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Tax Policy in a Simple General Oligopoly Equilibrium Model with Pollution Permits

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JEL classification : D43 - D51 - H2.

Keywords : oligopoly equilibrium - taxation policy – pollution.

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A first version of this paper has been presented in the workshop "Markets, strategic interactions and environment" which held in Dijon in December 2012. We are grateful to some participants for their comments and suggestions. We also acknowledge the Regional Council of Burgundy for financial support (PARI 2011, contract 9201AAO047S5590). Two referees of the CEC Chair are acknowledged for their comments and remarks. Remaining deficiencies are ours.

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Abstract

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1 Introduction

In this paper we build up a static general oligopoly equilibrium model with a competitive pollution permits market. Introducing a competitive permit market, however, is not sufficient to eliminate market distortions and to reach a Pareto optimal allocation. The problem at stake may thus be stated as follows: how to enhance the gains from trade without increasing pollution in an economy with strategic interactions within interrelated markets?

More specifically, we consider a two–commodity economy with one productive sector. The economy embodies traders who consume both goods. All traders have log-linear utility functions. One representative trader is endowed with the second commodity only and behaves competitively. The other traders, namely the oligopolists, behave strategically. The oligopolists are not endowed with any commodity but with a linear technology which enables them to produce the first commodity. This production is performed by using the second commodity as input and is a polluting activity. To limit the emissions, a competitive permits market is introduced. An amount of pollution permits is freely and exogenously spread among all strategic traders. These traders make two decisions: the quantity to produce, which determines the emissions (and whether they are net buyer or net seller of pollution permits), and also the part of their production which is bring to the market of the second good. Since the strategic traders manipulate the relative prices by restricting the production sent to the market, the general oligopoly equilibria of this economy generally display an inefficiency. To cure this inefficiency, we consider a balanced-budget policy which consists in subsidizing the strategic traders and taxing the competitive trader. We show that this subsidy policy is Pareto improving provided that the size of the subsidy is sufficiently small.

This paper contributes to three strands of literature. First, it contributes to the literature on taxation in general equilibrium models with imperfect competition. In these models, taxation can achieve two objectives. On the one hand, it aims at correcting the distortions generated by the market mechanism in the presence of imperfectly competitive behavior. On the other hand, it serves as a tool for redistributive purposes (see Guesnerie and Laffont, 1978; Myles, 1989, 1996; Delipalla and Keen, 1992; Gabszewicz and Grazzini, 1999; Reinhorn, 2005; and Grazzini, 2006). Nevertheless, our contribution differs in two ways from those cited above. First, we do not determine whether taxation can decentralize Pareto efficient allocations. We rather investigate the problem of taxation in an economy with pollution permits and with strategic interactions in interrelated markets. In particular, we pay attention to the indirect effects of a tax policy on the permit markets. Second, the comparison between different kinds of taxes is beyond the scope of this paper. We study neither per unit and ad valorem taxations as in Delipalla and Keen (1992), and Grazzini (2006), nor the commodity taxation as in Gabszewicz and Grazzini (1999). The main reason for this is that in general, these taxation schemes lead to a restriction of the quantity the oligopolists are willing to trade, thereby reinforcing the original market distortions. In addition, the redistribution of the product of the tax is done by resorting to an outside agent who possesses no initial wealth and receives the product of the tax. We rather consider a policy which consists in taxing the competitive traders endowed with the polluting input and in subsidizing the strategic traders' supply under a balanced budget rule constraint. We thus focus on the redistribution of the collected resources, but in favor of the strategic traders. The tax/subsidize policy is a means to enlarge the size of trades, and not for encouraging production, and thus emissions.[1]

Second, our paper also contributes to the literature on general oligopoly equilibrium.[2] This approach aims at studying the consequences of market power into a general equilibrium framework.[3] In this approach, the market embodies strategic traders and competitive agents and prices are determined according to the Walrasian pricing rule. The strategic traders determine their strategies taking into account the Walras price correspondence as in the Cournot game. They manipulate the price system via the amount of commodity they send to the market for trade. In this paper, we contribute to the general oligopoly literature by introducing a pollution permits market and by studying the conditions under which there exists a Pareto-improving subsidy policy.[4]

Third, our model extends the literature on pollution permits to strategic multilateral exchange with production.[5] This exercise, however, is not a simple transposition of partial equilibrium results to a general equilibrium setting. Indeed, our results depend on some fundamentals which cannot be captured in a single market framework. In particular, agents' preferences matter for three reasons. First, we rely on agents' preferences to provide some micro-foundations for the market demand. On the contrary, partial equilibrium models with pollution permits usually assume some given exogenous (often linear) market demand function.[6] Second, for the market to clear when the demand is given, the quan-

[1] In industrial organization models, all the production is generally brought to the market. By contrast this is not the case here since oligopolists are also consumers.

[2] This literature was initiated by Gabszewicz and Vial (1972), Roberts (1980), Mas-Colell (1982), and pursued later by Codognato and Gabszewicz (1991), (1993), Gabszewicz and Michel (1997), Grazzini (2006) and Julien (2011).

[3] One interest of the models belonging to this field of research is that they circumvent the problems associated with the normalization of prices which occur frequently with imperfect competition (see Dierker and Grodal, 1986).

[4] Taxation on emissions under oligopolistic interactions is studied notably in Lahiri and Ono (2007). See also Metcalf (2009) who discusses the advantages and shortcomings of using marketable emission permits.

[5] There is a huge literature devoted to emission permits market under imperfect competition (for a survey, see notably Montero, 2009). But following Sartzetakis (1997), (2004), we rather make the assumption of a competitive permits market. Indeed, according to Montero: "Finally, an empirical revision of the functioning of past and existing permit markets shows no indication of market power that can be of concern" (pp. 26). Anyway, under imperfect competition, the regulator would need more instruments (Montero, 2009)). See also the approach developed by Schwartz and Stahn (2013), who consider perfect competition on the permits market and Cournot competition in an eco-industry.

[6] See for instance Hahn, 1984; Sartzadakis, 1997, 2004; Montero, 2002, 2009; Chen and Hobbs, 2005; Kato, 2006; Ehrhart *et al.*, 2008; and Sanin and Zanaj, 2011, 2012.

tity produced by the firms must be equal to the quantity sold. Under this circumstance, all production is brought to the market. In this paper, by contrast, oligopolists are also consumers: they bring to the market the difference between the quantity produced and the quantity they consume. This surplus brought to the market matches the demand of the competitive consumer. Therefore, the source of market distortions cannot be cured in the usual way since there is no exogenous competitive demand side facing a strategic supply side. In this connection, the efficacy of the subsidy policy depends critically on the value of the preferences' parameters. When the parameter of preferences relating to the produced commodity is sufficiently high, the subsidy policy increases the supply of the oligopolists. This increase in supply is bought by the competitive agent despite the fact that he must pay the tax which finances the subsidy. Moreover, under the same conditions on preferences, the subsidy/tax policy is welfare improving (at least for small values of the subsidy). Third, our model departs from the representative agent framework: it displays behavioral heterogeneity, as several strategic traders interact with a representative competitive trader.

The paper unfolds as follows. In section 2, we describe the economy and we compute the (standard) competitive equilibrium. Section 3 is devoted to the analysis of the oligopoly equilibrium with pollution permits. We notably prove that when the economy is replicated an infinite number of times, the limit oligopoly equilibrium prices, allocations and individual decisions coincide with those of the competitive equilibrium of the finite economy. Section 4 deals with a comparative welfare analysis of the competitive and oligopoly equilibria. In section 5, we consider a Pareto improving subsidizing policy. Section 6 offers some concluding remarks.

2 The Economy

We consider a two-commodity economy with one productive sector. Let us denote p_1 and p_2 the prices of good 1 and good 2 respectively. Let good 2 be the *numéraire*, $p = \frac{p_1}{p_2}$ the relative price of good 1 in terms of good 2 and $\mathbf{p}$ the normalized price vector for the commodities: $\mathbf{p} = (p, 1)$.

The economy includes $n + 1$ traders: one representative competitive trader who is price-taker and n strategic traders, each being indexed by i, $i = 1, \ldots, n$. Let $\mathcal{O}$ be the set of strategic traders. All agents consume the two goods. The competitive trader supplies inelastically the good 2 while the strategic traders produce good 1 using good 2 as input. Production is a polluting activity. To control the pollution, there is an emissions permits market. Each strategic trader i has initially $\lambda^i E$ pollution permits, where E is the legal maximum aggregate level of pollution, $0 \leq \lambda_i \leq 1$ and $\sum_i \lambda^i = 1$. We let q denote the permit price in terms of good 2. The price system is therefore given by $(\mathbf{p}, q)$.

2.1 Preferences, endowments and technologies

All the agents have the same utility function:

$$U(x_1, x_2) = \alpha \ln x_1 + (1-\alpha) \ln x_2, \tag{1}$$

where x_1 and x_2 are the consumptions of goods 1 and 2.[7]

The initial endowments of both types of agents are as follows:

$$\boldsymbol{\omega}^i = (0,0), \ \forall i \in \mathcal{O}, \tag{2}$$

$$\boldsymbol{\omega} = (0,1), \text{ for the competitive trader.} \tag{3}$$

So, as in Gabszewicz and Michel (1997), an oligopolist must produce to consume. With a quantity z^i of good 2, an oligopolist i in $\mathcal{O}$ produces a quantity y^i of good 1 according to a linear technology[8]:

$$y^i = \frac{1}{\beta} z^i, \ \beta > 0, \ \forall i \in \mathcal{O}. \tag{4}$$

Following Sanin and Zanaj (2011)-(2012), the polluting input z^i generates a quantity of emissions e^i as follows:

$$e^i = \frac{1}{\gamma} z^i, \ \gamma > 0, \ \forall i \in \mathcal{O}, \tag{5}$$

where γ measures the magnitude of pollution.

