Les Cahiers de la Chaire Economie du Climat

Working Paper Series

n° 2013-04

Modeling of Emission Allowance Markets: A Literature Review

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This paper reviews the development of emission trading models from the earliest to recent contributions. First, we introduce the economics of pollution control and the origins of emission trading. We give a brief description of policy instruments for the control of pollution, and explain why economic instruments (Pigouvian tax and emission trading) produce better results than "command-and-control" approaches. Second, we review several papers on modeling of emission trading systems, with a focus on dynamic models in case of perfect competition. We begin with the earliest static models, investigating a number of factor that can affect the effectiveness of emission trading (e.g. market power, transaction-costs, political pressures, etc). Next, we present dynamic models of permit markets, analysing questions such as banking/borrowing, relationship between spot and future markets, exogenous factors influencing the marginal abatement cost, etc. Finally, we end the paper with recent studies that model the main features of the European Emission Trading Scheme (EU ETS) in a dynamic framework with stochastic emissions.

Keywords: Emission Trading, EU ETS, Partial Equilibrium Modeling

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Abstract

This paper reviews the development of emission trading models from the earliest to recent contributions. First, we introduce the economics of pollution control and the origins of emission trading. We give a brief description of policy instruments for the control of pollution, and explain why economic instruments (Pigouvian tax and emission trading) produce better results than "command-and-control" approaches. Second, we review several papers on modeling of emission trading systems, with a focus on dynamic models in case of perfect competition. We begin with the earliest static models, investigating a number of factor that can affect the effectiveness of emission trading (e.g. market power, transaction-costs, political pressures, etc). Next, we present dynamic models of permit markets, analysing questions such as banking/borrowing, relationship between spot and future markets, exogenous factors influencing the marginal abatement cost, etc. Finally, we end the paper with recent studies that model the main features of the European Emission Trading Scheme (EU ETS) in a dynamic framework with stochastic emissions.

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1. Introduction

The problem of excessive pollution had long been a part of economic theory. It is considered as the consequence of absence of price on emissions, which results in higher volumes than socially optimal levels, and associated inefficient outcomes. From an economic point of view, the solution consists in putting a price on pollution in order to provide a signal for polluters about the social value of their private decisions.

Among the policy measures that help tackle this problem, emission trading systems are very popular, and are often considered as the best instrument. They have been widely applied to different kind of environmental issues in the last decades, and, in particular, to greenhouse gas (GHG) emissions since greenhouse effect is a global process, and thus local differences in air concentrations do not matter.[1] This mechanism offers a number of advantages which are attractive for business and policymakers. Among them, it gives the environmental authority a direct control on the overall quantity of emissions. It also increases the acceptability of environmental policy for covered companies, due to flexibility and possibility to make profit from it.

The interest for emission trading has produced an extensive literature. Some previous work have provided useful synthesis of many contributions to this literature. Notably, papers of Springer (2003) and Taschini (2010) give a wide overview of environmental economics and permit market studies, including both theoretical and empirical research. While very instructive, the large scope of these papers make them sometimes very general on some specific points. Other papers have adopted a different approach, focusing on precise questions about emission trading. Among them, Montero (2009) reviews studies on market power in permit markets. One can also mention the survey of Chevallier (2012), about banking and borrowing in the European Emission Trading Scheme (EU ETS).

Compared with these reviews, we focus on contributions dealing with dynamic partial equilibrium modeling, in case of perfect competition. The aim of this paper is to provide basic knowledge and the main results from this literature. After a brief overview of the economics of pollution control and the origins of emission trading, we begin with a review of the earliest static models of permit markets. Those papers reviewed investigate a number of factors that can affect the effectiveness of emission trading (e.g. market power, transaction-costs, political pressures, etc). Next, we present dynamic models of permit markets, analysing questions such as banking/borrowing, relationship between spot and future markets, exogenous factors influencing

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[1] See Bertrand (2012a) for a review of emission trading applications for different kind of environmental problems.

the marginal abatement cost, etc. Finally, we end the paper with recent studies that model the main features of the EU ETS in a dynamic framework with stochastic emissions.

2. The economics of pollution control and emission trading

An externality exists when an agent takes decisions that affect other agents' well-being, without this being accounted for in a market price. Accordingly, producers of externalities do not have any incentives to take into account the effects of their decisions on others. Pollution is generally considered as a negative externality. A negative externality causes divergence between social and private costs. The private cost of polluting activities is under-estimated with respect to the social cost, since it neglects the "external" cost of damages created by pollution. As a result, the chosen level of pollution is higher than the socially optimal level (i.e. the level which equalizes the social marginal cost to the social marginal benefit of pollution).

As a result of a negative externality, excessive pollution leads to a market failure and inefficient outcomes. For economists, the solution consists in putting a price on pollution in order to "internalize" the cost of pollution in private decisions. Basically, there are two categories of economic instruments to internalize pollution: Pigouvian tax and emissions trading system ("cap-and-trade"). Both have been advocated by economists because they minimize the overall cost of environmental regulation compared to rigid "command-and-control" approaches. Command-and-control regulations generally apply uniform emissions limits on regulated firms, regardless of firms' efficiency to reduce emissions. By contrast, with economic instruments, individual firms are free to choose how much they will reduce their emissions by comparing their abatement costs with the price of pollution. As a consequence, firms with lower costs make higher share of the overall effort of emissions reduction, and vice versa. This leads to the "least-cost solution" in which each firm equalizes its marginal abatement cost to the price of pollution.

Pigouvian tax was introduced by Pigou (1920) as a way to restore market efficiency in presence of negative externalities. In his famous example, Pigou explains that social benefits of railway services in the England of the 19th century was over-estimated due to negligence of damages caused by sparks from engines. To correct the negative externality, Pigou proposed to place a tax on railway companies varying with the amount of smoke produced and equivalent to the monetary value of the externality. Hence, by making companies financially liable for the damages created by sparks, the Pigouvian tax gives an incentive to reduce the output to the socially optimal level.

The concept of the emission trading was introduced by Dales (1968),[2] based on the Coase solution. Coase (1960) proposed a solution that consists in establishing property rights on emission of externalities. If transaction costs are negligible, Coase shows that parties – “disrupters” and “victims” – can achieve a socially optimal level of externality by bargaining, regardless of who initially received the property rights. The socially optimal level of externality is attained when the marginal benefit of the externality (i.e. profits arising from the activity which generates the externality) is equal to the marginal cost of the externality.[3] Moreover, a market price emerges for the externality. Based on the Coasian approach, market-based instruments (MBIs) have been popularized as an efficient way to reduce pollution. They work with a central authority which sets a cap on the total amount of pollutant that can be emitted. The cap is converted into allowances that give the right to emit a certain amount of pollutant. Allowances are allocated to polluters, and they can be traded on a secondary market. A market price emerges and buyers pay that price to increase their emissions, while sellers can earn money by selling unused allowances. Thus, polluters with low abatement costs have an incentive to reduce their emissions by more than needed, and those with high abatement costs can buy more allowances rather than engage in costly emission reductions. Accordingly, MBIs theoretically achieve emission reduction targets at the lowest cost to society. Such a “least-cost” solution implies equalization of marginal cost of abatement among polluters. Montgomery (1972) formalized this result and showed that it is verified in the equilibrium of the market for allowances, under certain conditions (competitive market, no transaction-costs, etc).

3. Modeling of permit markets

3.1. Modeling of permit markets in a static framework

Numerous theoretical studies on modeling of emission allowance markets have developed since the pioneering work of Montgomery (1972). Montgomery proves that in a competitive permit market with perfect information and no transaction costs, an efficient equilibrium exists. This equilibrium achieves any emission reduction target at the lowest cost for society (i.e. at the least total cost over

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[2] First references to emission trading can be found in Crocker (1966).

