we assume that future generations may have a probability to change their preferences all the higher since the stock of the resource becomes low. We describe within this model more clearly the behavior of the central planner facing this type of uncertainty.

Keywords: preservation of natural resources, uncertainty, preferences.

JEL Classification Numbers: O4, Q2.

1. Université Paris-Ouest Nanterre La Défense, Chaire Economie du Climat
adayong@univ-paris1.fr

Climate Economics Chair
Paris-Dauphine University
CDC Climat

Preservation and Endogenous Uncertain Future Preferences

Alain Ayong Le Kama*

EQUIPPE, Universités de Lille

Address: Université de Lille I, Faculté de Sciences économiques et sociales
59655 - Villeneuve d'Ascq, France

(*E-mail* : adayong@univ-paris1.fr)

Summary. We extend the Beltratti, Chichilnisky and Heal's (1993) and (1998) continuous-time stochastic dynamic framework to analyze the optimal depletion of an environmental asset whose consumption is irreversible, in the face of an exogenous uncertainty about future preferences. We introduce an *endogenous* uncertainty about future preferences. The idea is that the ability of the future generations to change their preferences will depend on the state of the asset. More precisely, we assume that future generations may have a probability to change their preferences all the higher since the stock of the resource becomes low. We describe within this model more clearly the behavior of the central planner facing this type of uncertainty.

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1 Introduction

Following Dasgupta, P. and Heal (1974), who analyze optimal depletion of a fixed stock of an environmental good, Beltratti, Chichilnisky and Heal (1993) and (1998) introduce a dynamic optimization model in which the decision maker is uncertain about the preferences of future generations. They review within this model a question that has been raised often in connection with the conservation of natural resources. The question is the following. Does the possibility that future generations may have preferences that are significantly different from ours have implications for the conservation of natural resources. In particular, if there is some chance that they may value these resources more than we do, does this imply that we should be more conservative than otherwise in their depletion?

Beltratti, Chichilnisky and Heal (1993) and (1998) formalize this question in a continuous-time stochastic dynamic framework to analyze the optimal depletion of an asset whose consumption is irreversible, in the face of an *exogenous* uncertainty about future preferences. Consumption is the sole source of welfare. The utility of present generations is then described by a function $u(C)$. A change in preferences may take place in the future at an unknown date and it modifies in an unknown way the level of utility associated to any level of consumption. Ayong Le Kama (2001) extends this model. He shows that in some interesting cases it is possible to solve their model analytically. Nevertheless, all these papers, because they assume an exogenous probability of the change in preferences, find that the mere existence of uncertainty about future preferences does not provide a conservation motive: such a motive exists only if there is a "drift" in the stochastic process governing preference evolution, and this drift is towards a strengthening of the preference for natural resources[2].

Cunha-E-Sà and Costa-Duarte (2000) propose an extension of these models. In this extension, they no longer consider exogenous uncertainty, but they render it endogenous by assuming that the probability of switching instantaneous utility at some random date T depends on the state of the environment at this date. They motivated their extension as follows: "[...] depending on the state of the environment, as represented by the level of stock of the environmental asset at the time the change in preferences occurs [...], the economic agents may choose to be more or less conservative with respect to the depletion of the environmental asset. The larger the stock viewable at time T is, the smaller is the probability that the agents choose more conservative preferences and vice-versa". But their model is rather general and so the results are general qualitative theorems[3].

The aim of this paper is to simplify their model and just focus on some interesting cases where it is possible to offer a finer resolution of the model. The cases involve constant elasticity utility functions and the assumption of a Poisson process for the date at which

[2] Ayong Le Kama and Schubert (2004) address the same question in a growth model. They come out with the same result in the case of a separability between consumption and environmental quality in the utility function. But, things are less simple when preferences are non-separable: the attitude of the society now depends not only on the expectation of the change in preferences but also on the characteristics of the economy (impatience, intertemporal flexibility, natural capacities of regeneration of the environment, relative preference for the environment), on its history (initial level of the environmental quality) and on the date at which preferences are expected to change (near or far future).

[3] Furtheremore, Cunha-E-Sà and Costa-Duarte (2000) develop a more general general framework by considering some amenity valuation of the resource in addition to the standard utilitarian one.

the switch in preferences takes place. Both are widely used approximations and it is nice to see analytical solutions in these cases.

Therefore, following Cunha-E-Sà and Costa-Duarte (2000), we consider that: the ability of the future generations to change their preferences will depend on the state of the natural resource. More precisely, we assume that the probability of a change in the preferences of future generations, $q(.)$, is a function of the the level of the stock of the resource remaining at the time the change in preferences occurs, S.

It is important to notice that the choice of the sign of the marginal probability q' should be tied to some different economic scenario. If q' is assumed to be positive, this implies that the more we have used, the more is the economy geared towards enjoying consumption C, which indicates some sort of history dependence. This leads to a strand of the literature which studies the question of habit formation and introduces for example a utility discount factor depending on past consumption levels (Epstein (1987) and Obstfeld (1990))[4]. But this is not the claim of this paper.

Here, as Cunha-E-Sà and Costa-Duarte (2000), we assume a negative marginal probability of a change in the preferences of future generations. We consider that future generations may have a probability to change their preferences all the higher since the stock of the resource becomes low. For example, until, at least, the early 70s, nobody cared about the depletion of oil resources. But, since then we have worried about this depletion and our consumption has became more and more conservative. The change in preferences is therefore modeled as follow: the utility function may become $(1+\alpha)u(C)$ with the endogenous probability $q(S)$ and $(1-\beta)u(C)$ with probability $[1-q(S)]$.

We first study the case of preference uncertainty alone (*i.e.* we consider that there is only uncertainty about future preferences, the date at which this change occurs is given). We show that, contrary to the case studies in the previous papers, the definition of the *symmetric uncertainty,* which was the situation where the decision maker optimally ignores the uncertainty about future preferences and adopts the policy suggested by the certain problem, is no longer the same. In fact, in the previous papers, because they assumed an exogenous probability of a change in preferences, the symmetric uncertainty means that, although there is uncertainty about future preferences, on average we expect no change. But, with an endogenous probability of a change, as introduced in this paper, this result corresponds to a specific level of the probability of the change in preferences and then to a given level of the stock.

We can therefore deduce that the introduction of an endogenous probability of a change in preferences extends and clarifies the definition of a symmetric uncertainty, which is now given in marginal terms. That is: uncertainty about future preferences alone has no impact at all on optimal current consumption levels if the expected marginal value of the stock is equal to the marginal utility of consumption at the time the change in preferences occurs.