From the last two equations, we can express the production y^i of good 1 by each strategic trader in terms of the emissions e^i, namely:

$$y^i = \frac{\gamma}{\beta} e^i, \ \forall i \in \mathcal{O}. \tag{6}$$

Thus to increase their supply the oligopolists must increase the emissions.[9]

The aggregate quantity of emissions results from the production decisions of the strategic traders, and thereby depends on the market structure. When the law determines the maximal aggregate quantity of emissions, an emissions market allocate these emissions across the strategic traders. This market is useful when the emissions of the strategic traders differ from their emissions rights.

[7] For simplicity, pollution is not taken into account in the utility function. This does not matter since we can assume that pollution enters the utility function in a separable way and that for most the analysis, the emissions ceiling is considered as being constant.

[8] As in Gabszewicz and Michel (1997), this specification is sufficient to capture some relevant features of strategic interactions in a general equilibrium framework.

[9] There is a huge literature which considers emissions as free inputs. To limit these emissions some exogenous upper bound can be envisaged as in Jouvet *et al.* (2000). For quotas and taxes, see notably Ono (2002). Otherwise, Montgomery (1972) considers a non monotonic production function with respect to emissions and Stokey (1998) studies the limit on emissions via an index technology.

2.2 Strategy sets

Under Cournot competition, any strategic trader $i \in \mathcal{O}$ chooses both the quantity e^i of emissions (which determines through (3) and (4) the quantity of input z^i of good 2 and the quantity y^i of good 1), and the quantity s^i of good 1 for market sales. The income of the strategic agent is equal to his profit $\Pi^i\left(s^i, e^i\right)$, where:

$$\Pi^i\left(s^i, e^i\right) = ps^i - \gamma e^i + q\left(\lambda^i E - e^i\right),\, i \in \mathcal{O}. \tag{7}$$

This profit is the sum of the proceeds of the sales of good 1, ps^i (where $s^i \leq y^i$, and $y_i = (\gamma/\beta)e^i$), minus the input cost, which is given by $z_i = \beta y^i = \gamma e^i$, minus the net purchases of emissions rights, *i.e.*, $q(e^i - \lambda^i E)$.

Agent's i profit finances his purchases x_2^i of good 2. His consumption of good 1, x_1^i equals the quantity $y^i - s^i$ of its production that is not sold. The strategy set of trader i is therefore given by:

$$S^i = \left\{\left(s^i, e^i\right) \in \mathbb{R}^2_+ \mid 0 \leq s^i \leq \frac{\gamma}{\beta} e^i\right\}, i \in \mathcal{O}. \tag{8}$$

Each trader $i \in \mathcal{O}$ recognizes his influence on the (equilibrium) price system. More precisely, a strategic trader takes into account his influence on the price of good 1, but take the unit price q of emission as given (we assume that the permits market is competitive).

Taking the price vector $(p, 1)$ as given, the competitive agent chooses his demands for both goods 1 and 2 by solving the standard problem $\max\limits_{(x_1,x_2)} \alpha \ln x_1 + (1-\alpha)\ln x_2$, s.t. $px_1 + x_2 \leq 1$. His demands are therefore equal to $(x_1, x_2) = \left(\frac{\alpha}{p}, 1-\alpha\right)$.

2.3 The competitive equilibrium

As a benchmark case, let us determine the competitive equilibrium for a given allocations of emissions rights $\left(\lambda^i\right)_{i\in\mathcal{O}}$. In a competitive equilibrium, strategic agents take the price system $(\mathbf{p}, q)$ as given and sell all their production of good 1. Therefore they solve the following problem:

$$\begin{aligned} &\max_{x_1^i,\, x_2^i} \ \alpha \ln x_1^i + (1-\alpha)\ln x_2^i \qquad \text{(P1)} \\ \text{s.t.}\quad & px_1^i + x_2^i \leq p\frac{\gamma}{\beta}e^i - \gamma e^i + q(\lambda^i E - e^i), \\ & \text{with } x_1^i \geq 0,\, x_2^i \geq 0. \end{aligned}$$

A competitive equilibrium (CE) is a price vector $(\mathbf{p}^*, q^*)$ and a set of individual decisions such that all the decisions solve the corresponding agents' problems, and all the markets are balanced.[10]

[10] There are three markets: the market for good 1, the market for good 2, and the market for the pollution permits.

As agent i's profit ($i \in \mathcal{O}$) is linear in e^i, there exists an equilibrium only if $p = \beta(1 + \frac{q}{\gamma})$. With such a price, the demands of this agent i may be written $\big((x_1^i)^*, (x_2^i)^*\big) = \big(\alpha \frac{q}{p} \lambda^i E, (1-\alpha) q \lambda^i E\big)$. From Walras' law and equation (4), we obtain the market-clearing condition for good 2, *i.e.*, $(1-\alpha)qE + \gamma E + 1 - \alpha = 1$. The term on the left hand side of this equation represents the aggregate demand of commodity 2, where $(1-\alpha)qE$ and γE are respectively the demands for consumption and for the input by the n traders, and $1-\alpha$ is the demand by the representative consumer. The term on the right hand side is the total supply of commodity 2. From the market clearing condition, we obtain the equilibrium relative price for permits, namely:

$$q^* = \frac{\alpha - \gamma E}{(1-\alpha)E}. \tag{9}$$

One remarks that the numerator in (9) must satisfy $\alpha - \gamma E > 0$ for the equilibrium price to be positive. The term $\alpha - \gamma E$ represents the net supply of good 2 by the competitive trader, *i.e.*, after having taken into account the demand of this trader. The lower α the higher the demand of good 2 by the competitive trader. The higher γ the higher the demand of good 2 as an input by the strategic traders.[11] For the consumption of good 2 by the strategic traders to be positive, it is necessary that the quantity $\alpha - \gamma E$ be positive. The price of permits q adjusts in such a way that the demand of good 2 by the strategic traders equals the net supply of this good. So, α/γ represents a threshold for E above which the permits market cannot balance.

From $p = \beta(1 + \frac{q}{\gamma})$ and the above expression of q, one deduces the relative price of good 1 in terms of good 2:

$$p^* = \frac{\alpha\beta(1 - \gamma E)}{(1-\alpha)\gamma E}. \tag{10}$$

We can see that the competitive prices decrease with the aggregate amount of permits issued: more rights to pollute leads to lower prices (because more rights to pollute means more production).

The allocation of production across the strategic traders is indeterminate since their profit function is linear with respect to e^i and the equilibrium prices p and q are such that the net receipts are nil.

The following equations give the competitive equilibrium allocations and the

[11] These demands stem from the fact that preferences are log-linear and the technology is linear.

corresponding utility levels:

$$\left((x_1^i)^*, (x_2^i)^*\right) = \left(\frac{\gamma}{\beta}\left(\frac{\alpha-\gamma E}{1-\gamma E}\right)\lambda^i E, \lambda^i(\alpha-\gamma E)\right), i \in \mathcal{O}, \quad (11)$$

$$(x_1^*, x_2^*) = \left(\frac{(1-\alpha)\gamma E}{\beta(1-\gamma E)}, 1-\alpha\right), \quad (12)$$

$$(U^i)^* = \alpha \ln\left(\frac{\gamma E}{\beta(1-\gamma E)}\right) + \ln \lambda^i(\alpha-\gamma E),\ i \in \mathcal{O}, \quad (13)$$

$$U^* = \alpha \ln\left(\frac{\gamma E}{\beta(1-\gamma E)}\right) + \ln(1-\alpha). \quad (14)$$

3 Oligopoly Equilibrium with Pollution Permits

The study of the oligopoly equilibrium (OE) is made in four parts. First, we describe the logic of the OE. Second, we determine the equilibrium outcome. Third, we study the effect of an increase in the emissions ceiling E on the oligopoly equilibrium. Fourth, we determine to what extent the OE coincides with the competitive equilibrium.

3.1 The oligopoly equilibrium

The equilibrium prices and allocations depend on the decisions of both the competitive and strategic agents. The computation of the equilibrium is based on a two-step procedure.[12] In the first step, the competitive market clearing prices are computed given the decisions of both the competitive and strategic agents. In the second step, the oligopolists play a Cournot game on quantities by taking into account the value of the competitive market clearing prices. We then deduce the OE relative prices and allocations.

Consider first the determination of the market clearing prices for a given profile of strategies $\mathbf{s} = \left(s^1, s^2, ..., s^n\right)$, with $\mathbf{s} \in \prod_i S^i$, and emissions $\tilde{\mathbf{e}} = \left(\tilde{e}^1, \tilde{e}^2, ..., \tilde{e}^n\right)$, with $\tilde{e}^i$ satisfying (5) for each $i \in \mathcal{O}$. The vector of competitive demand functions for commodities 1 and 2 by the competitive agent is given by: $\mathbf{x} = \left(\frac{\alpha}{p}, 1-\alpha\right)$ (see subsection 2.3). The aggregate demand function in good 1 which is addressed to the oligopolists is thus $x_1 = \alpha/p$. Given $\mathbf{s} = \left(s^1, s^2, ..., s^n\right)$, the market-clearing condition for good 1 is written $\alpha/p = \sum_{i=1}^n s^i$, which leads to:

$$p = \frac{\alpha}{\sum_{i=1}^n s^i}. \quad (15)$$

[12] Our oligopoly equilibrium concept does not exactly belong to the class of Cournot-Walras equilibria (see notably Codognato and Gabszewicz 1991, 1993) since the strategic traders (the oligopolists) do not directly manipulate the permits' price, *i.e.*, they do not behave strategically on the permits market.

From (15), we see that the oligopolists can get a better price by restricting their supplies.