[3] In his 1960 paper, Coase argued that the traditional Pigouvian approach may lead to results which are not necessarily the true social optimum, because it neglects the “reciprocal nature” of externalities: inducing disrupters to reduce harm on victims also inflicts harm on disrupters. He proposed his solution as a way to overcome this problem.

all firms) and is independent of initial allocation of allowances. This "least-cost" solution implies equalization of marginal cost of abatement among polluters. Hence, the price of allowances must always be equal to marginal abatement costs in market equilibrium: $C_i'(a_i)=p$, $\forall\ i$, where $C_i'(a_i)$ is the marginal abatement cost of firm *i* associated with abatement effort a_i , and p is the price of allowances. This statement underpins that emission trading induces firms to exploit any differences between the price of allowances and their marginal costs of abatement. On the one hand, firms with lower abatement costs can make profits by abating more CO_2 than they would need to comply with the regulation. This allows them to sell unused allowances at a higher price than their marginal abatement costs. On the other hand, firms with higher abatement costs can reduce their compliance costs by abating less CO_2 than they would need to comply with the regulation, and then buying the lacking allowances on the market at a lower price than their marginal abatement costs.

Montgomery (1972) also investigates the case of ambient permit markets, i.e. permit markets for pollutants with non-uniform assimilation rates among different affected regions (see also Atkinson (1983) and Tietenberg (1985)). In this case the location of pollution sources is crucial because a same volume of emissions does not produce the same effect in all locations. Thus, a target has to be specified for each specific location in terms of a ceiling on concentration of the pollutant at this specific region. This is equivalent to say that there are as much permit markets as the number of locations affected by pollution. So, an equilibrium on permit markets exists and leads to the least-cost solution which implies that each firm equates its marginal abatement cost with a weighted sum of prices of permits at each location: $C_i'(a_i)=\sum_j h_{ij} p_j$, $\forall\ i$, where h_{ij} are the transfer coefficients associated with each affected location[4] and p_j is the price of allowances in location *j*.

Based on static models similar to the one introduced by Montgomery (1972), many papers have investigated a number of factors that can affect the market equilibrium or even prevent permit market from achieving efficiency. Among important issues, the question of market power in the permit market has been addressed by Hahn (1984). He shows that the market equilibrium can deviate from the least-cost solution obtained by Montgomery (1972) in a competitive market. Moreover, the author identifies that the degree of inefficiency observed in the market is related to the initial distribution of permits, whereas Montgomery found that optimality is independent of initial allocations in the perfect case. Hahn demonstrates that optimality can be restored by distributing to firms with dominant position a number of permits exactly equal to what they need to cover their emissions (which necessitates to know cost functions, and may be very costly). By contrast, there is no restriction about initial allocations for firms in the competitive fringe. Stavins

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[4] The coefficients h_{ij} translate emission increases by firm *i* into changes in the concentration at location *j*.

(1995) has investigated the presence of transaction costs in the permit market. The author shows that significant transaction costs reduce the trading volume. As a consequence, the market equilibrium can deviate from the optimum and is sensitive to initial distributions of permits. Note that this result is consistent with the Coase theorem which states that the optimum is achieved regardless of who initially received the permits, if there is no transaction costs. Conrad and Kohn (1996) have provided a formal treatment of factors that explained the low price of SO_2 permits in the early years of the US Acid Rain Program. They show that the price was lower than expected because of excess allowances resulting from political decisions. These surpluses were explained by the creation and distribution of more permits than were initially authorized – due to political pressures – and more stringent air quality standards in some areas (e.g. near national parks) preventing high cost abaters in those areas from buying more permits in order to increase their emissions. Maeda (2004) was the first who formally includes random GHG emissions in a one-period equilibrium model. He pointed out that GHG emissions – and especially carbon emissions – are closely related to energy use which, in turn, is closely related to random factors such as economic activity and weather conditions. He assumed a single random variable reflecting macro-factors that affect emissions. Emissions from various firms are all correlated with this random variable. It can be a GDP of one or more countries, an industrial production index, temperatures, rainfall, etc. In addition to this "single factor", Maeda (2004) models firm-specific random variables reflecting uncertainties that have no correlation to each other. Maeda found that uncertainties about the price of allowances depends entirely on uncertainties about emissions. More importantly, he showed that uncertainties that are specific to each firm are diversified and disappear when there is a large number of firms in the market. This indicates that, for a large number of emission sources, the probabilistic nature of the price of allowances would only depend on a "single factor" to which emissions of all firms are correlated. Accordingly, random macro-factors such as economic activity and weather conditions are of major importance to explain fluctuations of the permit price in a multi-period setting.

3.2. Modeling of permit markets in a dynamic framework

In the wake of papers dealing with modeling of permit markets in a static framework, several authors have developed multi-period models to study the theoretical properties of inter-temporal trading. The first contributions on this topic are those of Tietenberg (1985) and Cronshaw and Kruse (1996). Both consider a dynamic equilibrium model with banking, in discrete-time and non-stochastic environments (i.e. without introducing uncertainty in emissions). Tietenberg (1985) characterizes the joint least-cost allocation of abatement efforts, given a single constraint on the total amount of emissions over time.[5] He also states that a decentralized permit market can yield the same least-cost allocation. In this case, the permit price must rise at the rate of interest. Tietenberg assumes that all permits are issued at the beginning of first period, so that some permits will always be in the bank. By contrast, Cronshaw and Kruse (1996) consider that permits are allocated to firms in each of *T* periods. Additionally, they investigate the effect of profit regulation on the firms' behavior. They find that the permit market achieves the least-cost solution if there is no profit regulation, but may not do so if firms are subject to profit regulation in their output market. Cronshaw and Kruse also show that, without profit regulation, firms are willing to bank permits if the permit price rises over time with the rate of interest. However, firms do not desire to bank if the price rises by less than the rate of interest.

Rubin (1996) extends the work of Tietenberg (1985) and Cronshaw and Kruse (1996) by providing a more general treatment of inter-temporal trading in continuous time through the use of optimal-control theory. Instead of limiting inter-temporal trading to banking, Rubin allows both borrowing and banking. He analyses the case of a central planner minimizing the inter-temporal joint-cost of reducing pollution of *N* heterogeneous firms subject to emission constraints. He considers a finite time horizon with deterministic emissions. As a special case, Rubin also investigates the consequences of restrictions on borrowing. While the constraint on borrowing is not explicitly taken into account in optimization, some insight into the effect of the inability to borrow are derived. Rubin (1996) defines $S_{i,t}$, the endowment of permits received by a firm *i*, $-C_i(e_{i,t})$, the abatement cost function (increasing and convex) associated with the chosen level of emissions $e_{i,t}$,[6] and $B_{i,t}$, the number of permits that are in the bank. Thus, $e_{i,t}$ is a control variable, while $B_{i,t}$ is a state variable. Finally, defining $B=\sum_{i=1}^{N} B_{i,t}$ as the aggregate stock of banked permits, $\dot{B}$ as the rate of change of B (where dots denote time derivatives), and *T* as the terminal time period,

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[5] Tietenberg (1985) considers a single constraint on the total amount of emissions over the *T* time periods, and all permits are issued at the beginning of first period. Therefore, firms can freely transfer permits across time periods. In other words, both banking and borrowing are allowed.