Moreover, introducing the two types of uncertainty (the date and the change of the level of the utility function), we also show that the results of the previous papers, from which the optimal solution only depends on the comparison between the expected marginal value of the initial stock with respect to the marginal utility of this stock, are only relevant

[4]This literature has been extend more recently to environmental concerns by Ayong Le Kama and Schubert (2007).

here in the short run. That is, in the short run, the central planner chooses to consume more than the deterministic case if at time zero the expected marginal value of the stock is smaller than the marginal utility of consumption.

With an endogenous uncertainty, the behavior of the central planner will also depend in the long run on the probability of a change in preferences for a level of the stock becoming very low. That is, if the probability of a change for a stock going towards zero is high enough the optimal choice is to consume more conservatively in the long run. More precisely, if the central planner considers that the future generations will value the stock more than we do, he will choose to be more conservative in its depletion.

Main results: combining these conditions, we show clearly that three different scenarios can occur in this overall model.

First, if the expected marginal value of the initial stock is low enough with respect to the marginal utility of the initial level of consumption and in the same time the probability of the change in preferences for a stock going towards zero is low, the optimal choice is to consume less conservatively (with respect to the model that ignores uncertainty) all along the time horizon, that is from the beginning to the time the change occurs. The level of consumption is scaled down when the change in preferences occurs.

Second, if now the expected marginal value of the initial stock is high with respect to the marginal utility of the initial level of consumption and also the probability of the change in preferences for a stock going towards zero is high enough, the central planner will choose to consume more conservatively all along the time horizon. The level of consumption rises when the change in preferences occurs.

Last, if the expected marginal value of the initial stock is low with respect to the marginal utility of the initial level of consumption but the probability of the change in preferences for a stock going towards zero is high, the central planner chooses in this case to consume less conservatively in the short run, but more in the long term. Thus, if the change in preferences takes place in the near future, the level of consumption is scaled down, but if it occurs later the consumption will scale up.

This last scenario gives a clearer picture of what is happening and of course open the way to some quantitative estimates. In fact, it shows that even if the present generations are not so conservative in the depletion of natural resources, they can optimally set conservatives objectives in the long run and decrease their consumption smoothly.

2 The model

As in Beltratti, Chichilnisky and Heal (1993 and 1998), Cunha-E-Sà and Costa-Duarte (2000) and Ayong Le Kama (2001) we consider an environmental good of which there is at time t a stock S_t. This good is consumed at a rate C_t, so that the rate of change of the stock is given by:

$$\frac{dS_t}{dt} \equiv \dot{S}_t = -C_t \tag{1}$$

At time zero society derives utility from the consumption of this good according to the function $U(C_t)$ which is assumed to have negative values, to be increasing, twice continuously differentiable and to possess the following properties.

Assumption 1: $-\infty < U(.) < 0$; $U' > 0$, $U'' < 0$; with $U^{'}(0) \rightarrow +\infty$.

2.1 The change in preferences

We also suppose that there is a possibility that at a random future date T, with marginal density w_t, the utility of consuming this good will change. The change in preferences is assumed to be a once-and-for-all phenomenon. So, the function $U(C_t)$ will become equal to $(1+\alpha)\,U(C_t)$ with an endogenous probability $q(S_t)$ depending on the level of the stock remaining at the time the change occurs, or to $(1-\beta)\,U(C_t)$ with probability $[1-q(S_t)]$, for $\alpha \geq 0$ and $0 \leq \beta \leq 1$. With the probability function $q(.)$ which satisfies the following.

Assumption 2: *(i)* $0 \leq q(S_t) \leq 1, \forall\, S_t \in [0, S_0]$; *(ii)* $q' < 0$, with $\lim\limits_{S\to 0} q'(S) = q'(0) > -\infty$; $q'' > 0$.

Given this, the overall planner problem is defined as follow[5]:

$$\mathcal{P} \quad \begin{cases} \max \displaystyle\int_0^\infty w_T \left[\int_0^T U(C_t)e^{-\delta t}dt + e^{-\delta T} EW(S_T)\right] dT \\ st. \left| \begin{array}{l} \dot{S}_t = -C_t \\ S_0 \text{ given ;} \quad S_t \geq 0, C_t \geq 0 \quad \forall t \end{array} \right. \end{cases} \tag{2}$$

where $\delta > 0$ is the discount factor and $EW(S_T)$ is the expected state valuation function, which values the stock S_T remaining at time T at which the change in preferences occurs.

$$EW(S_T) = q(S_T)\,(1+\alpha)\,W(S_T) + [1-q(S_T)]\,(1-\beta)\,W(S_T) = \Gamma(S_T)W(S_T) \tag{3}$$

with $\Gamma(S) = (\alpha+\beta)\,q(S) + 1 - \beta \geq 0$ and by assumption 2 *(i)* we also have $1-\beta \leq \Gamma(S) \leq 1+\alpha$. Besides, by assumption 2 *(ii)* we can deduce that: $\Gamma'(S) = (\alpha+\beta)\,q'(S) \leq 0$ and $\Gamma''(S) = (\alpha+\beta)\,q''(S) \geq 0$[6].

2.2 The deterministic solution

Let us first consider, as the benchmark, the solution of the case where there is no uncertainty. Therefore, the problem of the decision maker can be formulated as:

$$\mathcal{P}\,(1) \quad W(S_0) = \max \int_0^\infty U(C_t)e^{-\delta t}dt \quad st. \int_0^\infty C_t dt \leq S_0. \tag{4}$$

If we denote $\eta = \frac{CU''(C)}{U'(C)} < 0$ the elasticity of the marginal utility of the function $U\,(.)$ with respect to consumption, which we assumed to be constant, and p_t the shadow price of the stock S_t at time t, then the solution to the problem $\mathcal{P}\,(1)$ is (see Beltratti, Chichilnisky and Heal (1998) or Ayong Le Kama (2001)):

$$\frac{\dot{C}_t^{(1)}}{C_t^{(1)}} = g = \frac{\delta}{\eta} < 0 \tag{5}$$

Thus, we have, $C_t^{(1)} = C_0^{(1)} e^{gt} = -gS_0 e^{gt}$, $S_t^{(1)} = S_0 e^{gt}$ and $p_t^{(1)} = p_0 e^{\delta t} = U'(C_t^{(1)})$.

[5] The definition and interpretation of this problem are given in Beltratti, Chichilnisky and Heal (1993) and (1998).

[6] It is easy to see that if the probability of the change in preferences is exogenous, as in Beltratti, Chichilnisky and Heal (1993 and 1998) and Ayong Le Kama (2001): $q'(S) = 0 \Rightarrow \Gamma'(S) = \Gamma''(S) = 0$.