Consider now the oligopolists as consumers. For a given vector $(\mathbf{p}, q)$ and for given strategy $s^i \in S^i$ and emission e^i satisfying (5), the strategic trader i consumes a quantity $x_1^i = \frac{\gamma}{\beta} e^i - s^i$ of good 1 and an amount $x_2^i = \Pi^i\big(s^i, e^i\big) = ps^i - \gamma e^i + q(\lambda^i E - e^i)$ of good 2. Consequently, the indirect utility function of any trader i may be written as follows:

$$V^i(s^i, e^i) \equiv \alpha \ln\left(\frac{\gamma}{\beta} e^i - s^i\right) + (1-\alpha)\ln\Big(ps^i - \gamma e^i + q(\lambda^i E - e^i)\Big),\ i \in \mathcal{O}. \quad (16)$$

Let us now introduce a game associated with this economy. While the representative consumer takes the prices as given and recognizes that he exerts no influence on market prices, the oligopolists exert a partial control on equilibrium prices by manipulating strategically their supply. An oligopoly equilibrium is a non cooperative equilibrium of the game in which the players are the oligopolists, whose strategies are the quantity of good 1 they bring to the market, and whose payoffs are the utility level they achieve through the equilibrium price system $(\tilde{\mathbf{p}}, \tilde{q})$ corresponding to the market clearing conditions.

Definition 1 *An oligopoly equilibrium is given by a vector of strategies* $\tilde{\mathbf{s}} = \big(\tilde{s}^1, \tilde{s}^2, ..., \tilde{s}^n\big)$, *with* $\tilde{\mathbf{s}} \in \prod_i S^i$, *and a vector of emissions* $\tilde{\mathbf{e}} = \big(\tilde{e}^1, \tilde{e}^2, ..., \tilde{e}^n\big)$, *with* $\tilde{e}^i$ *satisfying (5) for each* $i \in \mathcal{O}$ *such that* $V^i(\tilde{s}^i, \tilde{e}^i, \tilde{\mathbf{s}}^{-i}, \tilde{\mathbf{e}}^{-i}) \geq V^i(s^i, e^i, \tilde{\mathbf{s}}^{-i}, \tilde{\mathbf{e}}^{-i})$, $\forall \tilde{s}^i \in S^i$ *and* $\forall e^i$ *satisfying (5), where* $\tilde{\mathbf{s}}^{-i}$ *and* $\tilde{\mathbf{e}}^{-i}$ *denote respectively the vector of strategies and emissions of all strategic traders but* i.

3.2 Equilibrium analysis

We now compute the OE. The problem of trader i consists in maximizing $V^i(s^i, e^i)$ with respect to s^i and e^i:

$$\max_{(s^i, e^i)} \ln\left(\frac{\gamma e^i}{\beta} - s^i\right)^{\alpha}\left(\frac{\alpha s^i}{\sum\limits_{k=1}^{n} s^k} - (\gamma + q)\, e^i + q\lambda^i E\right)^{1-\alpha}. \quad \text{(P2)}$$

Using (15), the conditions $\partial V^i/\partial s^i = 0$ and $\partial V^i/\partial e^i = 0$ lead to:

$$\frac{-\alpha}{\frac{\gamma}{\beta}e^i - s^i} + \frac{\alpha(1-\alpha)\left(\dfrac{S - s^i}{S^2}\right)}{ps^i - \gamma e^i - q\left(e^i - \lambda^i E\right)} = 0, \quad (17)$$

$$\frac{\alpha\frac{\gamma}{\beta}}{\frac{\gamma}{\beta}e^i - s^i} - \frac{(1-\alpha)(\gamma + q)}{ps^i - \gamma e^i - q\left(e^i - \lambda^i E\right)} = 0, \quad (18)$$

where $S \equiv \sum\limits_i s^i$ et $p = \alpha / \sum\limits_{i=1}^{n} s^i$.

Equating the two preceding expressions, we obtain $\frac{\beta}{\gamma}(\gamma+q) = \alpha\dfrac{S-s^i}{S^2}$. Summing across i yields $\gamma + q = \frac{\alpha\gamma}{\beta}\left(\frac{n-1}{nS}\right)$. It follows that the equilibrium strategy of any trader i is:

$$\tilde{s}^i = \left(\frac{1-\alpha}{\beta}\right)\left(\frac{n-1}{n^2}\right)\frac{\gamma E}{\frac{n-(1-\alpha)}{n} - \gamma E},\ i \in \mathcal{O}. \tag{19}$$

The equilibrium supply of a strategic trader i does not depend upon i because the marginal rate of substitution of good 1 for good 2 is equal to $\frac{\alpha x_2^i}{(1-\alpha)x_1^i} = \frac{\beta}{\gamma}(\gamma+q)$ and is constant across agent.

As $\gamma + q = \frac{\alpha\gamma}{\beta}\left(\frac{n-1}{nS}\right)$, the equilibrium relative permits' price is then:

$$\tilde{q} = \frac{\alpha\dfrac{n-(1-\alpha)}{n} - \gamma E}{(1-\alpha)E}. \tag{20}$$

One remarks that the expression of the equilibrium value of $\tilde{q}$ is meaningful if the numerator in (20) is positive, that is if $\alpha\dfrac{n-(1-\alpha)}{n} - \gamma E > 0$. This implies that an equilibrium exists only if E is not too high.

To understand this condition, note that since $\tilde{s}^i$ does not depend on i, the equation $\gamma + q = \frac{\alpha\gamma}{\beta}\left(\frac{n-1}{nS}\right)$ can be written $\frac{\gamma}{\beta}\alpha\dfrac{n-1}{n^2 s} = \gamma + q$. In mark-up terms this is equivalent to state that $\tilde{p}\frac{\gamma}{\beta}\left(1-\frac{1}{n}\right) = \gamma + \tilde{q}$. This last equation means that the marginal value of pollution equals the marginal cost of pollution.[13] From (19), the higher is E, the higher is s^i. But for high value of s^i, q must be low (the higher s^i, the lower the marginal value of pollution). However, $\tilde{q}$ cannot be negative, so there is threshold for E above which the permits market cannot balance, which requires $\alpha\dfrac{n-(1-\alpha)}{n} - \gamma E > 0$.

On the limit the sign of $\tilde{q}$ equals the sign of $\lim_{n\to\infty}\left(\alpha\dfrac{n-(1-\alpha)}{n} - \gamma E\right) = \alpha - \gamma E > 0$ (this limit is equivalent to $p^*\frac{\gamma}{\beta} = \gamma + q^*$). The threshold is higher under perfect competition. Using (9) and (20), we see that $\tilde{q} - q^* = -\frac{\alpha\frac{n-(1-\alpha)}{n}}{(1-\alpha)E} < 0$: the relative price for permits is lower in the OE than in the CE. So, by behaving strategically the strategic traders pay a lower permits' price.

[13] The term $\frac{\gamma}{\beta}\alpha\dfrac{n-1}{n^2 s}$ is the marginal value of an additional unit of pollution (the increase in production due to a unit increase in pollution, *i.e.*, $\frac{\gamma}{\beta}$ times the marginal value of a unit increase of production, *i.e.*, $\alpha\dfrac{n-1}{n^2 s}$). The term $\gamma + q$ represents the marginal cost of a unit increase in pollution. To increase pollution by one unit requires to increase the input used by γ units, which results in an increase by γ ($p_2 = 1$) (the cost increases). Moreover, to pollute one more unit, a trader must buy an additional permit (or decrease its permits sales by one unit), and therefore has to pay (or to bear a loss equal to) q.

The equilibrium value of the emissions of any trader $i \in O$ is given by:

$$\tilde{e}^i = \frac{\left\{\dfrac{n-(1-\alpha)}{n^2} + \dfrac{\lambda^i}{1-\alpha}\left[\alpha\dfrac{n-(1-\alpha)}{n} - \gamma E\right]\right\}(1-\alpha)E}{\dfrac{n-(1-\alpha)}{n} - \gamma E}. \tag{21}$$

From (21), one remarks that more rights to pollute trivially increases the emissions of oligopolists in equilibrium. Thus, if a government were to give a higher amount of permits $\lambda^i E$ to oligopolist $i \in \mathcal{O}$, this oligopolist would have an interest to pollute more. In addition, if an oligopolist initially receives less than a proportion $\frac{1}{n}$, he has a positive excess demand for permits. Indeed, we can check that $\tilde{e}^i - \lambda^i E = (1-\alpha)\frac{n-(1-\alpha)}{n}\left(\frac{1}{n} - \lambda^i\right) E > 0$ (resp. < 0) whenever $\lambda^i < \frac{1}{n}$ (resp. $\lambda^i > \frac{1}{n}$).

As $\tilde{p} = \frac{\alpha}{\sum_i \tilde{s}^i}$, the equilibrium relative price of good 1 is:

$$\tilde{p} = \left(\frac{\alpha}{1-\alpha}\right)\left(\frac{\beta n}{n-1}\right)\frac{\dfrac{n-(1-\alpha)}{n} - \gamma E}{\gamma E}. \tag{22}$$

The equilibrium allocations follow:

$$(\tilde{x}_1^i, \tilde{x}_2^i) = \left(\frac{\Lambda}{\Delta}, \frac{\alpha(1-\alpha)}{n^2} + \lambda^i \psi\right), \ i \in \mathcal{O}, \tag{23}$$

$$(\tilde{x}_1, \tilde{x}_2) = \left(\frac{\left(\frac{1-\alpha}{\beta}\right)\left(\frac{n-1}{n}\right)\gamma E}{\Delta}, 1-\alpha\right), \tag{24}$$

where we define Δ, ψ and Λ as $\Delta \equiv \frac{n-(1-\alpha)}{n} - \gamma E > 0$, $\psi \equiv \alpha\frac{n-(1-\alpha)}{n} - \gamma E > 0$ and $\Lambda \equiv \left(\frac{1-\alpha}{\beta}\right)\gamma E\left(\frac{\alpha}{n^2} + \frac{\lambda^i}{1-\alpha}\psi\right)$.

We summarize the preceding discussion with the following proposition.