[6] Following Montgomery (1972), Rubin defines $C_i(e_{i,t})$ as the difference between unconstrained profits and profits under the cap-and-trade regime. This difference is equal to $C_i(e_{i,t})+P_t y_{i,t}$ when trading is allowed (see problem **(2)** below). However, he does not explicitly define abatements ($a_{i,t}$) and "business-as-usual" emissions ($u_{i,t}$), even though they are implicitly taken into account, since $e_{i,t}=u_{i,t}-a_{i,t}$ with $e_{i,t}\leq u_{i,t}$. Accordingly, the optimization problem is solved by minimizing $C_i(e_{i,t})$, i.e. by lowering emissions $e_{i,t}$ so as to minimize the difference between constrained and unconstrained profits, with $C_i'(e_{i,t})<0$ and $C_i''(e_{i,t})>0$. Equivalently, the problem could be solved by minimizing $C_i(a_{i,t})$, where $C_i(a_{i,t})$ is an abatement cost function. In this case, the action of minimizing the difference between constrained and unconstrained profits would be controlled by choosing an abatement effort, $a_{i,t}$, with $C_i'(a_{i,t})>0$ and $C_i''(a_{i,t})>0$. As pointed out by Rubin, using the cost function $C_i(e_{i,t})$, the abatement cost is defined by $-C_i(e_{i,t})$, and therefore the marginal abatement cost is $-C_i'(e_{i,t})$.

the joint-cost problem of a central planner can be written as:

$$\begin{cases} \underset{e_{i,t}}{\min} \quad \int_0^T \mathrm{e}^{-rt} \sum_{i=1}^{N} C_i(e_{i,t})\,dt \\ \text{s.t} \quad \dot{B} = \sum_{i=1}^{N} \left(S_{i,t} - e_{i,t}\right) \\ B_0 = 0\,,\ B_T \geq 0\,, \\ e_{i,t} \geq 0\,,\ \forall\, i\,, \end{cases} \qquad \textbf{(1)}$$

where *r* is a risk-free rate of interest. Solving the problem yields necessary conditions that indicate that the regulator allocates abatement efforts so that all firms have equal present discounted marginal abatement costs: $-e^{-rt}C'_1(e_{1,t}) = -e^{-rt}C'_2(e_{2,t}) = \cdots = -e^{-rt}C'_N(e_{N,t})$. Besides, all firms have present discounted marginal abatement costs equal to the costate variable on the state equation $\dot{B} = \sum_{i=1}^{T}\left(S_{i,t} - e_{i,t}\right)$, which reflects the shadow value of an additional unit of banked emissions. Thus, the abatement effort of each firm is increased as long as the cost of one more unit of abatement is lower than its value in the bank. Finally, results show that if, in total, permits are banked and borrowed over time, then the discounted marginal abatement cost is constant in time. In this case, the marginal abatement cost rises over time with the rate of interest. By contrast, if firms, in total, would like to borrow but are not allowed to do so, the discounted marginal abatement cost would be decreasing in time.[7] In this case, the rate of growth in the marginal abatement cost must be less than the interest rate.

Next, Rubin (1996) explores the consequences of introducing emission trading in the model, with price taking firms. Formally, letting P_t be the instantaneous price of permits $y_{i,t}$ purchased or sold by a firm *i* at period *t* (where $y_{i,t} > 0$ if permits are bought, and $y_{i,t} < 0$ if permits are sold), and $A_{i,t}$ and $D_{i,t}$ be bounds on $y_{i,t}$,[8] the problem of a firm *i* can be characterized. Thus, the joint-cost problem **(1)** is modified as follows:

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[7] Here, Rubin assumes a central planner that would like to borrow but which is not allowed to do so. So, the author looks at the impact of an "ex-post" constraint $B_t \geq 0$ (i.e. that is not explicitly taken into account in optimization) on necessary conditions.

[8] A firm *i* cannot buy (sell, respectively) more than $D_{i,t}$ ($A_{i,t}$, respectively) permits at any period *t*. Assuming these bounds is a technical requirement, as explained below.

$$\begin{cases} \min\limits_{y_{i,t},\, e_{i,t}} \quad \int_0^T \mathrm{e}^{-rt}\left[C_i(e_{i,t})+P_t y_{i,t}\right]dt \\ \text{s.t} \quad \dot{B}_i = S_{i,t} - e_{i,t} + y_{i,t} \\ B_{i,0}=0\ ,\ B_{i,T}\geq 0 \\ e_{i,t}\geq 0\ , \\ -A_{i,t}\leq y_{i,t}\leq D_{i,t}\ ,\ A_{i,t}>0\ ,\ D_{i,t}>0\ . \end{cases} \qquad \textbf{(2)}$$

The last constraint provides bounds ($A_{i,t}$ and $D_{i,t}$) on the maximum number of permits that can be instantaneously bought and sold by a firm *i*. This is a necessary technical requirement to avoid corner solutions, since the objective function is linear in $y_{i,t}$ (see also Cronshaw and Kruse (1996) and Kling and Rubin (1997)). As pointed out by Rubin (1996), rather than explicitly taking into account this constraint in the resolution, an alternative approach is to consider a price path for which an internal solution exists. This is equivalent to assuming a non-bounded solution over the entire time horizon, so that the firms have an internal solution in each period. In this case, the permit price must rise at the rate of interest.

Solving problem **(2)**, Rubin shows that an inter-temporal market equilibrium exists, and that, in equilibrium, each firm equates the marginal cost of pollution abatement with the price of permits. Thus, when allowed to trade with one another, firms collectively behave like a central planner who efficiently allocates emission permits to each firm so as to minimize the overall compliance cost. In other words, a decentralized equilibrium solution exists, and it is efficient in the sense of achieving the least-cost solution attained by a central planner. Moreover, as for the joint-cost problem, all firms have present discounted marginal abatement costs equal to the marginal value of an additional permit in the bank. Finally, Rubin shows that, on the one hand, for each firm to have a non-bounded solution, the permit prices must grow at the rate of interest when firms can bank and borrow permits. In this case the present-value price of permits must be constant in time. On the other hand, if firms face a constraint on borrowing ($B_{i,t}\geq 0$, which is not explicitly taken into account in optimization, but the author investigates the impact of this constraint on necessary conditions), the rate of growth in prices must be less than the interest rate. In this case, the present-value price of permits is decreasing through time.[9] Note that this required price path has the same shape as the rate

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[9] This results on the required price path for an internal solution is close to the one obtained by Cronshaw and Kruse

of growth in the marginal abatement cost in the case of a central planner.

Using the same deterministic continuous time model as in Rubin (1996), Kling and Rubin (1997) have explored consequences of inter-temporal trading on social damages of pollution.[10] They identify the socially optimal emission path and show that, in many cases, firms have an incentive to borrow more permits than needed at the social optimum. To restore the social optimality, Kling and Rubin propose a modified inter-temporal trading system, which provides firms with disincentives to borrow to much permits. Their solution consists in allowing borrowing but only at a discount rate. Thus, for one permit borrowed in the current period more than one permit must be surrendered in a subsequent time period. Therefore, if the permit discount rate is chosen so as to match the private decisions with the socially optimal emission path, the social optimum can be restored.

Schennach (2000) explores the consequences of constraints on borrowing. She wants to take into account an important feature of the Title IV of the US Clean Air Act Amendments of 1990: borrowing of permit is not allowed. The author considers a continuous time model with a single central planner who faces an infinite-horizon optimization problem. Moreover, she explicitly takes into account a non-negativity constraint on banking (i.e. $B_t \geq 0$) meaning that borrowing is not allowed. Her aim is to identify the consequences of this constraint on the path of the permit price and emissions. Following previous studies, Schennach observes that whether firms have an incentive to bank/borrow or not depends on the difference between the interest rate and the rate of growth of the marginal abatement cost (MAC) if inter-temporal trading is not allowed. Inter-temporal trading enables firms to smooth out any possible jump in the MAC, by exploiting differences between the interest rate and the rate of growth of the MAC. Thus, under an unrestricted inter-temporal trading regime, the rate of growth of the permit price must satisfy the Hotelling's rule. However, any restriction on inter-temporal trading is expected to yield a different price path.