2.3 The model after the change in preferences

After preferences have changed, the problem is the same as the one of certainty $\mathcal{P}(1)$, the only difference is the initial level of the stock of the environmental good. The state valuation function of a given remaining stock S_T from time T onwards is therefore given as follow:

$$\mathcal{P}(2) \quad W(S_T) = \max \int_T^{\infty} U(C_t)e^{-\delta(t-T)}dt \quad st.\ \dot{S}_t = -C_t \qquad ;\ t \geq T \tag{6}$$

By analogy, with the solution of the problem $\mathcal{P}(1)$ above, we obtain: $\frac{\dot{C}_t^{(2)}}{C_t^{(2)}} = g$, $C_t^{(2)} = C_T^{(2)}e^{g(t-T)} = -gS_Te^{g(t-T)}$, $S_t^{(2)} = S_Te^{g(t-T)}$ and $p_t^{(2)} = p_T^{(2)}e^{\delta(t-T)} = U'(C_t^{(2)})$. We also know that the marginal valuation of the stock is equal to the marginal utility of consumption at time T at which the change in preferences takes place[7]:

$$\frac{dW(S_T)}{dS_T} = W'(S_T) = p_T^{(2)} = U'(C_T^{(2)}) \tag{7}$$

2.4 The model with preference uncertainty alone

Let us now consider the case where there is only uncertainty about future preferences. The time T at which the change in preferences occurs is given. The problem $\mathcal{P}$ becomes:

$$\mathcal{P}(3) \qquad \begin{cases} \max \int_0^T U(C_t)e^{-\delta t}dt + e^{-\delta T}EW(S_T) \\ st. \left| \begin{array}{l} \dot{S}_t = -C_t \\ S_0 \text{ given };\ S_t \geq 0, C_t \geq 0 \end{array} \right. \end{cases} \tag{8}$$

The necessary conditions of problem $\mathcal{P}(3)$ are the same as those of the one of certainty $\mathcal{P}(1)$, apart from the transversality condition which is:

$$P_T^{(3)} = \frac{\partial}{\partial S_T}EW(S_T) = \frac{\partial}{\partial S_T}\left[\Gamma(S_T)W(S_T)\right]$$

this condition[8] yields that the shadow price of the stock being equal to its expected marginal valuation at time T. We also know that $P_T^{(3)} = U'\left(C_T^{(3)}\right)$ and we obtained in the problem $\mathcal{P}(2)$ above, equation (7), that $W'(S_T) = U'(C_T^{(2)})$. The transversality condition becomes:

$$U'\left(C_T^{(3)}\right) = \frac{\partial}{\partial S_T}EW(S_T) = \Gamma(S_T)U'\left(C_T^{(2)}\right) + \underbrace{\underbrace{\Gamma'(S_T)}_{<0}\underbrace{W(S_T)}_{<0}}_{\textbf{endogenous effect} >0} \tag{9}$$

This relation shows that, contrary to the case with exogenous probability where there is a jump in the consumption path depending only if the parameter Γ is different to 1, there will be an additional effect: *"the endogenous probability effect"*, which increases the

[7] For the proof, see Beltratti, Chichilnisky and Heal (1998) proposition 6.

[8] This transversality condition is standard in a problem with stochastic scrap value.

expected marginal value of the stock. This result seems intuitive. In fact, because we assume that the probability of a change in preferences is a decreasing function of the stock, the depletion of this stock will increase its marginal valuation. We therefore can deduce the optimal consumption path:

$$\begin{cases} C_t^{(3)} = C_0^{(3)} e^{gt}, & t < T \quad ; \\ U'\left(C_T^{(3)}\right) = \Gamma\left(S_T\right) U'\left(C_T^{(2)}\right) + \Gamma'\left(S_T\right) W\left(S_T\right), & t = T \quad ; \\ C_t^{(2)} = C_T^{(2)} e^{g(t-T)}, & t > T. \end{cases} \tag{10}$$

2.4.1 Existence and uniqueness of the optimal path

The problem here is to find the necessary and sufficient conditions for the existence of a unique level of the stock S_T^* at the time the change in preferences occurs which satisfies the transversality condition (9).

First, by integrating the resource constraint between 0 and T, we obtain: $S_T - S_0 = -\int_0^T C_t^{(3)} dt = -C_0^{(3)} \int_0^T e^{gt} dt = \frac{C_0^{(3)}}{g}\left[1 - e^{gT}\right] \Longrightarrow C_0^{(3)} = \frac{(-g)(S_0 - S_T)}{(e^{-gT}-1)} e^{-gT}$. We therefore can deduce: $C_T^{(3)} = C_0^{(3)} e^{gT} = \frac{(-g)(S_0 - S_T)}{(e^{-gT}-1)}$ and we also know that $C_T^{(2)} = -gS_T$. Now, substituting these consumption levels into (9), we see that there will exist a unique solution of the problem $\mathcal{P}(3)$ if a unique value S_T^* exists such that $h\left(S_T^*\right) = f\left(S_T^*\right)$, with

$$\left| \begin{array}{l} h\left(S_T\right) = U'\left(C_T^{(3)}\right) = U'\left(\frac{(-g)(S_0 - S_T)}{(e^{-gT}-1)}\right) > 0 \\ f\left(S_T\right) = \frac{\partial}{\partial S_T} EW(S_T) = \Gamma\left(S_T\right) U'\left(-gS_T\right) + \Gamma'\left(S_T\right) W\left(S_T\right) > 0 \end{array} \right. \tag{11}$$

Proposition 1: *Under assumptions 1 and 2, there always exists a unique solution S_T^* of the problem $\mathcal{P}(3)$ with preference uncertainty alone.*

Proof. First, we have $h'\left(S_T\right) = \left(\frac{g}{(e^{-gT}-1)}\right) U''\left(\frac{(-g)(S_0 - S_T)}{(e^{-gT}-1)}\right) > 0$, function $h(.)$ is strictly increasing. In addition, the limits of function $h\left(S_T\right)$ are: $\lim_{S_T \to 0} h\left(S_T\right) = U'\left(\frac{(-g)S_0}{(e^{-gT}-1)}\right) > 0$ and $\lim_{S_T \to S_0} h\left(S_T\right) = U'(0) \to +\infty$. Thus, function $h\left(S_T\right)$ increases monotically from the positive value $U'\left(\frac{(-g)S_0}{(e^{-gT}-1)}\right)$ for $S_T = 0$ to $+\infty$ when $S_T = S_0$.