Proposition 2 *The equilibrium strategies and prices are as follows:*

$$(\tilde{s}^i, \tilde{e}^i) = \left(\left(\frac{1-\alpha}{\beta}\right)\left(\frac{n-1}{n^2}\right)\frac{\gamma E}{\Delta}, \frac{\left\{\dfrac{n-(1-\alpha)}{n^2} + \dfrac{\lambda^i \psi}{1-\alpha}\right\}(1-\alpha)E}{\Delta}\right),$$

$$(\tilde{p}, \tilde{q}) = \left(\left(\frac{\alpha}{1-\alpha}\right)\left(\frac{\beta n}{n-1}\right)\frac{\Delta}{\gamma E}, \frac{\psi}{(1-\alpha)E}\right).$$

The corresponding payments may be deduced from (23) and (24):

$$\tilde{U}^i = \alpha \ln\left(\frac{\Lambda}{\Delta}\right) + (1-\alpha)\ln\left(\frac{\alpha(1-\alpha)}{n^2} + \lambda^i \psi\right), \ i \in \mathcal{O}, \tag{25}$$

$$\tilde{U} = \alpha \ln \left(\frac{\frac{n-1}{\beta n}\gamma E}{\Delta} \right) + \ln(1-\alpha). \tag{26}$$

3.3 The oligopoly equilibrium and the emissions ceiling E

We now consider the effects of an increase in the emissions ceiling E on the oligopoly equilibrium. These effects are as follows:

Proposition 3 *The equilibrium relative prices decrease with the emissions ceiling E. But, though the equilibrium supply s^i of good 1 is always increasing with E, the equilibrium emissions strategies are not always increasing with E.*

Proof. The first part of the Proposition stems from (19) and (22). From this, one easily sees that $\frac{\partial \tilde{s}^i}{\partial E} > 0$ since $\tilde{s}^i = \frac{\alpha}{n}\frac{1}{\tilde{p}}$. Finally, from (21) one deduces that $\frac{\partial \tilde{e}^i}{\partial E} = \frac{\gamma}{\frac{n-(1-\alpha)}{n} - \gamma E}\tilde{e}^i + \frac{1}{E}\tilde{e}^i - \lambda^i \frac{\gamma E}{\frac{n-(1-\alpha)}{n} - \gamma E}$. Using the equilibrium value of e^i, the inequality $\frac{\partial \tilde{e}^i}{\partial E} > 0$ reduces to:

$$(1-\alpha)\left(\frac{n-(1-\alpha)}{n^2}\right) + \lambda^i \left(\alpha(\frac{n-(1-\alpha)}{n}) - 2\gamma E + \frac{\gamma^2 E^2 n}{n-(1-\alpha)}\right) > 0. \tag{27}$$

But if this expression is negative, it must be so with $\lambda^i = 1$. This leads us to study the sign of the following polynomial:

$$P(E) = (1-\alpha)\left(\frac{n-(1-\alpha)}{n^2}\right) + \left(\alpha(\frac{n-(1-\alpha)}{n}) - 2\gamma E + \frac{\gamma^2 E^2 n}{n-(1-\alpha)}\right). \tag{28}$$

We observe that:

$$P(\frac{\alpha(n-(1-\alpha))}{\gamma n}) = \frac{n-(1-\alpha)}{n}(1-\alpha)(\frac{1}{n} - \alpha). \tag{29}$$

Moreover:

$$P'(\frac{\alpha(n-(1-\alpha))}{\gamma n}) = 2\gamma(\alpha - 1) < 0. \tag{30}$$

Therefore when $1/n > \alpha$ we always have $\frac{\partial \tilde{e}^i}{\partial E} > 0$, whatever the value of λ^i. If on the other hand $1/n < \alpha$, then for λ^i close enough to 1 and E close enough to $\frac{\alpha(n-(1-\alpha))}{\gamma n}$, we have: $\frac{\partial \tilde{e}^i}{\partial E} < 0$. ■

When there is an increase in the emissions ceiling E, there is a usual direct supply effect on the relative price of permits. Indeed as the total supply of permits exceeds the demand, the relative price $\tilde{q}$ goes downward (see (20)). In addition there is also an indirect decreasing effect on $\tilde{p}$ because the strategic traders can increase the quantity of emissions, and thereby sell more units of good 1.

3.4 Relation between the OE and the CE

We now investigate whether the OE of the replicated economy coincides with the CE of the same replicated economy. To do this we consider a replication procedure *à la* Debreu-Scarf (1963). This procedure consists in "cloning" symmetrically both sides of the market of the basic economy, while keeping the emissions ceiling E constant.

Proposition 4 *When the economy $\xi(r)$ is replicated an infinite number of times, the limit oligopoly equilibrium is the competitive equilibrium of the finite economy.*

Proof. The replicated economy comprises of nr oligopolists. More specifically, there are r sets $\mathcal{O}^j$, $j = 1, ..., r$, with $\mathcal{O}(r) = \cup_{j=1}^{r}\mathcal{O}^j$. The index ij denotes oligopolist ij, $ij \in \mathcal{O}(r)$. In addition, one has for all $j = 1, ..., r$: $\omega^{ij} = \omega^i$, $y^{ij} = \frac{\gamma}{\beta}e^{ij}$ and $S^{ij} = S^i$, $ij \in \mathcal{O}$ and $\omega^j = \omega$.

The complete characterization of the OE of the replicated economy is presented in Appendix B. The equilibrium strategies are given by:

$$\tilde{s}^{\,ij} = \left(\frac{1-\alpha}{\beta}\right)\left(\frac{rn-1}{rn^2}\right)\frac{\gamma E}{\frac{rn-(1-\alpha)}{rn} - \gamma E}, \tag{31}$$

$$\tilde{e}^{\,ij} = \frac{\left\{\frac{rn-(1-\alpha)}{rn^2} + \frac{\lambda^{ij}r}{1-\alpha}\left[\alpha\frac{rn-(1-\alpha)}{rn} - \gamma E\right]\right\}(1-\alpha)E}{\frac{rn-(1-\alpha)}{rn} - \gamma E}, ij \in \mathcal{O}(r), \tag{32}$$

and

$$\tilde{\mathbf{x}}^{\,ij} = \left(\frac{\left(\frac{1-\alpha}{\beta}\right)\gamma E\left[\frac{\alpha}{n^2} + \frac{\lambda^{ij}r}{1-\alpha}\psi(r)\right]}{\Delta(r)}, \frac{\alpha(1-\alpha)}{n^2} + \lambda^{ij}r\psi(r)\right), \; ij \in \mathcal{O}(r), \tag{33}$$

$$\tilde{\mathbf{x}}^{\,j} = \left(\frac{\left(\frac{1-\alpha}{\beta}\right)\left(\frac{rn-1}{rn}\right)\gamma E}{\Delta(r)}, 1-\alpha\right), \; j = 1, \ldots, r, \tag{34}$$

where $\Delta(r) \equiv \frac{rn-(1-\alpha)}{rn} - \gamma E > 0$ and $\psi(r) \equiv \alpha\frac{rn-(1-\alpha)}{rn} - \gamma E > 0$.

The equilibrium values of the prices are:

$$\tilde{q}(r) = \frac{\alpha\frac{rn-(1-\alpha)}{rn} - \gamma E}{(1-\alpha)E}, \tag{35}$$

$$\tilde{p}(r) = \left(\frac{\alpha}{1-\alpha}\right)\left(\frac{\beta rn}{rn-1}\right)\frac{\frac{rn-(1-\alpha)}{rn} - \gamma E}{\gamma E}. \tag{36}$$

Using the assumption that $\lambda^{ij} = \frac{\lambda^i}{r}$, we have:

$$\lim_{r\to\infty} \tilde{s}^{\,ij} = \frac{(1-\alpha)\gamma E}{(1-\gamma E)\beta n}, \tag{37}$$

$$\lim_{r\to\infty} \tilde{e}^{\,ij} = \frac{\left[\frac{1}{n} + \frac{\lambda^i}{1-\alpha}(\alpha - \gamma E)\right](1-\alpha)E}{1-\gamma E}, \tag{38}$$

$$\lim_{r\to\infty} \tilde{q}(r) = \frac{\alpha - \gamma E}{(1-\alpha)E}, \tag{39}$$

$$\lim_{r\to\infty} p^*(r) = \alpha\left(\frac{\beta}{1-\alpha}\right)\frac{(1-\gamma E)}{\gamma E}, \tag{40}$$

$$\lim_{n\to\infty}\left(\tilde{x}_1^{ij}, \tilde{x}_2^{ij}\right) = \left(\frac{\gamma}{\beta}\left(\frac{\alpha-\gamma E}{1-\gamma E}\right)\lambda^i E, \lambda^i(\alpha - \gamma E)\right), \tag{41}$$

$$\lim_{n\to\infty}\left(\tilde{x}_1^{j}, \tilde{x}_2^{j}\right) = \left(\frac{(1-\alpha)\gamma E}{\beta(1-\gamma E)}, 1-\alpha\right). \tag{42}$$

One can conclude that $\lim_{r\to\infty}(\tilde{p}(r), \tilde{q}(r)) = (p^*(r), q^*(r))$ and, $\lim_{r\to\infty} \tilde{\mathbf{x}}^{\,ij} = (\mathbf{x}^{ij})^*$, $ij \in \mathcal{O}(r)$, and $\lim_{n\to\infty} \tilde{\mathbf{x}}^{j} = (\mathbf{x}^{j})^*$. The sum across each type i of $\lim_{r\to+\infty} e^{ij}$ is equal to E and is therefore feasible for the finite competitive economy. We have thus shown that the limit decisions and limit prices in the limit oligopolistic equilibrium correspond to the competitive equilibrium for the finite economy. ■

We can deduce from the above proposition that when the number of agents increases unboundedly on each side of the market, the OE strategies-prices-allocations tend to coincide with those of a CE of the enlarged economy. In such a case, the market power of each oligopolist vanishes. The strategic traders can no longer exert any influence, even negligible, on the equilibrium relative prices.

4 Welfare Comparisons

In this section we study and compare the agents' welfare across different equilibria. In addition, we study the Pareto optimality of the OE.