Schennach considers the effect of different factors that may influence the evolution of the MAC across time, and, hence, the banking and borrowing behavior of affected sources. Those factors are the availability of low-sulfur coal, technological progress, and growth in electricity demand.[11] In the same spirit, the author investigates consequences of reduction in the number of

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(1996) in a discrete-time model with banking. Cronshaw and Kruse show that the permit price can rise no faster than the rate of interest (regardless of whether banking is allowed or not) in a perfectly competitive market equilibrium. Otherwise, there could be corner solutions.

[10] Inter-temporal trading may increase damages from pollution by concentrating emissions in one time period. Indeed, concentration of emissions in one time period may be a concern due to possible interactions with other pollutants. Moreover, this may induce unfavorable effects (e.g. irreversibility, acceleration of damages, etc) creating more and more damages for subsequent time periods.

[11] Schennach (2000) considers the case of power producers whose SO_2 emissions are constrained under the US Title IV. Thus, in each period, the demand for electricity stands for the SO_2 emissions.

issued allowances between Phase 1 and Phase 2 of the Title IV, that translates in increase of the MAC in Phase 2. Solving the problem in the case of deterministic emissions, Schennach (2000) gets the following condition: $\dot{MC}_t(a_t)=r\,MC_t(a_t)-\lambda_t$, where $MC_t(a_t)$ is the MAC associated with abatement a_t, λ_t is the Lagrange multiplier associated with the constraint $B_t\geq 0$, *r* is a riskless rate of interest, and the dot denotes time derivative. According to this equation, the MAC increases at the rate of interest when $\lambda_t=0$, and thus the constraint on borrowing is not binding. In this case, the permit price follows the Hotelling's rule. However, when the constraint is binding, $\lambda_t>0$, and then the MAC increases at a rate less than the interest rate. Note that this result depends on the assumption that the rate of growth of the MAC is smaller than the interest rate before inter-temporal trading intervenes, meaning that firms would have an incentive to borrow permits, but the constraint $B_t\geq 0$ makes this impossible. In this situation, if borrowing was allowed, firms would put back the permit price on the path of the Hotelling's rule, by exploiting any difference between the interest rate and the rate of growth of the MAC. However, the banning of borrowing prevents such arbitrages, and the rate of growth of the permit price remains smaller than the interest rate.

Schennach also demonstrates that, under certain assumptions, the evolution of emissions and permit prices can be divided into two periods. The first is a banking period where part of permits allocations are saved for future use and the permit price must grow at the rate of interest. This is followed by a period in which, each year, all allocated permits are used immediately (banking stops) and the permit price follows changes in electricity demand and MAC function. There is an incentive to save in period *t* for a period *t'* in a distant future only if the rate of growth of the MAC, before inter-temporal trading intervenes, satisfies the following condition: the MAC in *t'*, discounted to time *t*, must be higher than the MAC of *t*. In other words, $i > r$, where *i* stands for the rate of growth of the MAC. Moreover, if *i* becomes smaller than *r* beyond *t'* (e.g. due to some technological innovations or a lowering power demand), the incentive to save does not exist anymore, and banking has to stop.

Another important contribution of the paper consists in introducing uncertainty in the model, by considering stochastic emissions and random events that may affect the MAC function. Thus, Schennach (2000) provides the first attempt to model the permit price dynamic in continuous time with stochastic emissions. Though uncertainties are not explicitly taken into account in the resolution, the dynamic behavior of the permit price is implicitly analysed in this situation. The author derives the following condition: $E_s[\dot{MC}_t(a_t)]=E_s[\mu\,MC_t(a_t)-\lambda_t]$, $\forall t\geq s$, where $\mu=r+\rho$ is a discount rate in which ρ accounts for the asset-specific risk premium in the spirit of the CAPM. Here again, as under certainty, this equation shows that the expected permit price rises at a rate equal to the discount rate, μ, when the constraint on borrowing is not binding. When the constraint is binding, the expected price will rise at a rate less than μ. As pointed out by the author, an important difference under uncertainty is that there should always be a positive probability of having an increase of the MAC in the future, translating in stockout and a binding constraint if borrowing is not allowed. Accordingly, $E_s[\lambda_t]$ should always be non-zero, and the expected permit price must grow at a rate less than the discount rate. Schennach also conjectures that the actual price path needs to be continuously updated as new information becomes available, which may generate discontinuity.

Interestingly, a consequence of the result regarding the expected price path under uncertainty is that there is a convenience yield associated with holding permits rather than taking position on future contracts. This reflects the benefits associated with having permits in the bank to absorb unexpected fluctuations, when the market cannot borrow allowances from future allocations.[12] As noted by Schennach, this can be better seen when considering the previous condition as analogous to an equation of the following form: $i=\mu-\Psi_t$, where $i=\frac{E_t[P_{t+1}]-P_t}{P_t}$, and $\Psi_t=\frac{\lambda_t}{P_t}$ is the convenience yield. In this case, $E_t[P_{t+1}]$ should be understood as the price of a future contract.

Innes (2003) and Maeda (2004) are among the first studies that explicitly took into account the stochastic nature of emissions in a multi-period setting. Innes (2003) considers the impact of costly government enforcement actions in a two-period model with stochastic emissions. He shows that when pollution is stochastic and inter-temporal trading is not allowed, emission trading necessarily leads to some regulatory violations (i.e. some firms will necessarily have higher emissions than their number of permits). In such a situation, regulatory fines must be imposed to non-compliant firms. However, inter-temporal trading can avoid regulatory fines (by allowing non-compliant firms to borrow lacking permits rather than being sanctioned) and the costs of their imposition. Accordingly, Innes (2003) concludes that when emissions are stochastic, if regulatory sanctions and other government enforcement actions are costly, regulators can increase economic efficiency by allowing unrestricted inter-temporal trading of permits.[13] In another two-period model with stochastic emissions, Maeda (2004) analyses the permit price behavior in a trading system with "emitters" (i.e. regulated firms) and "non-emitters" (i.e. unregulated firms which operate in the permit market only to make money). Moreover, he assumes that banking is allowed while

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[12] The existence of a convenience yield in the context of the SO_2 market was also attributed to transaction costs associated with trading of allowances (Bailey, 1996). Schennach notes that both explanations (transaction costs and constraint on borrowing) are not mutully exclusive, and can influence the market for allowances.

[13] This conclusion contrasts with the results of Kling and Rubin (1997), that were obtained under assumptions of deterministic emissions and non-costly government enforcement actions. However, Innes does not consider higher pollution damages that may result from unrestricted inter-temporal trading.

borrowing is prohibited. Interestingly, Maeda shows that the permit price is increasing with respect to the number of regulated firms, and decreasing with respect to the number of non-emitters. He also finds that the permit price volatility depends on the proportion of non-emitters over the total number of market participants. Maeda (2004) also investigates consequences of different ratios between the number of non-emitters and the total number of market participants (m), in a context where both spot and forward trading are possible. The results indicate that the forward price is greater than the expected value of the future spot price, when there are no non-emitters in the market ($m = 0$). By contrast, the forward price is less than the expected value of the future spot price, when the number of non-emitters overwhelms that of emitters (m tends to 1). To explain this result, the author remarks that holding a permit has a completely different meaning depending on the type of market participant. Whereas non-emitters are only motivated by the perspective of making money, emitters are obliged to hold permits for the purpose of compliance. Moreover, permits are exposed to a systematic risk that emitters want to hedge.[14] Hence, the use of forward contracts gives them a way to hedge their positions and reduce this risk. Accordingly, emitters are willing to pay higher prices for forward contracts, reflecting the real option value of hedging. This results in a higher forward price, in the case $m = 0$. This property, which is specific to forward contract for emissions permits, weakened and gradually disappears as the number non-emitter market participants increases.