Besides, we know that $f(.) > 0$. It is easy to show that $f'\left(S_T\right) = 2\Gamma'\left(S_T\right) U'\left(-gS_T\right) - g\Gamma'\left(S_T\right) U''\left(-gS_T\right) + \Gamma''\left(S_T\right) W\left(S_T\right) < 0$, function $f(.)$ is strictly decreasing under assumptions 1 and 2. Thus, the limits of function $f\left(S_T\right)$ are: $\lim_{S_T \to 0} f\left(S_T\right) = f(0) = \Gamma(0) U'(0) + \Gamma'(0) W(0) \to +\infty$ and $\lim_{S_T \to S_0} f\left(S_T\right) = f\left(S_0\right) = \Gamma\left(S_0\right) U'\left(S_0\right) + \Gamma'\left(S_0\right) W\left(S_0\right) > 0$. Thus, function $f\left(S_T\right)$ is positive and decreases monotically from $+\infty$ for $S_T = 0$ to $f\left(S_0\right)$ when $S_T = S_0$. There exists a unique solution. ■

2.4.2 A comparison with the deterministic solution

The possibility of a change in the valuation of the environmental good in the future may give rise to an increase (or a decrease) of its shadow price depending on the level of the constraint on its availability. We therefore can compare the optimal levels of consumption of this problem with the corresponding levels in the case where the decision maker ignores uncertainty about future preferences and assumes that the utility function is never going to change. The calculus are given in appendix (see. *Appendix 1),* we just summarize the results here.

- If $U'\left(C_T^{(1)}\right) > \frac{\partial EW(S_T^*)}{\partial S_T}$, *i.e.* if the expected marginal value of the stock is lower than the marginal utility of consumption at the time the change in preferences occurs, then: $S_T^{(1)} > S_T^*$. The level of the stock is higher with the stochastic solution than the deterministic one. In addition, we have $C_0^{(1)} < C_0^{(3)}$, the optimal choice is to consume less conservatively at the beginning of the time horizon (with respect to the model that ignores uncertainty). Besides, we have $C_T^{(3)} > C_T^{(1)} > C_T^{(2)}$, the level of consumption is scaled down when the change in preferences occurs.

- Now, when $U'\left(C_T^{(1)}\right) < \frac{\partial EW(S_T^*)}{\partial S_T}$, *i.e.* the expected marginal value of the stock is higher than the marginal utility of consumption at time T, $S_T^{(1)} < S_T^*$. It is easy to show that in this case $C_0^{(1)} > C_0^{(3)}$, it is now appropriate to consume more conservatively at the beginning of the time horizon. In addition, we have $C_T^{(3)} < C_T^{(1)} < C_T^{(2)}$, and this makes the level of consumption rise when the change in preferences occurs.

- At the end, for $U'\left(C_T^{(1)}\right) = \frac{\partial EW(S_T^*)}{\partial S_T}$, then we have $C_0^{(1)} = C_0^{(3)}$ and $C_T^{(1)} = C_T^{(2)} = C_T^{(3)}$. We obtain the case that Beltratti, Chichilnisky and Heal (1993) and (1998) describe as *symmetric uncertainty,* in which the decision maker optimally ignores the uncertainty about future preferences and adopts the policy suggested by the certain problem.

  But the definition of this symmetric uncertainty is no longer the same. In the Beltratti, Chichilnisky and Heal model, because they assumed an exogenous probability of a change in preferences, the symmetric uncertainty means that, although there is uncertainty about future preferences, on average we expect no change. That is $EW(S_T) = W(S_T)$. With an endogenous probability of a change, as introduced in this paper, this result corresponds to a specific level of the probability and then to a given level of the stock: $EW(\overline{S}_T) = W(\overline{S}_T) \Rightarrow q\left(\overline{S}_T\right) = \frac{\beta}{\alpha+\beta}$. We can therefore deduce that the introduction of an endogenous probability of a change in preferences extends and clarifies the definition of a symmetric uncertainty, which is now in marginal terms. That is: *uncertainty about future preferences alone has no impact at all on optimal current consumption levels if the expected marginal value of the stock is equal to the marginal utility of consumption at the time the change in preferences occurs.*

2.5 The overall model

By integrating by parts the maximand in (2) the problem $\mathcal{P}$ can be reformulated as:

$$\mathcal{P}(4) \quad \begin{cases} \max \int_0^\infty e^{-\delta t}\left[\Omega_t U(C_t) + w_t EW(S_t)\right] dt \\ st. \left| \begin{array}{l} \dot{S}_t = -C_t \\ S_0 : \text{ given } ; S_t \geq 0, C_t \geq 0 \end{array} \right. \end{cases} \tag{12}$$

where $\Omega_t = \int_t^\infty w_\tau d\tau$. By analogy with the solution obtained by Beltratti, Chichilnisky and Heal (1998), it is easy to show that:

$$g^{(4)} = \frac{\dot{C}_t^{(4)}}{C_t^{(4)}} = g + \frac{w_t}{\eta \Omega_t}\left[1 - \frac{\partial EW(S_t)/\partial S_t}{U'\left(C_t^{(4)}\right)}\right]$$

If we assume that the distribution w_t is a Poisson distribution with parameter $\theta \geq 0$ so that $\frac{w_t}{\Omega_t} = \theta \; \forall t$ and if we recall the function f defined above (11), $f(S_t) = \partial EW(S_t)/\partial S_t$, this optimal growth rate becomes:

$$g^{(4)} = \frac{\dot{C}_t^{(4)}}{C_t^{(4)}} = g + \frac{\theta}{\eta}\left[1 - \frac{f(S_t)}{U'\left(C_t^{(4)}\right)}\right] \tag{13}$$

We introduce a stationary variable denoted $x_t = \frac{C_t^{(4)}}{S_t} \geq 0$, the complete dynamic system characterizing the evolution of this economy with endogenous uncertain preferences is then given by:

$$\begin{cases} \frac{\dot{x}}{x} = g + x + \frac{\theta}{\eta}\left[1 - \frac{f(S)}{U'(xS)}\right] \\ \frac{\dot{S}}{S} = -x \end{cases} \tag{14}$$

As we can see, this dynamic system can no longer be reduce to a single equation in x, as it is the case in Beltratti, Chichilnisky and Heal (1998) or Ayong Le Kama (2001) where the uncertainty is exogenous. Two types of equilibrium can therefore occur in this case:

- a stationary solution (with $\frac{\dot{x}}{x} = \frac{\dot{S}}{S} = 0$) at which the consumption of the flow is zero, $C^* = x^* = 0$, and the stock S^* is constant;
- or an asymptotical depletion, where we suppose that there exists a constant and negative depletion rate $\overline{g}$ such that $\lim_{t\to\infty} \frac{\dot{S}}{S} = \lim_{t\to\infty} \frac{\dot{C}}{C} = \overline{g}$ and $\lim_{t\to\infty} \frac{\dot{x}}{x} = 0$, thus $x_t = \frac{C_t^{(4)}}{S_t} = \overline{x}$, $\forall t$.