4.1 Welfare comparisons

We here focus on the negative implications caused by imperfectly competitive behavior. Two results are obtained. Proposition 4 considers a case in which the emissions ceiling is held constant, while Proposition 5 assumes that this emissions ceiling varies across the different equilibria considered.

Proposition 5 *The OE with pollution permits is not Pareto dominated by the CE with pollution permits.*

Proof. We first compare $\tilde{U}^i$ with $(U^i)^*$. From (25) and (13), one has $\tilde{U}^i - (U^i)^* = \alpha \ln(1-\gamma E) + \ln\left(\frac{\alpha(1-\alpha)}{n^2} + \lambda^i\left[\alpha\frac{n-(1-\alpha)}{n} - \gamma E\right]\right) - \alpha\ln\left(\frac{n-(1-\alpha)}{n} - \gamma E\right) - \ln\lambda^i(\alpha - \gamma E)$. So $\tilde{U}^i - (U^i)^* = \alpha\ln\left(\frac{1-\gamma E}{\frac{n-(1-\alpha)}{n} - \gamma E}\right) + \ln\left(\frac{\frac{\alpha(1-\alpha)}{n^2} + \lambda^i\left[\alpha\frac{n-(1-\alpha)}{n} - \gamma E\right]}{\lambda^i(\alpha-\gamma E)}\right)$. The first term in brackets is positive. The second term is positive if $\lambda^{ij} \leq \frac{1}{n}$. Then for all $i \in \mathcal{O}$ for which $\lambda^{ij} \leq \frac{1}{n}$, it is certainly true that $\tilde{U}^i - (U^i)^* > 0$. Finally, one can have $\tilde{U}^i < (U^i)^*$ for the oligopolists for which $\lambda^i > \frac{1}{n}$.

We now compare $\tilde{U}$ with U^*. From (26) and (14), one gets $\tilde{U} - U^* = \alpha\ln\left((\frac{n-1}{n})\frac{1-\gamma E}{\frac{n-(1-\alpha)}{n} - \gamma E}\right)$. We see that this expression is negative since all the terms in the log are lower than 1. ■

The absence of Pareto domination between both equilibria is due to the fact that when the number of strategic traders increases, or when competition is fiercer, they exert less control on the equilibrium prices. As a result they obtain a lower level of utility. The converse is true for the representative competitive agent.

Comparing the equilibria with pollution permits with the equilibria without pollution permits is also instructive (see Appendixes A1 and A2 for a computation of the equilibria without pollution permits).[14]

Proposition 6 *The OE (CE) with pollution permits is not Pareto dominated by the OE (CE) without pollution permits.*

Proof. First, consider the comparisons between the OE with pollution permits and the OE without pollution permits. For $i \in \mathcal{O}$, simple calculations lead to:

$$\tilde{U}^i - \hat{U}^i = \alpha\ln\left(\frac{(1-\alpha)\gamma E\left\{\frac{\alpha}{n^2} + \frac{\lambda^i}{1-\alpha}\psi\right\}}{\left(\frac{\alpha}{n}\right)^2\Delta}\right) + (1-\alpha)\ln\left(1 + \frac{\lambda^i\psi}{\frac{\alpha(1-\alpha)}{n^2}}\right). \tag{43}$$

The sign of this expression is negative when $\lambda^i = 0$. It is positive when $\lambda^i = 1$ and n is high enough. Therefore it is in general indeterminate. Little algebra yields $\tilde{U} - \hat{U} = \alpha\ln\left(\frac{(1-\alpha)\gamma E}{\alpha\left(\frac{n-(1-\alpha)}{n} - \gamma E\right)}\right)$. In addition one has $\frac{(1-\alpha)\gamma E}{\alpha\left(\frac{n-(1-\alpha)}{n} - \gamma E\right)} < 1$ since $\alpha\frac{n-(1-\alpha)}{n} - \gamma E > 0$. We deduce from this that $\tilde{U} < \hat{U}$.

Second, we compare the competitive equilibria. It is immediate that $(U^i)^* - \breve{U}^i = +\infty$, so $(U^i)^* > \breve{U}^i$ for $i \in \mathcal{O}$. In addition, one has $U^* - \breve{U} = \alpha\ln\left(\frac{\gamma E}{\alpha(1-\gamma E)}\right) + \alpha\ln(1-\alpha)$. One can therefore see that $U^* < \breve{U}$. ■

[14] Here, we notice that pollution is not taken into account in the utility function. Alternatively, we can assume that when pollution enters the utility function, but in a separable way, the difference in the pollution levels obtained in both kinds of equilibria is not important.

The pollution permits create a limit to the extent of the production activities of the strategic traders. The exchange of permits, however, can allow some oligopolists to produce more quantity of good 1 by buying pollution permits. This happens when their preference for this good is high enough. The quantity of commodity one brought to the market, however, is lower than the quantity the strategic traders would have brought without restrictions on emissions: from (19) and (80) (in the appendix), in any case, one gets for all $i \in \mathcal{O}$, $\tilde{s}^i - \hat{s}^i = \frac{1}{\beta}\left(\frac{n-1}{n^2}\right)\frac{\gamma E - \alpha\frac{n-(1-\alpha)}{n}}{\frac{n-(1-\alpha)}{n} - \gamma E} < 0$.

In the case of perfect competition, the relative price p^* exceeds the competitive price without permits $\breve{p}$, since $p^* - \breve{p} = \beta\left(\frac{\alpha - \gamma E}{(1-\alpha)\gamma E}\right) > 0$. The strategic traders consume a positive amount of both goods since their supply is less than their production (the permit market limits the supply). The competitive trader faces a higher price but the quantity effect driven by higher consumption in commodity 1 cancels the price effect. This, again, prevails if the preference for good 1 is high enough.

4.2 Non-Pareto Optimality of the OE with Pollution Permits

We now study the Pareto-optimality of the OE. We can state the next proposition:

Proposition 7 *The OE is not Pareto optimal.*

Proof. We first study the Pareto optimal allocations where all agents have positive consumptions (this is meaningful since this is the case in an OE and we want to study the Pareto-optimality of the later). Second we show that an OE is not Pareto-optimal.

All the Pareto-optimal allocations where all agents have positive consumptions solve the next program for a given set of social weights θ^i, where $\theta^i > 0$, for $i = 1, \ldots, n$:

$$\begin{aligned}
\underset{x_1^i,\, x_2^i,\, x_1,\, x_2,\, z^i}{\text{maximize}} \quad & \sum_{i=1}^{n} \theta^i \Big(\alpha \ln x_1^i + (1-\alpha)\ln x_2^i\Big) + \alpha \ln x_1 + (1-\alpha)\ln x_2, \\
\text{subject to} \quad & \sum_{i=1}^{n} x_1^i + x_1 = Y, \\
& \sum_{i=1}^{n} x_2^i + x_2 + \sum_{i=1}^{n} z^i = 1, \\
& Y = \frac{\sum_{i=1}^{n} z^i}{\beta}, \\
& \frac{\sum_{i=1}^{n} z^i}{\gamma} = E.
\end{aligned} \tag{P3}$$

The first order optimality conditions for this problem are:

$$\frac{\theta^i}{x_1^i} = \frac{1}{\frac{\gamma}{\beta}E - \sum_{j=1}^n x_1^j}, \tag{44}$$

$$\frac{\theta^i}{x_2^i} = \frac{1}{1 - \gamma E - \sum_{j=1}^n x_2^j}. \tag{45}$$

Summing the first condition across j yields after a little algebra:

$$x_1^i = \frac{\theta_i \frac{\gamma E}{\beta}}{1 + \sum_{j=1}^n \theta^j}. \tag{46}$$

By symmetry, we also obtain:

$$x_2^i = \frac{\theta^i (1 - \gamma E)}{1 + \sum_{j=1}^n \theta^j}. \tag{47}$$

Using the two above equations, one gets:

$$\frac{x_2^i}{x_1^i} = \frac{1 - \gamma E}{\frac{\gamma}{\beta}E}. \tag{48}$$

Using (23) to compute the ratio of the consumptions of an oligopolist in an oligopoly equilibrium, we obtain after some algebra:

$$\frac{\tilde{x}_2^i}{\tilde{x}_1^i} = \frac{\frac{n-(1-\alpha)}{n} - \gamma E}{\frac{\gamma}{\beta}E}. \tag{49}$$

Comparing (48) with (49), we see that:

$$\frac{\tilde{x}_2^i}{\tilde{x}_1^i} < \frac{x_2^i}{x_1^i}. \tag{50}$$

Therefore, the OE is no Pareto-optimal. ■

In an oligopoly equilibrium, any oligopolist consumes relatively more of good 1 than in a Pareto-optimal allocation. The reason stems from their market power. By rationing the supply of good 1, they distort the relative prices and they obtain a relatively cheap price for the good 2. In addition, the supply of commodity 1 is lower in the OE with pollution permits. So the emissions permits market is not sufficient to eliminate market distortions. This leads us to investigate whether subsidizing the supply of good 1 may be Pareto-improving. This issue is taken up in the next section.

5 A Pareto-Improving Subsidizing Policy

In this section, we study a subsidy to the supply of good 1 by the oligopolists. This subsidy is financed by taxing the endowments of the representative trader. Such a policy is in contrast with the taxation of the oligopolists' endowments studied by Gabszewicz and Grazzini (1999). We will show that this subsidy policy is Pareto-improving when the amount of the subsidy is small (and some mild conditions are met). To ease the analysis, we assume that the oligopolists are endowed with the same amount of pollution permits, namely, $\lambda^i = 1/n$, $i = 1, \ldots, n$.