Based on literature about inter-temporal trading, some authors have developed dynamic equilibrium models to investigate various factors that affect the price dynamic of allowances in the EU ETS. Presenting these models is the object of the following section.

3.3. Theoretical equilibrium models for the EU ETS

Seifert et al. (2008) were the first to develop a dynamic equilibrium model reflecting the main features of the EU ETS, with stochastic emissions and continuous time. They rely on results of Rubin (1996) to represent the market equilibrium obtained with emission trading, using a central planner minimizing the total cost of reducing emissions. Accordingly, the authors assume that all market participants are aggregated into one representative agent who choose the optimal abatement trajectory, $\{u_t\}_{t\in[0,T]}$, so as to minimize the overall expected compliance cost over time horizon T.

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[14] In a previous section of his paper, Maeda (2004) demonstrates that, for a large number of emission sources, fluctuations of the permit price would only depend on random macro-factors (e.g. economic activity or weather), and uncertainties that are specific to each firm are diversified and disappear. This is discussed in section 3.1 of this paper.

The representative agent has an initial endowment of EUAs, e_0, at the beginning of the T periods, and continuously emits CO2 over the whole Phase $[0,T]$, at a rate given by a continuous stochastic process y_t. At every time period *t*, he decides whether to costly abate some of the CO2 emissions or not. At the end of the Phase $[0,T]$, realized accumulated emissions (net of abatements), x_T, are determined. For every tonne of CO2 not covered by an EUA from the initial endowment, a penalty has to be paid. Formally, the central planner minimizes the overall compliance cost (the authors choose to maximize the negative cost, which is equivalent):

$$\max_{\{u_t\}_{t\in[0,T]}} \quad E_0\left[\int_0^T e^{-rt} C(t,u_t)dt + e^{-rT} P(x_T)\right]$$

where r is a risk-free rate of interest, $E_t[.]$ denotes the expectation operator conditional on the information set F_t available at time *t*,

$$C(t,u_t) = -\frac{1}{2} c\, u_t^2$$

describes the abatement costs per unit of time, where c is a constant cost coefficient;

$$P(x_T) = \min\{0, p(e_0 - x_T)\}$$

stands for the potential penalty cost at the end of *T*. In the EU ETS, if an installation fails to surrender enough allowances to cover its verified emissions at the end of a year, it must pay a penalty for each uncovered tonne of CO2, and, in addition, cover excess emissions with the next year's allocation of allowances. Accordingly, the penalty cost, *p*, that has to be paid for each lacking EUA, does not represent the penalty payment only. It describes all costs a company faces when it fails to comply in the EU ETS (i.e. when x_T is higher than e_0), including the cost of having to deliver lacking EUAs at a later point.

Given the emission rate y_t for “business-as-usual” or “uncontrolled” emissions, the author derive

$$x_t = -\int_0^t u_s\, ds + E_t\left[\int_0^T y_s\, ds\right],$$

the total expected emissions (net of abatements) over the whole Phase $[0,T]$. The uncontrolled emission evolves according to a stochastic process of the general form $dy_t = \mu\, dt + \sigma\, dW_t$ (an arithmetic Brownian motion in this case), where μ is a drift coefficient, σ is the empirical variance of y_t and dW_t is the stochastic increment of a standard Wiener process. Given a stochastic process for y_t and the above equation for x_t, the authors get a stochastic process for x_t (applying the Itô's Lemma) which is given by $dx_t = -u_t dt + G(t) dW_t$, where $G(t)$ depends on the stochastic process chosen for the underlying emission rate y_t.[15] Next, applying the Hamilton-Jacobi-Bellman approach, and using the equation obtained for dx_t, the authors derive a partial differential equation which describes the dynamic of chosen emission x_t. The partial differential equation is:

$$V^{(t)} = -\frac{1}{2}(G(t))^2 V^{(xx)} - \frac{1}{2c}(V^{(x)})^2,$$

with boundary condition $V(T, x_T) = e^{-rT} P(x_T)$, where $V(t, x_t)$ is the expected value of the optimal abatement trajectory $\{u_t\}_{t \in [0, T]}$ expressed with x_t (i.e. the “optimal cost to go” from *t* to *T*). $V^{(t)}$, $V^{(x)}$ and $V^{(xx)}$ denote the partial derivatives of $V(t, x_t)$. Maximizing the Hamilton-Jacobi-Bellman equation with respect to u_t, the optimal value $u_t = -\frac{1}{c} \mathrm{e}^{rt} V^{(x)}$ is derived, and the expression of the price of EUAs, $S(t, x_t)$, using the marginal abatement cost: $-\partial C(t, u_t) / \partial u_t = c\, u_t = -e^{rt} V^{(x)}$. Hence, the optimal price behavior of EUAs can be obtained by solving the partial differential equation. The partial differential equation can be solved analytically when $r = 0$ and y_t follows a white noise process. Numerical techniques are required for other cases.

Based on graphical representations for numerical and analytical solutions, the authors get several insights about the solution (see Figure 1).[16] Notably, the price of EUAs is shown to be bounded on the interval $(0\ ,\ pe^{-r(T-t)})$ at each instant $t \in [0, T]$. On the one hand, the carbon price may not rise above $p\, e^{-r(T-t)}$ because when the carbon price reaches the discounted penalty cost, the representative agent would no longer increase efforts but would rather pay the cheaper penalty.[17] On the other hand, the carbon price never reaches zero, because the probability of having fewer allowances than realized emissions is always positive due to stochastic nature of emissions.

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[15] They consider three different processes for y_t: white noise, arithmetic Brownian motion, and Ornstein-Uhlenbeck process. See Neftci (1996) for an overview on stochastic processes.

[16] The authors cannot get a tractable expression when an analytical solution can be obtained. Hence, they rely on graphical representation to interpret the solution. Values of parameters are chosen so as to remind some stylized facts in the EU ETS.

[17] Let us remind that the penalty cost, *p*, corresponds to the price of the next year's allowances plus the penalty.

Seifert et al. (2008) also detect that the allowance price becomes more sensitive to x_t when we move toward the end of the Phase. In other words, shocks on uncontrolled emissions have a stronger impact on the price of EUAs if they occur in a period *t* which is closer to the last period *T*. The logic arises from the fact that the ability to adapt to a rise in uncontrolled emissions – by smoothing abatements across time – is smaller in periods that are close to the end of the Phase. Graphically, it appears in the slopes of the *x*-directional characteristic curves of the surface representing the solution for $S(t, x_t)$ (Figure 1). In the zone where x_t is around e_0, we observe an increasing *x*-directional steepness when we move along the *t*-axis toward *T*. Finally, at time *T*, when any uncertainty is resolved, $S(t, x_t)$ is either zero ($x_T < e_0$) or $p e^{-r(T-t)}$ ($x_T > e_0$).

[insert Figure 1]

With regard to price volatility, Seifert et al. (2008) show that it increases when coming closer to *T*, while, at the same time, it decreases when the price is close to its bounds.[18] Another interesting result comes from the partial differential equation. The authors show that the price $S(t, x_t)$ follows a martingale, which indicates that the stochastic process followed by the carbon price is not affected by any trend.[19] In summary, Seifert et al. (2008) conclude that an adequate process for the price of EUAs does not have to follow any trend or seasonal patterns, and should exhibit a time- and price-dependent volatility structure.

As in Seifert et al. (2008), Hintermann (2010) shows that the equilibrium price of allowances exhibits time dependency. Hintermann identifies that shocks on exogenous variables that influence "business-as-usual" (BAU) emissions increasingly affect the permit price as we move towards the end of the Phase. Following the same strategy as in Maeda (2004), the author uses the fact that, in equilibrium, each firm equalizes its marginal abatement cost to the price of permits, to derive an expression for the carbon price. Moreover, he extends the model of Maeda by considering several time periods.[20] Hintermann (2010) considers a permit market with *N* participants and a fixed time horizon *T*, reflecting a Phase in the EU ETS. The marginal abatement cost function of each

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[18] Dependence of the price volatility on the price level can be observed in Figure 1. We see that the slope of the *x*-directional characteristic curves approaches zero when departing from the region around e_0. This is equivalent to saying that the price volatility decreases and finally reaches zero when the price moves toward either of its bounds.