2.5.1 The stationary solution

Let us first look at the existence of a stationary state of the dynamic system (14). The aim here is to find if there exists an optimal solution where the planner can choose to stabilize the depletion of the stock before the change of preferences occurs.

A stationary solution (x^*, S^*) of this dynamic system is characterized by $\dot{x} = \dot{S} = 0$. This implies $x^* = 0$. Substituting this into the first equation of the system (14) and given that we assumed that $U'(0) \to +\infty$ (assumption 1), we easily see that stationary solutions will be such that $g = -\frac{\theta}{\eta} > 0$, which is impossible[9].

[9] As we can notice, even if the dynamic system (14) has two equations and two variables x and S, a stationary state with a constant and positive stock does not exist in the overall problem $P(4)$, before the change in preferences occurs.

2.5.2 The asymptotical depletion

We now consider the second feasible case, the asymptotical depletion of the stock.

Existence and convergence

Let us suppose that there exists a constant and negative depletion rate $\overline{g}$ such that $\lim\limits_{t\to\infty}\frac{\dot{S}}{S} = \lim\limits_{t\to\infty}\frac{\dot{C}}{C} = \overline{g}$. We then have $\lim\limits_{t\to\infty}\frac{\dot{x}}{x} = 0$. Besides, because $\overline{g} < 0$, we have $\lim\limits_{t\to\infty} S_t = 0$.

Proposition 2: *If* $\Gamma(0) \leq 1 + \frac{\delta}{\theta}$, *i.e.* $q(0) \leq \min\left(\frac{\beta+\frac{\delta}{\theta}}{\alpha+\beta}, 1\right)$,

(i) an asymptotical depletion of the resource occurs in the overall problem $\mathcal{P}(4)$, *before the change in preferences takes place, with*[10]*:*

$$\begin{cases} \overline{g} = g + \frac{\theta}{\eta}\left[1 - \Gamma(0)\right] < 0 \\ \overline{x} = -\overline{g} \end{cases} \tag{15}$$

(ii) moreover,

- if $1 \leq \Gamma(0) \leq 1 + \frac{\delta}{\theta}$, *i.e.* $\frac{\beta}{\alpha+\beta} \leq q(0) \leq \frac{\beta+\frac{\delta}{\theta}}{\alpha+\beta}$, *then:* $\overline{g} \geq g$. *The depletion is slower before the change in preferences occurs than after (or than in the deterministic case).*
- else if $\Gamma(0) < 1$, *i.e.* $0 \leq q(0) < \frac{\beta}{\alpha+\beta}$, *then:* $\overline{g} < g$.

Proof. When the stock of the resource decreases towards zero, the first equation of (14) becomes: $0 = g + \overline{x} + \frac{\theta}{\eta}\left[1 - \lim\limits_{S\to 0}\frac{f(S)}{U'(xS)}\right]$, with function $f(.)$ given in (11). We then have, under assumption 1, that: $\lim\limits_{S\to 0}\frac{f(S)}{U'(xS)} = \Gamma(0)$. The long term of the dynamic system (14) is therefore given by :

$$\begin{cases} 0 = g + \overline{x} + \frac{\theta}{\eta}\left[1 - \Gamma(0)\right] \\ \overline{x} = -\overline{g} \end{cases}$$

from which we deduce (15). Besides, by construction, this solution is valid if and only if $\overline{g} < 0$. Given that $g = \frac{\delta}{\eta}$, we have: $\overline{g} < 0 \Rightarrow \delta + \theta\left[1 - \Gamma(0)\right] > 0 \Rightarrow \Gamma(0) < 1 + \frac{\delta}{\theta}$.(this shows the first part of the proposition). Moreover, we have by (15) that: $\overline{g} \geq g \Rightarrow \frac{\theta}{\eta}\left[1 - \Gamma(0)\right] \geq 0 \Rightarrow \Gamma(0) \geq 1$ (knowing that $\eta < 0$). ■

Now, for the convergence we need the following assumption.

Assumption 3: $\frac{Sf'(S)}{f(S)} < \eta$

Knowing that[11]: $\frac{Sf'(S)}{f(S)} < \eta \Leftrightarrow \frac{\partial^2 EW(S)/\partial S^2}{\partial EW(S)/\partial S} < \eta = \frac{\partial^2 U(C)/\partial C^2}{\partial U(C)/\partial C}$, this assumption demands that the elasticity of the utility function is sensitive enough with respect to changes in the level of consumption than the one of the expected marginal value of the stock. Besides it is necessary to ensure the monotonicity of $x(.)|_{\dot{x}=0}$ as a function of the stock S, i.e. $\left.\frac{\partial x(S)}{\partial S}\right|_{\dot{x}=0} < 0$, along the locus $\dot{x} = 0$ (see below).

[10] With $\Gamma(S) = (\alpha + \beta)\, q(S) + 1 - \beta \geq 0$ defined above.

[11] Given that $f(S) = \frac{\partial}{\partial S} EW(S)$, we have $f'(S) = \frac{\partial^2}{\partial S^2} EW(S)$. Thus $\frac{Sf'(S)}{f(S)} = \frac{S\frac{\partial^2}{\partial S^2}EW(S)}{\frac{\partial}{\partial S}EW(S)}$.

Proposition 3: *Under assumptions 1–3, if* $\Gamma(0) \leq 1 + \frac{\delta}{\theta}$*, then along the path of asymptotical depletion of the environmental resource in the overall problem* $\mathcal{P}(4)$ *before the change in preferences occurs:*
(i) x is higher than its long run value, i.e. $x_t > \overline{x}$ $\forall t$;
(ii) the stock of the resource decreases more quickly than in the long run, i.e. $\dot{S}_t/S_t < \overline{g}$ $\forall t$.