5.1 The fiscal setup

We assume that each oligopolist receives a subsidy τ per unit of good 1 sold on the market. The sum of the subsidies are financed by a lump-sum tax T on the competitive agent. We thus have:

$$\tau \sum_{k=1}^{n} s^k = T. \tag{51}$$

The subsidy policy impacts the equilibrium of the market for good 1. In effect, the equilibrium condition can now be written:

$$\sum_{k=1}^{n} s^k = \frac{\alpha(1-T)}{p}, \tag{52}$$

from which we obtain:

$$p = \frac{\alpha(1-T)}{\sum_{k=1}^{n} s^k}. \tag{53}$$

This gives a new expression for the demand function of good 1. The corresponding new value of the oligopolists' profits is: $(p+\tau)s^i - (\gamma+q)e^i + q\frac{E}{n}$.

5.2 Study of the oligopoly equilibrium with a subsidy

Each oligopolist now solves the following problem:

$$\max_{s^i,\, e^i} \alpha \ln\left(\frac{\gamma}{\beta}e^i - s^i\right) + (1-\alpha)\ln\left((p+\tau)s^i - (\gamma+q)e^i + q\frac{E}{n}\right), \tag{P4}$$

where p is given by (53). The first-order conditions are given by the following two equations:

$$\frac{\alpha}{\frac{\gamma}{\beta}e^i - s^i} = \frac{(1-\alpha)\left(p+\tau - s^i \frac{\alpha(1-T)}{(\sum_{k=1}^{n} s^k)^2}\right)}{(p+\tau)s^i - (\gamma+q)e^i + q\frac{E}{n}}, \tag{54}$$

$$\frac{\gamma}{\beta}\frac{\alpha}{\frac{\gamma}{\beta}e^i - s^i} = \frac{(1-\alpha)(\gamma+q)}{(p+\tau)s^i - (\gamma+q)e^i + q\frac{E}{n}}. \tag{55}$$

Since the strategic traders are given the same amount of pollution permits, we shall concentrate on a symmetric equilibrium (*i.e.*, $s^i = s$ for all i).

Combining the above two first-order conditions, we get:

$$\gamma + q = \frac{\gamma}{\beta}\Big(p + \tau - s^i \frac{\alpha(1-T)}{\big(\sum_{k=1}^{n} s^k\big)^2}\Big). \tag{56}$$

Using the balanced budget condition and the symmetry assumption (that is $s^i = s$ for all i), we get:

$$\gamma + q = \frac{\gamma}{\beta}\Big(\frac{\alpha(1-ns\tau)}{ns}(1-\frac{1}{n}) + \tau\Big). \tag{57}$$

Using the first-order condition for the optimal choice of e with $e^i = E/n$ for all i, we have:

$$\frac{\gamma}{\beta}\frac{\alpha}{\frac{\gamma}{\beta}\frac{E}{n} - s} = \frac{(1-\alpha)(\gamma+q)}{\frac{\alpha-\gamma E}{n} + (1-\alpha)\tau s}. \tag{58}$$

Using the above equations, we obtain the following proposition:

Proposition 8 *There exists an oligopoly equilibrium with subsidy to good 1 when the amount of the subsidy is low.*

Proof. See Appendix C. ■

Remark. It can be shown with the l'Hospital's rules that:

$$\lim_{\tau \to 0+} s(\tau) = \frac{(1-\alpha)(1-\frac{1}{n})\frac{\gamma E}{\beta n}}{\frac{n-(1-\alpha)}{n} - \gamma E} = \tilde{s}^i.$$

Since we may define $s(0) = \tilde{s}^i$ (the value of the oligopolists' supply of good 1 when the subsidy is nil is the value obtained in the oligopoly equilibrium without subsidy), we see that $s(.)$ is continuous in τ (at least in a relevant neighborhood of 0).

We now investigate the effect of a subsidy on the value of supply $s(\tau)$. Applying the implicit function theorem to equation (58), we obtain:

$$\frac{\partial s}{\partial \tau} = \frac{\Big(\frac{\gamma E}{\beta n}(1-\alpha)\big((1-\alpha)n+\alpha\big) - s(\tau)(1-\alpha)(\alpha+n)\Big)s(\tau)}{2s\tau(1-\alpha)(\alpha+n) + \alpha\left(\frac{n-(1-\alpha)}{n} - \gamma E\right) - \tau\frac{\gamma E}{\beta n}(1-\alpha)\big((1-\alpha)n+\alpha\big)}.$$

Since $s(\tau)$ is continuous at 0, we may define:

$$\frac{\partial s}{\partial \tau}\Big|_{\tau=0} = \lim_{\tau \to 0+}\frac{\partial s}{\partial \tau} = \frac{\Big(\frac{\gamma E}{\beta n}(1-\alpha)\big((1-\alpha)n+\alpha\big) - s(0)(1-\alpha)(\alpha+n)\Big)s(0)}{\alpha\left(\frac{n-(1-\alpha)}{n} - \gamma E\right)}.$$

Using the expression of $s(0)$ in the above definition, we obtain:

$$\frac{\partial s}{\partial \tau}\mid_{\tau=0}= \frac{\frac{\gamma E}{\beta n}(1-\alpha)\Big(\alpha-\big((1-\alpha)n+\alpha\big)\gamma E\Big)s(0)}{\alpha\big(\frac{n-(1-\alpha)}{n}-\gamma E\big)^2}.$$

From this expression, we deduce the following proposition:

Proposition 9 *A necessary condition for a subsidy policy to have a positive effect on the oligopolists' supply is:*

$$\alpha > \frac{\gamma n E}{1+(n-1)\gamma E}. \tag{59}$$

To understand this condition, let us start again with the first-order condition regarding the choice of s^i, that is:

$$\frac{\alpha}{\frac{\gamma}{\beta}e^i - s^i} = \frac{(1-\alpha)\Big(p+\tau-s^i\frac{\alpha(1-T)}{(\sum_{j=1}^n s^j)^2}\Big)}{(p+\tau)s^i-(\gamma+q)e^i+q\frac{E}{n}}.$$

In a symmetric equilibrium all agents choose the same strategic supply s and the same level of emissions E/n. So the above expression reduces to:

$$\frac{\alpha}{\frac{\gamma}{\beta}\frac{E}{n}-s} = \frac{(1-\alpha)\Big(\frac{\alpha(1-ns\tau)}{ns}(1-\frac{1}{n})+\tau\Big)}{\frac{\alpha-\gamma E}{n}+(1-\alpha)s\tau}, \tag{60}$$

$$= \frac{(1-\alpha)\Big(\alpha(\frac{1}{ns}-\tau)(1-\frac{1}{n})+\tau\Big)}{\frac{\alpha-\gamma E}{n}+(1-\alpha)s\tau}, \tag{61}$$

$$= \frac{(1-\alpha)\Big(\frac{\alpha}{ns}(1-\frac{1}{n})+\big(1-\alpha(1-\frac{1}{n})\big)\tau\Big)}{\frac{\alpha-\gamma E}{n}+(1-\alpha)s\tau}. \tag{62}$$

This equation determines implicitly a relation between the subsidy rate τ and the strategic supply s. When the right-hand side is a decreasing function of τ, the strategic supply is decreasing with respect to τ (because the left-hand side of the equation is increasing with respect to s, whereas the right-hand side is decreasing with respect to s).[15] But in equilibrium we know that

[15] It is not always true, however, that the right hand side of the equation is decreasing with τ. First of all, let us notice that the marginal revenue of an oligopolist is always increasing with the subsidy rate (assuming that the strategic supply is constant – that is before the later reacts to a change in the subsidy value). This is because the increase in the subsidy is always higher than the decrease in the demand of the competitive agents (whose income is taxed to finance the increase in the subsidy). Second, the consumption of good 2 is also increasing with respect to the subsidy (the explanation is the same as that given for the increase in the marginal revenue of the strategic supply). Formally, both the numerator and the denominator of the right-hand side increase with the subsidy. So, an increase in the subsidy has an ambiguous effect on the marginal utility resulting from the consumption of good 2 (which is equal to the right-hand side of the equation).

$s = s(0) = \lim_{\tau \to 0+} s(\tau) = \tilde{s}^i$. Therefore, when the subsidy rate is small, *i.e.*, when τ is close to 0, the strategic supply is a decreasing function of α (and an increasing function of E). Looking at equation (62), we see that when s is close to zero, the right-hand side is an increasing function of the subsidy. Thus, when α is relatively high (and E relatively low), the right-hand side is an increasing function of the subsidy (and so is the strategic supply). The condition (59) states the precise relation between α and E under which the strategic supply is an increasing function of the subsidy.

5.3 Welfare effects of the subsidy policy

5.3.1 The competitive agent

Let W be the value of the indirect utility of the representative competitive agent. Using (51)-(53), we have:

$$W = \alpha \ln\Big(\frac{\alpha(1-T)}{p}\Big) + (1-\alpha)\ln\Big((1-\alpha)(1-T)\Big), \tag{63}$$

$$= (1-\alpha)\ln(1-n\tau s) + \alpha \ln ns + \kappa, \tag{64}$$

where $\kappa = (1-\alpha)\ln(1-\alpha)$. We thus have:

$$\frac{\partial W}{\partial \tau} = \frac{-(1-\alpha)n(\tau\frac{\partial s}{\partial \tau} + s)}{1-n\tau s} + \alpha\frac{\frac{\partial s}{\partial \tau}}{s}. \tag{65}$$

Evaluating the above expression at $\tau = 0$, we get:

$$\frac{\partial W}{\partial \tau}\,|_{\tau=0} = \lim_{\tau \to 0+} \frac{-(1-\alpha)n(\tau\frac{\partial s}{\partial \tau} + s)}{1-n\tau s} + \alpha\frac{\frac{\partial s}{\partial \tau}}{s}, \tag{66}$$

$$= -(1-\alpha)ns(0) + \alpha\frac{\frac{\partial s}{\partial \tau}\,|_{\tau=0}}{s(0)}. \tag{67}$$

Using equation the expression of $\frac{\partial s}{\partial \tau}\,|_{\tau=0}$, we find after some tedious algebra:

$$\frac{\partial W}{\partial \tau}\,|_{\tau=0} \sim \alpha - (1-\alpha)(n-1)\frac{n-(1-\alpha)}{n} - \gamma E. \tag{68}$$

5.3.2 The oligopolists

Let $W^i(\tau)$ be the indirect utility of an oligopolist in equilibrium. We then have:

$$W^i(\tau) = \alpha \ln\left(\frac{\gamma}{\beta}\frac{E}{n} - s\right) + (1-\alpha)\ln\left(\frac{\alpha-\gamma E}{n} + \tau s(1-\alpha)\right). \tag{69}$$

It can be shown that a sufficient condition for the oligopolists' welfare to increase with the subsidy is:

$$(1-\alpha)(1-\frac{1-\alpha}{n}) - \frac{\alpha}{n} + \frac{\alpha}{n}E > 0. \tag{70}$$

Inspecting the previous expression leads to the following Proposition:

Proposition 10 *Suppose that* $\alpha \in]\frac{n-1}{n}, \frac{-(n-1)+\sqrt{(n-1)(n+3)}}{2}[$. *Then if* E *is small enough so that:*

$$\alpha - (1-\alpha)(n-1)\frac{n-(1-\alpha)}{n} - \gamma E > 0, \tag{71}$$

$$(1-\alpha)(1-\frac{1-\alpha}{n}) - \frac{\alpha}{n} + \frac{\alpha}{n}E > 0, \tag{72}$$

a subsidy policy is Pareto-improving provided that the size of the subsidy be small.