[19] A martingale is a stochastic process without drift or trend. It has the property that its expected value for any future time is equal to its value today. Therefore, the expected change in a martingale process over a time interval is zero. Formally speaking, a stochastic process S_t is a martingale if $E_t[d S_t]=0$, or $E_t[S_{t+s}-S_t]=0$ in discrete-time. See Neftci (1996).

[20] Maeda (2004) first considers a static model in which he investigates consequences of including random GHG emissions. Next, he develops a two-period model to analyse the permit price behavior in a trading system with "emitters" and "non-emitters". Hintermann (2010) extends this work by including dynamic with more than two-periods, and the effect of fuel prices.

firm *i* in each time *t* is given by:

$$MAC_{it}(a_{it}, G_t, C_t, BAU_{it}) = b\,a_{it} + d_1 G_t + d_2 C_t + g\,BAU_{it}\,, \tag{3}$$

where the time index $t = 1, \ldots, T$ refers to days, BAU_{it} are BAU emissions, a_{it} denotes abatements ($a_{it} = BAU_{it} - e_{it}$, where e_{it} is the chosen level of emissions), and C_t and G_t are coal and gas prices. $b > 0$, $d_1 > 0$, $d_2 < 0$ and $g > 0$ are parameters. BAU_{it} is modeled as a stochastic variable which is a function of a stochastic risk factor Ψ_t, shared by all firms, and firm-specific uncertainties, v_{it}, that have no correlation to each other:

$$BAU_{it}(\Psi_t) = E_{t-1}\left[BAU_{it}(\Psi_t)\right] + \beta_{it}\left(\Psi_t - E_{t-1}[\Psi_t]\right) + v_{it}\,, \tag{4}$$

where $\beta_{it} = \dfrac{Cov(BAU_{it}, \Psi_t)}{Var(\Psi_t)}$, $E\left[\Psi_t \cdot v_{it}\right] = E\left[v_{it} \cdot v_{jt}\right] = 0$ with $i \neq j$ and $E\left[v_{it}\right] = 0,\ \forall\, i$.

Finally, the environmental regulation requires that aggregate abatement has to equal the difference between aggregate BAU emissions and the emission cap *D*:

$$\sum_{k=1}^{T}\sum_{i=1}^{N} a_{ik} = \sum_{k=1}^{T}\sum_{i=1}^{N} BAU_{ik} - D\,. \tag{5}$$

Combining the expression of the optimal abatement, a_{it}^{*} (derived from **(3)**, using the fact that each firm has to equal its marginal abatement cost with the permit price, in the equilibrium), with **(5)**, aggregating and re-arranging yields:

$$\frac{1}{b}\sum_{k=1}^{T} p_k - \frac{d}{b}\sum_{k=1}^{T} F_k - \frac{g}{b}\sum_{k=1}^{T}\sum_{i=1}^{N} BAU_{ik} = \sum_{k=1}^{T}\sum_{i=1}^{N} BAU_{ik} - D\,. \tag{6}$$

where $d\,F_t \equiv d_1 G_t + d_2 C_t$, and p_t is the permit price.

Applying some transformations to **(6)** and substituting **(4)** for BAU_{it} in the new expression, we get:

$$\frac{1}{N}\sum_{k=t+1}^{T}\left(p_k - E_t[p_k]\right) = \frac{d}{N}\sum_{k=t+1}^{T}\left(F_k - E_t[F_k]\right) + \frac{(g+b)}{N}\sum_{k=1}^{T}\sum_{i=1}^{N}\beta_{it}\left(\Psi_k - E_{k-1}[\Psi_k]\right) + \frac{(g+b)}{N}\sum_{k=t+1}^{T}\sum_{i=1}^{N} v_{it}\,. \tag{7}$$

As shown in Maeda (2004), the variance of v_{it} goes to zero when *N* goes to infinity. This indicates that for a large number of emission sources, the probabilistic nature of the price of allowances would only depend on Ψ_t. Accordingly, the term $(\frac{g+b}{N})\sum_{k=t+1}^{T}\sum_{i=1}^{N} v_{it}$ can be neglected, which allows Hintermann to simplify **(7)** as follows:

$$\sum_{k=t+1}^{T} p_k = \sum_{k=t+1}^{T} E_t[p_k] + d \sum_{k=t+1}^{T} \left(F_k - E_t[F_k]\right) + h \sum_{k=1}^{T} \left(\Psi_k - E_{k-1}[\Psi_k]\right), \qquad \mathbf{(8)}$$

where $h = N(g+b)\bar{\beta}$ with $\bar{\beta} \equiv \bar{\beta}_t = (\frac{1}{N})\sum_{i=1}^{N} \beta_{it}$.

Applying conditions $E_t[P_{t+1}] = \rho P_t$ (where $\rho = 1 + r$ is a discount factor associated with the interest rate *r*, and P_t refers to any price) to **(8)**, and solving recursively for all $t \in [1, \dots, T]$, the author derive an expression for the equilibrium price of permit in any time[21] :

$$p_t = \rho p_{t-1} + d(F_t - \rho F_{t-1}) + h\left(\Psi_t^P - E_{t-1}[\Psi_t^P]\right) + h \cdot \frac{\Psi_t^{NP} - E_{t-1}[\Psi_t^{NP}]}{\sum_{k=t+1}^{T} \rho^{T-k}}. \qquad \mathbf{(9)}$$

where Ψ_t have been partitioned into prices (Ψ_t^P) and non-price determinants (Ψ_t^{NP}, such as weather), with $\Psi_t = \Psi_t^P + \Psi_t^{NP}$.

Equation **(9)** shows that the allowance price is determined by its own lagged value, changes in fuel prices and shocks on the common risk factor Ψ_t. More importantly, Hintermann (2010) identifies that shocks on Ψ_t increasingly affect the permit price as we move towards T (note that this applies only to non-price determinants of BAU emissions, Ψ_t^{NP}) . As for Seifert et al. (2008), this result can be explained by a smaller ability to adapt to rise in uncontrolled emissions that occur in periods *t* that are close to the end of the Phase. Likewise, one can also argue that if a shock appears in a period which is close to *T*, the probability that it will be neutralized by an opposite shock in a later period is smaller, and so it has a stronger impact.

More recently, a few papers have sought to extend the analysis of Seifert et al. (2008) by taking into new features of the EU ETS, namely the fact that inter-phase banking is now allowed (which allows firms to transfer allowances from Phase 2 to Phase 3). By contrast, inter-phase borrowing is still forbidden in the EU ETS. Hitzemann and Uhrig-Homburg (2010) propose a stochastic equilibrium model in continuous time, taking into account a sequence of consecutive finite trading periods (Phases) with inter-phase banking. However, inter-phase borrowing is not allowed. The authors find that the price of allowances and its volatility depend on upcoming Phases,

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[21] Conditions $E_t[P_{t+1}] = \rho P_t$ reflects the fact that markets are efficient with respect to information, and hence current prices fully incorporate all information concerning their future values. See Malkiel (2003).

and identify that each additional Phase leads to an additional component in the current price. Moreover, the relative share of each component depends on the relative share of expected emissions for that component.[22] Hitzemann and Uhrig-Homburg also identify an analogy between emission permits and options written on the risk of non-compliance (see also Chesney and Taschini (2008) and Peluchon (2011)). In their case, with several Phases and inter-phase banking, they show that an allowance is equivalent to "a strip of binary options", each option reflecting a Phase, and thus a risk of non-compliance. However, in contrast to classical financial options, the underlying process is not exogenous since it is derived endogenously through abatement measures.