Proof. Knowing (15), It is easy to show that the dynamic of x in (14) can be rewritten as :

$$\frac{\dot{x}}{x} = (x - \overline{x}) - \frac{\theta}{\eta}\left[\frac{f(S)}{U'(xS)} - \Gamma(0)\right]$$

Now given assumption 3, we can show that for any given x the function $\frac{f(S)}{U'(xS)}$ is decreasing in S. That is: $\frac{\partial\left(f(S)/U'(xS)\right)}{\partial S} = \frac{f'(S)U'(xS)-xf(S)U''(xS)}{\left[U'(xS)\right]^2} = \frac{1}{U'(xS)}\left[f'(S) - \eta\frac{f(S)}{S}\right] < 0$ (under assumption 3). We can deduce that $\frac{f(S)}{U'(xS)} \leq \lim_{S\to 0}\frac{f(S)}{U'(xS)} = \Gamma(0)$ for $x \geq 0$ and $S \in [0, S_0]$. We therefore can see that if $x \leq \overline{x}$, $\frac{\dot{x}}{x} < 0$ and x converges towards zero, which is impossible. We therefore deduce that $x_t > \overline{x}$, $\forall t$ (this shows the first part of the proposition). We then have $\frac{\dot{S}}{S} = -x = \overline{g} - (x - \overline{x}) < \overline{g}$ $\forall t$. ■

Propositions 2 and 3 indicate that when the probability of the change in preferences is low enough for a stock going towards zero, the optimal solution before the change in preferences occurs is an asymptotical depletion of the stock. Furthermore, the conservation motive is lower in the short run than in the long run. This implies that in the short run the stock decreases faster, which allows the society to consume more.

A comparison with the deterministic solution

Before comparing the optimal path which occurs before the change in preferences takes place with the one of the deterministic solution, let us first analyze the locus $\dot{x} = 0$ of the dynamic system (14).

We can show that the locus $\dot{x} = 0$ is a curve $x(S)$ which has the following properties.

Remark: By using the implicit functions theorem, we can easily show (see. *Appendix 2)* that $x(S)$ is positive and decreases monotically from $x(S_0)$ to $x(0)$ with the decreasing stock S under assumption 3, that is $\left.\frac{\partial x(S)}{\partial S}\right|_{\dot{x}=0} > 0$, along the locus $\dot{x} = 0$.

We want to compare function $x(S)$, along the locus $\dot{x} = 0$, and the line $x = -g$, the constant value of x in the deterministic case. Knowing by the remark above that $x(S)$ is monotonic, we only have to find the limits of $x(S)$ when S goes to zero or to its initial value $S = S_0$.

First, we know that when $S \to 0$ along the locus $\dot{x} = 0$, x will converge to its long run value $x(0) = \overline{x} = -\overline{g}$. Given the part (ii) of the proposition 2, we can deduce that:

a. if $\frac{\beta}{\alpha+\beta} \leq q(0) \leq \frac{\beta+\frac{\delta}{\theta}}{\alpha+\beta}$, then: $\overline{g} \geq g \Rightarrow \overline{x} \leq -g$;

b. else if $0 \leq q(0) < \frac{\beta}{\alpha+\beta}$, then: $\overline{g} < g \Rightarrow \overline{x} > -g$.

These conditions show that if the probability of a change for a stock going towards zero is high enough, that is if the central planner considers that the future generations will have a high valuation of the environmental good, the optimal choice is to consume more conservatively in the long run.

Now, for $S = S_0$, along the locus $\dot{x} = 0$ we know by (14) that: $x(S_0) = -g - \frac{\theta}{\eta}\left[1 - \frac{f(S_0)}{U'(x(S_0)S_0)}\right]$.We therefore have $x(S_0) > -g$ iff $f(S_0) < U'(x(S_0)S_0) \Leftrightarrow \frac{\partial EW(S_0)}{\partial S} < U'\left(C_0^{(4)}\right)$. This leads to the following conditions.

c. if $\frac{\partial EW(S_0)}{\partial S} < U'\left(C_0^{(4)}\right)$ then: $x(S_0) > -g$;

d. else if $\frac{\partial EW(S_0)}{\partial S} \geq U'\left(C_0^{(4)}\right)$, $x(S_0) \leq -g$.

Conditions c. and d. above show that the central planner will choose to consume more than in the deterministic case at the beginning of the time horizon only if he estimates that at time zero the expected marginal value of the stock is smaller than the marginal utility of consumption.

Combining these four conditions (a. to d.), we can now compare the optimal consumption paths of the overall problem $\mathcal{P}(4)$ with the deterministic one $\mathcal{P}(1)$. This leads to the following three[12] possible scenarios.

- If $\frac{\partial EW(S_0)}{\partial S} < U'\left(C_0^{(4)}\right)$ and $0 \leq q(0) < \frac{\beta}{\alpha+\beta}$, that is: $x(S_0) > -g$ and $\overline{x} > -g$. *i.e.* the expected marginal value of the initial stock is low enough with respect to the marginal utility of that level of consumption and also the probability of the change in preferences for a stock going towards zero is low. In this case the optimal choice is to consume less conservatively (with respect to the model that ignores uncertainty) all along the time horizon, that is from the beginning to the time the change occurs. The level of consumption is scaled down when the change in preferences occurs (see figure 1 below).

[12] It is impossible to combine conditions b. and d., given that $x(S)$ is a increasing function.

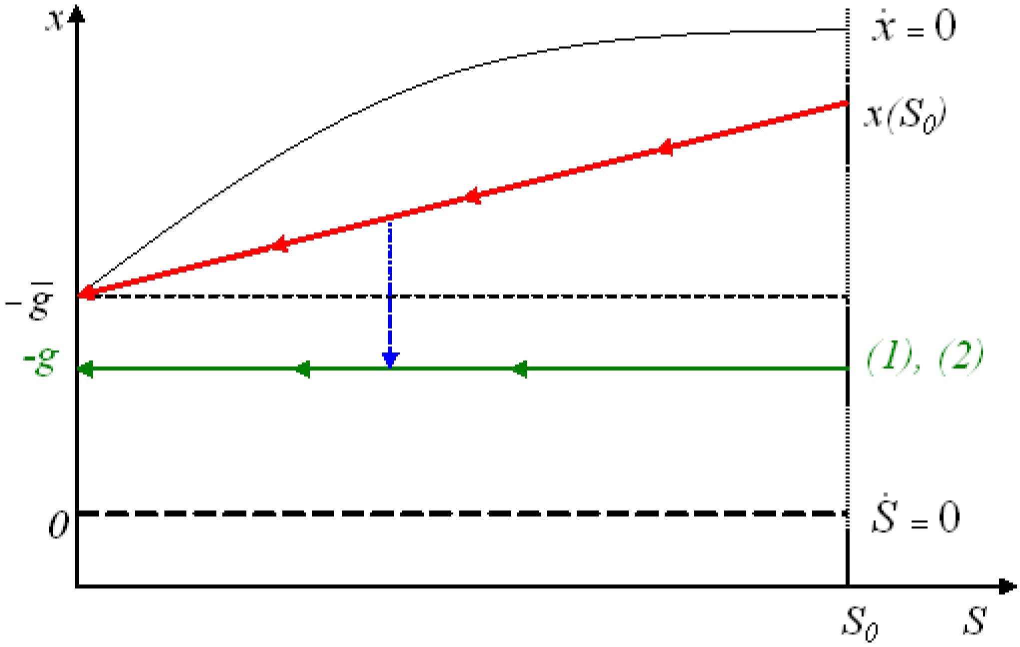

Figure 1: the consumption is less conservative

- If $\frac{\partial EW(S_0)}{\partial S} \geq U'\left(C_0^{(4)}\right)$ and $\frac{\beta}{\alpha+\beta} \leq q(0) \leq \frac{\beta+\frac{\delta}{\theta}}{\alpha+\beta}$, that is: $x(S_0) \leq -g$ and $\overline{x} \leq -g$. *i.e.* the expected marginal value of the initial stock is high with respect to the marginal utility of the initial level of consumption and also the probability of the change in preferences for a stock going towards zero is high enough. The central planner will choose to consume more conservatively all along the time horizon and the consumption will scale up when the change in preferences occurs (see figure 2 below).