Proof. See Appendix D. ■

Under our assumptions, we can see that when a subsidy policy is Pareto-improving then it must increase the supply of good 1. Indeed, we can show that if $\frac{n-(1-\alpha)}{n} - \gamma E > 0$ (a necessary condition for the existence of an oligopoly equilibrium), then we have:

$$\alpha - ((1-\alpha)n + \alpha)\gamma E > \alpha - (1-\alpha)(n-1)\left(\frac{n-(1-\alpha)}{n}\right) - \gamma E.$$

6 Conclusion

This paper provides a study of a second best policy in a general equilibrium model with a competitive tradable permits market and imperfect competition on the output market. Our study extends the literature on pollution permits to cover strategic interactions within interrelated markets. Several kinds of behavior take place between the two sides of the market: one side is competitive while the other side is strategic. Our model displays behavioral heterogeneity and throws light on its consequences on the welfare properties of general oligopoly equilibria. We have shown that subsidizing the supply of strategic traders increases the amount of trade (despite the fact that the subsidies are financed by a lump-sum tax on the competitive agent). This policy also increases individual welfare provided agents' preferences for the commodity produced with the polluting good is sufficiently high.

The analysis of this paper could be extended along different ways. First, imperfectly competitive behavior on the permits market could be taken into account. The analysis would be more complex since the distribution of permits across traders would matter. Second, we could also consider using more general specifications of the preferences and the technology instead of relying on log-linear utility functions and a linear production function.

A Equilibria without pollution permits

A.1 CE without pollution permits

Without pollution permits (10)-(14) become respectively:

$$\breve{p} = \beta, \tag{73}$$

$$\left(\breve{x}_1^i, \breve{x}_2^i\right) = (0,0), \; i \in \mathcal{O}, \tag{74}$$

$$\left(\breve{x}_1, \breve{x}_2\right) = \left(\frac{\alpha}{\beta}, 1-\alpha\right), \tag{75}$$

$$\breve{U}^i = -\infty, i \in \mathcal{O}, \tag{76}$$

$$\breve{U} = \alpha \ln\left(\frac{\alpha}{\beta}\right) + (1-\alpha)\ln(1-\alpha). \tag{77}$$

A.2 OE without pollution permits

Without pollution permits (17) and (18) become respectively:

$$\frac{-\alpha}{\frac{\gamma}{\beta}e^i - s^i} + \frac{\alpha(1-\alpha)\left(\frac{S-s^i}{S^2}\right)}{ps^i - \gamma e^i} = 0, \tag{78}$$

$$\frac{\alpha\frac{\gamma}{\beta}}{\frac{\gamma}{\beta}e^i - s^i} - \frac{(1-\alpha)(\gamma+q)}{ps^i - \gamma e^i} = 0. \tag{79}$$

From the above conditions we find that the equilibrium strategies are:

$$\hat{s}^i = \frac{\alpha}{\beta}\left(\frac{n-1}{n^2}\right), i \in \mathcal{O}, \tag{80}$$

$$\hat{e}^i = \frac{\alpha}{\gamma}\left[\frac{n-(1-\alpha)}{n^2}\right], i \in \mathcal{O}. \tag{81}$$

The equilibrium relative price follows:

$$\hat{p} = \beta\frac{n}{n-1}.$$

The corresponding equilibrium allocation is given by the following expressions:

$$\left(\hat{x}_1^i, \hat{x}_2^i\right) = \left(\frac{1}{\beta}\left(\frac{\alpha}{n}\right)^2, \alpha(1-\alpha)\frac{1}{n^2}\right), \; i \in \mathcal{O}, \tag{82}$$

$$\left(\hat{x}_1, \hat{x}_2\right) = \left(\frac{\alpha}{\beta}\frac{n-1}{n}, 1-\alpha\right). \tag{83}$$

The utility levels reached are then:

$$\hat{U}^i = \alpha\ln\left(\frac{1}{\beta}\left(\frac{\alpha}{n}\right)^2\right) + (1-\alpha)\ln\left(\frac{\alpha(1-\alpha)}{n^2}\right), \; i \in \mathcal{O}, \tag{84}$$

$$\hat{U} = \alpha\ln\left(\frac{\alpha}{\beta}\frac{n-1}{n}\right) + (1-\alpha)\ln(1-\alpha). \tag{85}$$

B Replication of the OE

The competitive step. The vector of competitive demand functions for commodities 1 and 2 by any j is given by: $\mathbf{x}^j = (\alpha, 1-\alpha)$, $j = 1, ..., r$. The aggregate demand function in good 1 which is addressed to the oligopolists is thus $\sum_{j=1}^{r} x_1^j = r\alpha\frac{1}{p}$. The market-clearing condition for good 1 then writes $r\alpha\frac{1}{p} = \sum_{i=1}^{n}\sum_{j=1}^{r} s^{ij}$. The market clearing condition (14) is now given by $p = \frac{\alpha r}{\sum_{i=1}^{n}\sum_{j=1}^{r} s^{ij}}$. For given prices $(\mathbf{p}, q)$ and for given strategies $\left(s^{ij}, e^{ij}\right)$ of the oligopolist ij, the objective function of oligopolist is written as: $V^{ij}(s^{ij}, e^{ij}) \equiv \alpha \ln\left(\frac{\gamma}{\beta}e^{ij} - s^{ij}\right) + (1-\alpha)\ln \Pi^{ij}(s^{ij}, e^{ij})$.

The strategic step. Given an nr-tuple of strategies $\mathbf{s} = \left(s^{11}, \ldots, s^{ij}, \ldots, s^{nr}\right)$, with $\mathbf{s} \in \prod_{i=1}^{nr} S^i$, the program of any oligopolist ij is given by:

$$\max_{(s^{ij}, e^{ij})} \ln\left(\frac{\gamma e^{ij}}{\beta} - s^{ij}\right)^{\alpha}\left(\frac{\alpha r s^{ij}}{\sum_{i=1}^{n}\sum_{j=1}^{r} s^{ij}} - (\gamma + q)\, e^{ij} + q\lambda^{ij} rE\right)^{1-\alpha}. \quad \text{(P5)}$$

The optimality conditions $\partial V^{ij}/\partial s^{ij} = 0$ and $\partial V^{ij}/\partial e^{ij} = 0$ now yield:

$$\frac{-\alpha}{\frac{\gamma}{\beta}e^{ij} - s^{ij}} + \frac{\alpha(1-\alpha)r\frac{\sum_{i=1}^{n}\sum_{j=1}^{r} s^{ij} - s^{ij}}{\left(\sum_{j=1}^{r}\sum_{i=1}^{n} s(a_{ij})\right)^2}}{ps^{ij} - \gamma e^{ij} - q\left(e^{ij} - \lambda^{ij} rE\right)} = 0, \quad (86)$$

$$\frac{\alpha\frac{\gamma}{\beta}}{\frac{\gamma}{\beta}e^{ij} - s^{ij}} - \frac{(1-\alpha)(\gamma+q)}{ps^{ij} - \gamma e^{ij} - q\left(e^{ij} - \lambda^{ij} rE\right)} = 0. \quad (87)$$

This leads to the equilibrium strategies:

$$\tilde{s}^{ij}(r) = \left(\frac{1-\alpha}{\beta}\right)\left(\frac{rn-1}{rn^2}\right)\frac{\gamma E}{\frac{rn-(1-\alpha)}{rn} - \gamma E},\ ij \in \mathcal{O}(r), \quad (88)$$

$$\tilde{e}^{ij}(r) = \frac{\left\{\frac{rn-(1-\alpha)}{rn^2} + \frac{\lambda^{ij} r}{1-\alpha}\left[\alpha\frac{rn-(1-\alpha)}{rn} - \gamma E\right]\right\}(1-\alpha)E}{\frac{rn-(1-\alpha)}{rn} - \gamma E}, \quad (89)$$

and to the equilibrium relative prices:

$$\tilde{q}(r) = \frac{\alpha\frac{rn-(1-\alpha)}{rn} - \gamma E}{(1-\alpha)E} \quad (90)$$

$$\tilde{p}(r) = \left(\frac{\alpha}{1-\alpha}\right)\left(\frac{\beta rn}{rn-1}\right)\frac{\frac{rn-(1-\alpha)}{rn} - \gamma E}{\gamma E}. \quad (91)$$

The equilibrium allocation follows:

$$\tilde{\mathbf{x}}^{ij}(r) = \left(\frac{\left(\frac{1-\alpha}{\beta}\right)\gamma E\left[\frac{\alpha}{n^2}+\frac{\lambda^{ij}r}{1-\alpha}\psi(r)\right]}{\Delta(r)}, \frac{\alpha(1-\alpha)}{n^2}+\lambda^{ij}r\psi(r)\right), \; ij \in \mathcal{O}(r), \tag{92}$$

$$\tilde{\mathbf{x}}^{j} = \left(\frac{\left(\frac{1-\alpha}{\beta}\right)\left(\frac{n-1}{n}\right)\gamma E}{\Delta(r)}, 1-\alpha\right), \; j = 1, ..., r. \tag{93}$$

where $\Delta(r) \equiv \frac{rn-(1-\alpha)}{rn} - \gamma E > 0$ and $\psi(r) \equiv \alpha\frac{rn-(1-\alpha)}{rn} - \gamma E > 0$.