The same results as in Hitzemann and Uhrig-Homburg (2010) are found in Peluchon (2011). The author follows a similar approach, using a dynamic model with several Phases and inter-phase banking. However, Peluchon (2011) works in a discrete-time setting, in which he shows that an allowance is equivalent to a sum of options. Each option has a different underlying, which is the net cumulative emissions until the end of a given Phase.

An alternative approach to Seifert et al. (2008) is taken by Carmona et al. (2009), who model a dynamic equilibrium with trading among market participants. Although the setting is more realistic, compared with the case of a central planner, the model only gives characterization of the carbon price behavior but does not provide explicit solution. Carmona et al. (2009) focus on the cheapest short-term abatement measures in the power sector: the coal-to-gas fuel switching. They consider a permit market with *N* firms and a fixed time horizon *T*, reflecting a Phase in the EU ETS. Each firm produces electricity from coal plants and Combined Cycle Gas Turbines (CCGTs), so that it can reduce part of its emissions by switching fuels from coal to gas. In order to comply with the regulation, a firm *i* can decides its abatement, $\xi_{t,i}$, at times $t \in [0,T]$. This corresponds to the fuel switching effort. Firms can also trade permits, $\theta_{t,i}$, at price A_t. The difference between allowances allocated at the beginning of the Phase and the expected uncontrolled carbon emissions over the whole Phase, Γ_i, is modeled as a simple random variable, whose realization is known at the end of *T*. This corresponds to the required level of effort for firm *i*, which can take either positive or negative values depending on realized uncontrolled emissions. Accordingly, at the end of *T*, each firm faces a penalty cost if $\Gamma_i - \theta_{t,i} - \sum_{t=0}^{T} \xi_{t,i} > 0$, where the penalty per tonne of CO_2 which is not covered by an allowance is equal to *p*. Finally, at time *t*, the fuel switching effort of firm *i* yields an expense equal to $\varepsilon_{t,i} \xi_{t,i}$, where $\varepsilon_{t,i}$ is the actualized value of the switching price, which represents

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[22] Hitzemann and Uhrig-Homburg (2010) point out that this result can explain the observed price of EUAs during the recession of 2008-2009. During this period, the carbon price did not reach zero because it was driven by expectations for Phase 3.

the cost of switching from coal plants to CCGTs to abate one tonne of CO_2. As a simplification, the authors assume a single type of CCGTs and a single type of coal plants for each firm, so that differences in the efficiency of power plants are not taken into account.[23] The marginal abatement cost, $\varepsilon_{t,i}$, is stochastic, because it depends on coal and gas prices which are stochastic variables. However, it does not depend on the level of switching effort.[24] Therefore, when coal and gas prices are fixed, the marginal abatement cost is constant, equal to $\varepsilon_{t,i}$, whatever the value of $\xi_{t,i}$.

For each firm, the profit/loss from emission trading over the whole Phase can be expressed as follows:

$$\Pi_i(\theta_i,\xi_i)=\sum_{t=0}^{T-1}\theta_{t,i}\left(A_{t+1}-A_t\right)-\theta_{T,i}A_T-p\left(\Gamma_i-\sum_{t=0}^{T}\xi_{t,i}-\theta_{T,i}\right)^{+}-\sum_{t=0}^{T}\varepsilon_{t,i}\xi_{t,i}, \qquad \mathbf{(10)}$$

where $\theta_i=\left(\theta_{t,i}\right)_{t\in[0,T]}$ and $\xi_i=\left(\xi_{t,i}\right)_{t\in[0,T]}$. $\theta_i=\left(\theta_{t,i}\right)_{t\in[0,T-1]}$ is defined as a trading strategy on forward contracts, while $\theta_{T,i}$ is the number of spot contracts which firm *i* purchases at time *T*. Interestingly, the penalty cost does not depends on positions held on forward contracts. Implicitly, this means that the strategy on forward contracts is a pure hedging strategy with financial settlement on each contract. Thus, in equation **(10)**, $\sum_{t=0}^{T-1}\theta_{t,i}\left(A_{t+1}-A_t\right)$ gives the wealth of hedging strategy $\theta_i=\left(\theta_{t,i}\right)_{t\in[0,T-1]}$, while $\theta_{T,i}$ corresponds to the number of allowances bought or sold for compliance purposes.

The individual optimization problem of a firm *i* is given by:

$$\max_{\theta_i\,,\,\xi_i}\; E_t\left[\Pi_i(\theta_i,\xi_i)\right]$$

where $E_t[.]$ denotes the expectation operator conditional on the information available at time *t*. Accordingly, an equilibrium carbon price process $A^*=\left(A_t^*\right)_{t\in[0,T]}$, given a fuel switching price process $\varepsilon_i=\left(\varepsilon_{t,i}\right)_{t\in[0,T]}$ for each firm *i*, can be defined as combinations of trading and switching strategies, $(\theta_i^*\,,\,\xi_i^*)$, so that:

$$E_t\left[\Pi_i(\theta_i^*,\xi_i^*)\right]\geq E_t\left[\Pi_i(\theta_i,\xi_i)\right],\ \forall\ (\theta_i,\xi_i)\ \text{with}\ i\in[1,\ldots,N],$$

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[23] Note that differences in the efficiency of power plants owned by a firm can inluence the cost of fuel switching and its dependence on fuel prices. See Bertrand (2012b).

[24] While previous papers considered marginal abatement cost as a deterministic function increasing in abatement efforts, Carmona et al. (2009) introduce a stochastic cost function which do not depend on abatement efforts. For an alternative approach, with a cost function for fuel switching which depends on the level of effort, see Bertrand (2010).

and

$\sum_{i=1}^{N} \theta^{*}_{t,i} = 0$, at any time $t \in [0, \ldots, T]$ (the market-clearing condition).

As in Rubin (1996), Carmona et al. (2009) show that this equilibrium is connected to the solution obtained by a central planner. They also characterize the shape of the equilibrium price as follows:

$$A_t^{*} \;=\; p \cdot E_t \left[\mathbf{1}_{\{\Gamma - \Xi^{*} \geq 0\}} \right], \tag{11}$$

where $\mathbf{1}_{\{\Gamma - \Xi^{*} \geq 0\}}$ is an indicator function, $\Gamma = \sum_{i=1}^{N} \Gamma_i$ and $\Xi^{*} = \sum_{i=1}^{N} \sum_{t=0}^{T} \xi^{*}_{t,i}$. Thus, the authors demonstrate that the equilibrium price of allowances depends on the difference between the aggregated required level of abatements, Γ, and the aggregated optimal switching effort, Ξ^{*}. Besides, as $\varepsilon_{t,i}$ is an increasing function of the gas price and a decreasing function of the coal price, equation **(11)** shows that the probability of having to pay a penalty at the end of the Phase (reflecting a positive carbon price) is an increasing function of the gas price and a decreasing function of the coal price. This demonstrates that, in equilibrium, the carbon price must be an increasing function of the gas price and a decreasing function of the coal price.

All the papers we have reviewed so far identify abatement- and production-decisions as key drivers of the carbon price. Chesney and Taschini (2008) belong to this literature. However, contrary to the papers mentioned above, the model of Chesney and Taschini accounts for the presence of asymmetric information regarding emission levels. Another particularity arises from the fact that no abatement measures are considered in the model, and thus, carbon emissions are fully exogenous to firms. The authors show that an equilibrium price for allowances exists. Though an explicit solution is not provided, they give some insights on the permit price behavior based on numerical simulations. Notably, Chesney and Taschini show that the higher the probability of each firm being in shortage by the end of the Phase, the higher the permit price. This confirms previous studies, and underlines the optional nature of permits as an insurance to avoid paying penalty costs.