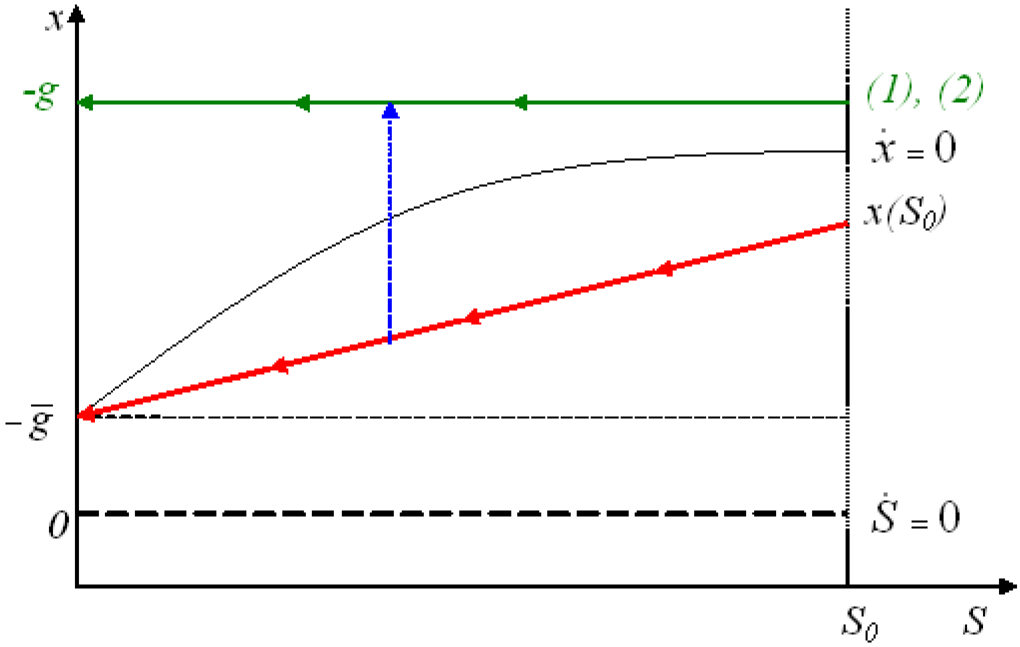

Figure 2: the consumption is more conservative

- If $\frac{\partial EW(S_0)}{\partial S} < U'\left(C_0^{(4)}\right)$ and $\frac{\beta}{\alpha+\beta} \leq q(0) \leq \frac{\beta+\frac{\delta}{\theta}}{\alpha+\beta}$, that is: $x(S_0) > -g$ and $\overline{x} \leq -g$. *i.e.* the expected marginal value of the initial stock is low with respect to the marginal utility of the initial level of consumption but the probability of the change in preferences for a stock going towards zero is high. The central planner chooses in this case to consume less conservatively at the beginning of the the time horizon, but more in the long term. Thus, if the change in preferences takes place very soon, the level of consumption is scaled down, but if it occurs later the consumption will scale up (see figure 3 below).

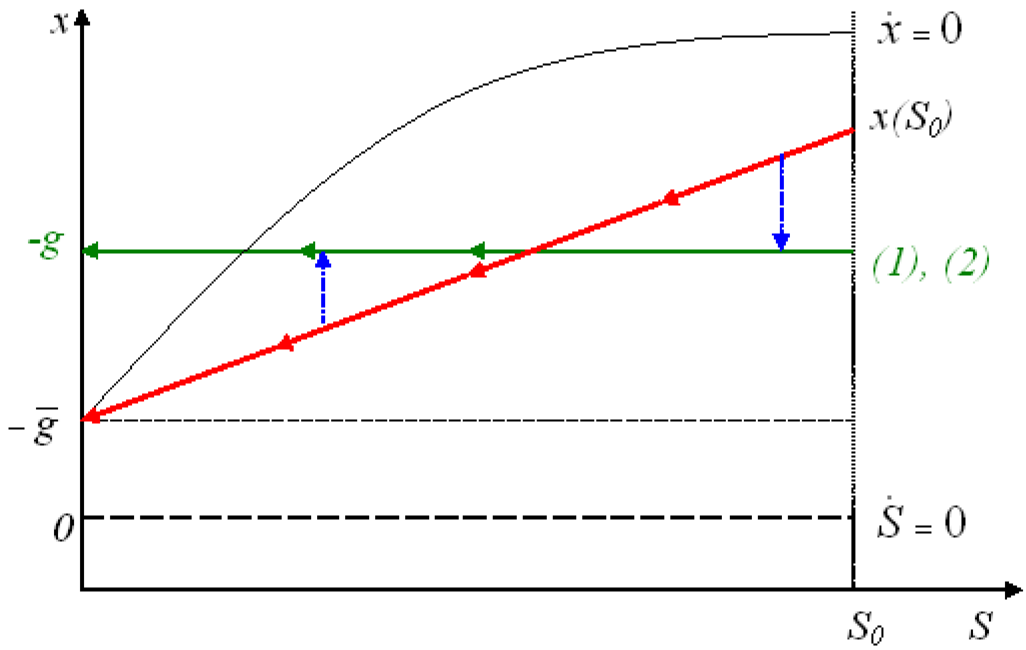

Figure 3: the consumption is less conservative at the beginning but more later

3 Concluding remarks

This paper introduces an endogenous uncertainty about the future preferences depending on the state of the environmental resource. We assume that the ability of the future generations to change their preferences will depend on the level of the remaining stock of the resource.