The corresponding payments are given by:

$$\tilde{U}^{ij} = \alpha \ln\left(\Lambda(r)\right) + (1-\alpha)\ln\left(\frac{\alpha(1-\alpha)}{n^2}+\lambda^{ij}r\psi(r)\right) - \alpha\ln\Delta(r), \; ij \in \mathcal{O}(r), \tag{94}$$

$$\tilde{U}^{j} = \alpha\ln\left(\frac{\gamma}{\beta}\left(\frac{n-1}{n}\right)E\right) + \ln(1-\alpha) - \alpha\ln\Delta(r), \; j \in T_1, \tag{95}$$

where $\Lambda(r) \equiv \left(\frac{1-\alpha}{\beta}\right)\gamma E\left[\frac{\alpha}{n^2}+\frac{r\lambda^{ij}}{1-\alpha}\psi\right]$.

C OE with a Subsidy

Substituting the value of $(\gamma+\tau)$ obtained in (57) in equation (58), we obtain after some tedious algebra a polynomial of degree two in the strategy s:

$$\begin{aligned} P(s) = {} & \tau(1-\alpha)(\alpha+n)s^2 \\ & + \Big(\alpha\big(\frac{n-(1-\alpha)}{n}-\gamma E\big) - \tau\frac{\gamma}{\beta}\frac{E}{n}(1-\alpha)\big((1-\alpha)\,n+\alpha\big)\Big)s \\ & - \alpha(1-\alpha)\frac{\gamma}{\beta}\frac{E}{n}(1-\frac{1}{n}) = 0. \end{aligned} \tag{96}$$

To prove the existence of an OE with a subsidy we must show that there is positive solution to the above equation which satisfies: $s < \min\{\frac{\gamma}{\beta n}E, \frac{1}{\tau n}\}$. This last condition ensures that the supply of the oligopolists are feasible and that the after tax income of the competitive agent is positive.

We first prove that there is a positive solution lower than $\gamma E/\beta n$. After some computations, we can find that:

$$P(0) = -\alpha(1-\alpha)\frac{\gamma}{\beta}\frac{E}{n}(1-\frac{1}{n}) < 0, \tag{97}$$

$$P\left(\frac{\gamma E}{\beta n}\right) = \alpha\frac{\gamma E}{\beta n}(\alpha-\gamma E) > 0. \tag{98}$$

It follows that there is a unique positive root of the polynomial function located in $]0, \frac{\gamma E}{\beta n}[$. The value of this root is:

$$s(\tau) = \frac{-\alpha\big(\frac{n-(1-\alpha)}{n}-\gamma E\big) + \tau\frac{\gamma E}{\beta n}(1-\alpha)\big((1-\alpha)\,n+\alpha\big) + \sqrt{\Omega(\tau)}}{2\tau(1-\alpha)(\alpha+n)}, \tag{99}$$

where:

$$\Omega(\tau) = \Big(\alpha\big(\frac{n-(1-\alpha)}{n} - \gamma E\big) - \tau\frac{\gamma}{\beta}\frac{E}{n}(1-\alpha)\big((1-\alpha)+\alpha\big)\Big)^2, \\ + 4\tau\alpha(1-\alpha)^2(\alpha+n)\frac{\gamma E}{\beta n}(1-\frac{1}{n}). \quad (100)$$

It remains to check that the positive root of the polynomial is lower than $1/\tau n$. We have:

$$P(\frac{1}{\tau n}) = \frac{1}{\tau n}\left(\frac{(1-\alpha)(1+n)}{n} + \frac{1}{\tau}\Big(\alpha\big(\frac{n-(1-\alpha)}{n}\big) - \gamma E\Big)\right) - (1-\alpha)\frac{\gamma E}{\beta n}. \quad (101)$$

Since we will only study the case where τ is close to 0, it follows that $P(\frac{1}{\tau n})$ will always be positive.

D Subsidy policy

Differentiating $W^i(\tau)$ with respect to τ yields:

$$\frac{\partial W^i(\tau)}{\partial\tau} = \frac{-\alpha\frac{\partial s}{\partial\tau}}{\frac{\gamma}{\beta}\frac{E}{n} - s} + (1-\alpha)^2\frac{s+\tau\frac{\partial s}{\partial\tau}}{\frac{\alpha-\gamma E}{n} + \tau s(1-\alpha)}, \quad (102)$$

and:

$$\lim_{\tau\to 0+}\frac{\partial W^i(\tau)}{\partial\tau} = \frac{-\alpha\frac{\partial s}{\partial\tau}\mid_{\tau=0}}{\frac{\gamma}{\beta}\frac{E}{n} - s} + (1-\alpha)^2\frac{s}{\frac{\alpha-\gamma E}{n}}. \quad (103)$$

From the first-order conditions for the oligopolists' problem, we know that:

$$\frac{\alpha}{\frac{\gamma}{\beta}e^i - s^i} = \frac{(1-\alpha)\Big(p+\tau - s^i\frac{\alpha(1-T)}{(\sum_{k=1}^n s^k)^2}\Big)}{(p+\tau)s^i - (\gamma+q)e^i + q\frac{E}{n}}. \quad (104)$$

In equilibrium we get:

$$\frac{\alpha}{\frac{\gamma}{\beta}\frac{E}{n} - s} = \frac{(1-\alpha)\Big(\frac{\alpha(1-ns\tau)}{ns}(1-\frac{1}{n}) + \tau\Big)}{\frac{\alpha-\gamma E}{n} + \tau s(1-\alpha)}, \quad (105)$$

and when τ goes to zero we obtain:

$$\frac{\alpha}{\frac{\gamma}{\beta}\frac{E}{n} - s} = \frac{(1-\alpha)\frac{\alpha}{ns}(1-\frac{1}{n})}{\frac{\alpha-\gamma E}{n}}. \quad (106)$$

Using this last equality, we have:

$$\lim_{\tau\to 0+}\frac{\partial W^i(\tau)}{\partial\tau} = -\frac{(1-\alpha)\frac{\alpha}{ns}(1-\frac{1}{n})}{\frac{\alpha-\gamma E}{n}}\frac{\partial s}{\partial\tau}\mid_{\tau=0} + (1-\alpha)^2\frac{s}{\frac{\alpha-\gamma E}{n}}. \quad (107)$$

Therefore, the sign of $\lim_{\tau \to 0+} \frac{\partial W^i(\tau)}{\partial \tau}$ is also that of:

$$\Sigma = -\frac{\alpha}{ns}(1 - \frac{1}{n})\frac{\partial s}{\partial \tau} \mid_{\tau=0} + (1-\alpha)s. \tag{108}$$

Substituting the expressions of s (see (59)) and $\frac{\partial s}{\partial \tau} \mid_{\tau=0}$ in that of Σ, we obtain:

$$\Sigma = -\frac{\alpha}{n}(1-\frac{1}{n})\frac{\frac{\gamma E}{\beta n}(1-\alpha)\Big(\alpha - \big((1-\alpha)n+\alpha\big)\gamma E\Big)}{\alpha\big(\frac{n-(1-\alpha)}{n} - \gamma E\big)^2} + (1-\alpha)\frac{(1-\alpha)(1-\frac{1}{n})\frac{\gamma E}{\beta n}}{\frac{n-(1-\alpha)}{n} - \gamma E}. \tag{109}$$

The sign of Σ is thus that of:

$$\Sigma' = -\frac{1}{n}\Big(\alpha - \big((1-\alpha)n+\alpha\big)\gamma E\Big) + (1-\alpha)\big(\frac{n-(1-\alpha)}{n} - \gamma E\big), \tag{110}$$

$$= (1-\alpha)(1 - \frac{1-\alpha}{n}) - \frac{\alpha}{n} + \frac{\alpha}{n}E. \tag{111}$$

We can now gather the condition ensuring that a subsidy policy is Pareto-improving. A necessary and sufficient condition for both the welfare of the competitive agent and of the oligopolists to increase with the subsidy τ (if τ is small) is (see again Appendix C):

$$\alpha - (1-\alpha)(n-1)\frac{n-(1-\alpha)}{n} - \gamma E > 0, \tag{112}$$

$$(1-\alpha)(1-\frac{1-\alpha}{n}) - \frac{\alpha}{n} + \frac{\alpha}{n}E > 0. \tag{113}$$

We can show that $(1-\alpha)(1-\frac{1-\alpha}{n}) - \frac{\alpha}{n}$ is positive for all $\alpha \in [0, \overline{\alpha}[$ where

$$\overline{\alpha} = \frac{-(n-1) + \sqrt{(n-1)(n+3)}}{2}, \tag{114}$$

is the positive root of the polynomial:

$$\alpha^2 + \alpha(n-1) - (n-1) = 0. \tag{115}$$

It can be shown that $\overline{\alpha}$ is lower than 1.

Now, a necessary condition for having a welfare improving subsidy policy is to satisfy the next inequation:

$$\alpha - (1-\alpha)(n-1)\frac{n-(1-\alpha)}{n} = 1 - (1-\alpha)n + (1-\alpha)^2\frac{n-1}{n} > 0. \tag{116}$$

A sufficient condition for this inequation to hold is that $\alpha > \frac{n-1}{n}$. Finally, we can furthermore show that $\frac{n-1}{n} < \overline{\alpha}$.

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