Similarly to Chesney and Taschini (2008), Barrieu and Fehr (2011) use a dynamic model in which carbon emissions are exogenous to firms. The authors propose an equilibrium analysis of compliance involving both CERs (Certified Emission Reduction, the certificates generated through the Clean Development Mechanisms) and EUAs, which enables them to analyse how CER and EUA prices should converge due to non-arbitrage strategies. In particular, they demonstrate that CER and EUA prices may diverge to some extend, because of limitation on the number of CER that can be used for compliance in the EU ETS. Barrieu and Fehr assume a representative agent who chooses an optimal compliance strategy for Phase 2, so that unused allowances can be transferred to Phase 3. They consider a situtation in which compliance takes place only at the end of the last year of Phase 2.[25] Then, the equilibrium is given by the arbitrage strategy between the prices of Phase 2 EUAs, Phase 3 EUAs (i.e. the future contracts maturing in Phase 3) and CERs.

As explained by the authors, those arbitrage strategies generate some bounds for the prices of CERs and EUAs. First, the current Phase's EUA price is bounded from above by the next Phase's EUA price plus the penalty, since emissions can always be covered by paying the penalty, and surrendering next year's allowances. There also exists a convergence between the price of Phase 2 EUAs and the price of Phase 3 EUAs, due to inter-phase banking. Hence, the next Phase's EUA price can be seen as a lower bound for the current Phase's EUA price. Besides, the current Phase's EUA price is also bounded from below by the CER price. Indeed, assuming that there is no import limit for CERs in the EU ETS, and that the CER price is lower than the EUA price, then firms will want to cover all their emissions with CERs, and bank all their EUAs. This would drive the CER price up, and the EUA price down until both prices converge. Moreover, the next period's allowance price and the CER price can also tend to converge due to inter-phase banking. In view of the EU ETS regulation, all Phase 2 EUAs that are not used for compliance at the end of Phase 2 are converted to Phase 3 EUAs. Thus, by controlling the number of CER used to offset Phase 2 emissions, and the resulting volume of banked Phase 2 EUAs, firms can control the amount of Phase 3 EUAs. Hence, firms can decide which asset, Phase 3 EUAs or CERs, they carry over to Phase 3. As long as one has a higher price than the other, firms will choose to transfert this asset rather than the other, and vice versa. Because of this arbitrage, the next Phase's EUA price and the CER price tend to converge.

As explained above, there may exist arbitrage opportunities between the prices of Phase 2 EUAs (A), Phase 3 EUAs (A') and CERs (C). These differences vanish if opportunities are all exploited, and, finally, the prices align together: $C = A' = A$. However, if there is an import limit for CERs, there may be arbitrage opportunities that cannot be exploited, so that differences between the prices can persist. In this case, the prices may not align together. This is shown by comparing the number of CERs that would be used for compliance in Phase 2 if no import limit for CERs

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[25] Since EUAs of various years, that are within the same Phase, cannot be distinguished, and emissions for a given year do not have to be covered until April of the subsequent year, the offset can be made by the subsequent year's EUAs, which are allocated in February. This corresponds to a borrowing, which is only allowed within the same Phase, but not between two Phases. As a consequence, firms have the ability, year by year, to postpone the compliance of each year to the end of the last year of the Phase. So, compliance in a Phase works as if there were only one constraint per Phase. Accordingly, one can assume that compliance takes place only at the end of the last year of the Phase. This is a usual assumption in modeling of the EU ETS.

would prevail (o^*), with the maximal number of CERs that the import limit allows to use (Γ). Using this setting, Barrieu and Fehr demonstrate that, when there is an import limit for CERs, the prices do not align together if $o^*>\Gamma$. This happens because the import limit induces firms to use more Phase 2 EUAs for compliance. This increases the scarcity of Phase 2 EUAs, but also the scarcity of Phase 3 EUAs by reducing the ability to carry over Phase 2 EUAs to Phase 3. Hence, A and A' tend to increase, so that $C<A$ and $C<A'$. Moreover, the greater use of Phase 2 EUAs drives their price up, so that it may reach its upper bound: $A=A'+\pi$, where π denotes the penalty to pay per lacking allowances at the end of Phase 2. In this case, $A'<A$.[26]

Next, using their analyse of difference between the CER and EUA prices, Barrieu and Fehr derive an expression for the pricing of EUA-CER spread options. As pointed out by the authors, these options enable regulated firms to trade their import limits for CERs, by allowing the buyers of such contracts to exchange a CER for an EUA. This is an interesting mechanism, because it may increase the flexibility, and thus the efficiency of the scheme. Indeed, allowing such trading between firms that are not equally efficient to exploit differences between the CER and EUA prices (e.g. a small firm, without trading department, is typically less efficient to exploit such differences), increases efficiency with respect to the solution without spread options, in which the regulator distribute the same import limit to each agent in the market, regardless of their efficiency. Using their expression for the price of spread options, Barrieu and Fehr run several sensitivity analysis to observe how the option price is affected when some parameters are modified. In particular, they analyse the influence of a price response parameter, which may be seen as an indicator of how efficient firms are to exploit arbitrage opportunities. Results indicate that the option price increases when the price response parameter decreases.

4. Conclusion

Among the policy measures that help tackle the problem of pollution, emission trading systems are very popular, and are often considered as the best instrument. This mechanism offers a number of advantages for business and policymakers, which have fostered the creation of emission trading systems, and the development of numerous theoretical models to analyse these markets and the price formation of allowances. The aim of this paper is to provide a wide overview of this literature,

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[26] Note that $A'<A$ may be seen as a consequence of the EU ETS rule that do not allow firms to borrow EUAs from Phase 3 to use them in Phase 2. If such inter-phase borrowing would be allowed, firms would be able to exploit any difference between A and A' . As a consequance, A and A' would converge. This mechanism, that has been fully described in the previous literature, is further discussed in section 3.2 of this paper.

with a focus on contributions dealing with dynamic partial equilibrium modeling, in case of perfect competition. After an introduction on the economics of pollution control and the origins of emission trading, we have begun with a review of the earliest static models investigating a number of factor that can affect the effectiveness of emission trading, such as market power, transaction-costs, political pressures, etc. Next, we have presented more recent dynamic models investigating implications of a range of questions including, inter-temporal trading, stochastic emissions, and the rules of the EU ETS.

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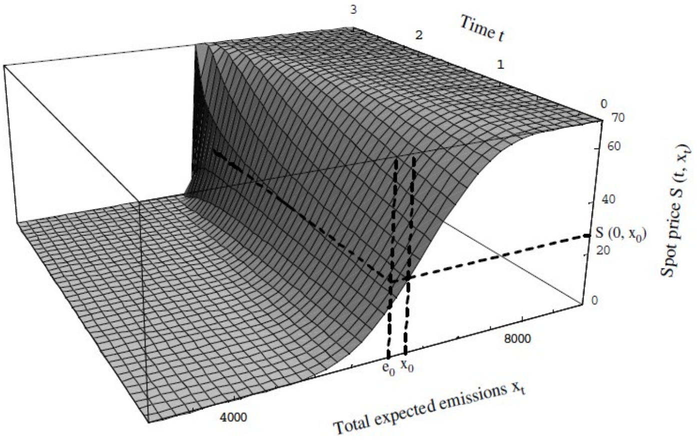
Figures

Figure 1: Equilibrium carbon price dynamic in Seifert et al. (2008). Values of parameters are chosen so as to remind the EU ETS.



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La Chaire Economie du Climat est une initiative de CDC Climat et de l'Université Paris-Dauphine sous l'égide de la Fondation Institut Europlace de Finance