We analyze within this framework how the existence of an endogenous uncertainty about future preferences can provide a conservation motive, when we consider the two types of uncertainty (the date and the level of the change). We show that, contrary to the case studies in the previous papers where the optimal solution only depends on the comparison of the expected marginal value of the initial stock with respect to the marginal utility of this stock (this is only relevant here in the short run), here the central planner has to take into account its optimal choices in the short run and also in the long run. In the short run, he will choose to consume more than in the deterministic case if at time zero the expected marginal value of the stock is smaller than the marginal utility of consumption. In the long run, the behavior of the central planner will depend on the probability of a change in preferences for a level of the stock becoming very low. That is if the probability of a change for a stock going towards zero is high enough the optimal choice is to consume more conservatively in the long run.

These results show that even if the present generations are less conservative in the depletion of natural resources, the central planner can optimally set conservative objectives in the long run and chooses a path were the level of consumption decreases smoothly.

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Appendix

Appendix 1 **: Comparison of optimal consumption paths of problems $\mathcal{P}(1)$ and $\mathcal{P}(3)$** The shadow price of the stock at the time the change in preferences occurs is characterized in the problem $\mathcal{P}(1)$ by $P_T^{(1)} = U'\left(C_T^{(1)}\right)$. While in $\mathcal{P}(3)$ it is given by $P_T^{(3)} = U'\left(C_T^{*(3)}\right) = \left(h\left(S_T^*\right) = f\left(S_T^*\right)\right) = \frac{\partial}{\partial S_T} EW(S_T^*)$.

Case 1: if $U'\left(C_T^{(1)}\right) > \frac{\partial}{\partial S_T} EW(S_T^*)$, *i.e.* if $P_T^{(1)} > P_T^{(3)}$, then:

$$U'\left(C_T^{(1)}\right) > U'\left(C_T^{*(3)}\right) \Rightarrow U'\left((-g)\, S_0 e^{gT}\right) > U'\left(\frac{(-g)\left(S_0 - S_T^*\right)}{\left(e^{-gT}-1\right)}\right).$$

Because by assumption 1 $U'' < 0$, we obtain: $(-g)\, S_0 e^{gT} < \frac{(-g)\left(S_0 - S_T^*\right)}{\left(e^{-gT}-1\right)} \Rightarrow S_0 e^{gT} > S_T^* \Rightarrow S_T^{(1)} > S_T^*$.

We can therefore deduce that: $(-g)\, S_0 e^{gT} > (-g)\, S_T^* \Rightarrow C_T^{(1)} > C_T^{(2)}$.

And also that: $S_T^* < S_0 e^{gT} \Rightarrow S_T^* < S_0 \left(\frac{e^{-gT} - e^{-gT} + 1}{e^{-gT}}\right) = S_0 \left(1 - \frac{e^{-gT}-1}{e^{-gT}}\right) \Rightarrow S_0 - S_T^* > S_0 \left(\frac{e^{-gT}-1}{e^{-gT}}\right) \Rightarrow \frac{(-g)\left(S_0 - S_T^*\right)e^{-gT}}{e^{-gT}-1} > (-g)\, S_0 \Rightarrow C_0^{(3)} > C_0^{(1)}$

We then have: $C_T^{(3)} = C_0^{(3)} e^{gT} > C_T^{(1)} = C_0^{(1)} e^{gT} > C_T^{(2)}$.

Case 2: if $U'\left(C_T^{(1)}\right) < \frac{\partial}{\partial S_T} EW(S_T^*)$, *i.e.* if $P_T^{(1)} < P_T^{(3)}$, then we have the opposite of the previous case. We therefore can deduce that: $S_T^{(1)} < S_T^*$ and $C_T^{(3)} < C_T^{(1)} < C_T^{(2)}$.

Case 3: if $U'\left(C_T^{(1)}\right) = \frac{\partial}{\partial S_T} EW(S_T^*)$, *i.e.* if $P_T^{(1)} = P_T^{(3)}$, then: $S_T^{(1)} = S_T^*$ and $C_T^{(3)} = C_T^{(1)} = C_T^{(2)}$.

Appendix 2 : The study of the curve $x\,(S)$ along the locus $\dot{x} = 0$

Along the locus $\dot{x} = 0$, the first equation of the dynamic system (14) can be written as a function $V\,(S,x)$, for any given (S,x), such that:

$$V\,(S,x) = g + x + \frac{\theta}{\eta}\left[1 - \frac{f\,(S)}{U'\,(xS)}\right] = 0$$

Besides, we have:

$$\frac{\partial V\,(S,x)}{\partial x} = 1 + \frac{\theta}{\eta}\left[\frac{Sf\,(S)\,U''\,(xS)}{\left[U'\,(xS)\right]^2}\right] > 1$$

That is for any given (S,x), $V\,(S,x) = 0$ and $\frac{\partial V(S,x)}{\partial x} \neq 0$, thus by the implicit functions theorem we can write x as a function of S, $x\,(S)$, with: $x'\,(S) = \frac{\partial x(S)}{\partial S} = \frac{-\frac{\partial V(S,x)}{\partial S}}{\frac{\partial V(S,x)}{\partial x}}$. Knowing that: $\frac{\partial V(S,x)}{\partial S} = -\frac{\theta}{\eta}\left[\frac{f'(S)U'(xS) - xf(S)U''(xS)}{\left[U'(xS)\right]^2}\right]$, we can deduce:

$$\begin{aligned}
x'\,(S) &= \frac{\theta}{\eta}\left[\frac{f'\,(S)\,U'\,(xS) - xf\,(S)\,U''\,(xS)}{\left[U'\,(xS)\right]^2 + \frac{\theta}{\eta}Sf\,(S)\,U''\,(xS)}\right] \\
&= \frac{\theta}{\eta}\left[\frac{f'\,(S) - f\,(S)\,\frac{xU''(xS)}{U'(xS)}}{U'\,(xS) + \frac{\theta}{\eta}f\,(S)\,\frac{SU''(xS)}{U'(xS)}}\right] \\
&= \frac{\theta}{\eta}\left[\frac{f'\,(S) - f\,(S)\,\frac{\eta}{S}}{U'\,(xS) + \frac{\theta}{\eta}f\,(S)\,\frac{\eta}{x}}\right] \\
&= \frac{\theta}{\eta}\left[\frac{f'\,(S) - \frac{\eta f(S)}{S}}{U'\,(xS) + \frac{\theta f(S)}{x}}\right]
\end{aligned}$$

thus, given that $U^{'}(xS) > 0$ under assumption 1 and $f(S) > 0$ (see. equation (11)), we know that $U^{'}(xS) + \frac{\theta f(S)}{x} > 0$. The sign of $x'(S)$ will depend on the one of: $\left(f'(S) - \frac{\eta f(S)}{S}\right)$, which is negative under assumption 3. We therefore obtain that $x'(S) > 0$ along the locus $\dot{x} = 0$.

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