Is shale gas a good bridge to renewables?
An application to Europe

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Abstract

This paper explores whether climate policy justifies developing more shale gas and addresses the question of a potential arbitrage between shale gas development and the transition to clean energy. We construct a Hotelling-like model where electricity may be produced by three perfectly substitutable sources: an abundant dirty resource (coal), a non-renewable less polluting resource (shale gas), and an abundant clean resource (solar). The resources differ by their carbon contents and their unit costs. Shale gas extraction's technology (fracking) generates local damages. Fixed costs must be paid to increase the quantity of shale gas extracted and to bring forward the arrival date of the clean resource. Climate policy takes the form of a ceiling on atmospheric carbon concentration. We show that, at the first best, a more stringent climate policy does not always go together with an increase of the quantity of shale gas extracted. Compared to a second best with a moratorium on fracking, shale gas extraction most often leads to postpone the development of the clean resource, but not always. Also, imposing a financial constraint on energy expenditures may lead to an over-investment in shale gas and to postpone the switch to the clean substitute. We calibrate the model for Europe and determine whether shale gas should be extracted and in which amount, depending on the magnitude of the local damage, the effect of a moratorium on extraction, and the potential extra amount of shale gas developed because of a financial constraint.

Keywords: shale gas, global warming, non-renewable resources, energy transition.

JEL Classification: H50, Q31, Q35, Q41, Q42, Q54.

1 Introduction

In France, the Jacob law of July 13th, 2011 banned hydraulic fracturing ("fracking"): "*Under the Environment Charter of 2004 and the principle of preventive and corrective action under Article L. 110-1 of the Environment Code, exploration and exploitation of hydrocarbon liquids or gas by drilling followed by hydraulic fracturing of the rock are prohibited on the national territory.*" Moreover, the exploration licences held by companies like the American Schuepbach or the French Total were cancelled. Schuepbach complained to the court that this law was unfair and unconstitutional, but the Constitutional Court confirmed the ban on October 8th, 2013, saying that the Jacob law conforms to the constitution and is not disproportionate. By the same time, French President François Hollande said France will not allow exploration of shale gas as long as he is in office.

This position, although supported by a majority of the population[1], may seem puzzling. France is the only one of the European Union's 28 countries besides Bulgaria to ban shale gas. The ban is grounded on two types of strong environmental arguments, that need to be examined closely. First, fracking is considered as dangerous and environmentally damaging. It pumps water, sand and chemical under high pressure deep underground to liberate the gas that is trapped in the rock. The main dangers are for surface water (through the disposal of the fracturing fluids) and groundwater (through the accidental leakage of fracking fluids from the pipe into potable aquifers). Also, seismic vibrations caused by the injection of water underground is feared. Finally, there are concerns over landscape, as the number of wells may be very important and their layout very dense. Second, it is argued that what should be done in the face of global warming is to reduce drastically the use of fossil fuels, not to find new ones, which will have the effect of postponing the transition to clean renewable energy. To these arguments, shale gas supporters answer that natural gas is less polluting than other fossil fuels (oil, and particularly coal), and that its substitution to coal and oil should be encouraged on environmental grounds. Anyway, coal resources are so large that they are more than sufficient by themselves to overtake any reasonable constraint on atmospheric carbon concentration. Adding to these resources new

[1] IFOP survey, Sept. 13th, 2012: 74% of the respondents are opposed to shale gas exploitation; BVA survey, Oct. 2nd, 2014: 62%. Note that this is greater than the opposition to nuclear energy, which provides most of France's electricity.

unconventional fossil fuel reserves is not an issue, as far as they help leaving ultimately more coal under the ground. Indeed, it seems impossible to fight global warming effectively without substantially reducing the use of coal, what shale gas could allow. According to the International Monetary Fund (2014), "*Natural gas is the cleanest source of energy among other fossil fuels (petroleum products and coal) and does not suffer from the other liabilities potentially associated with nuclear power generation. The abundance of natural gas could thus provide a "bridge" between where we are now in terms of the global energy mix and a hopeful future that would chiefly involve renewable energy sources.*"

The contrast between the position held by France and the situation of the United States is stunning. United States is at date the first natural gas producer in the world. Shale gas has risen from 2% of domestic energy production a decade ago to nearly 40% today (IMF, 2014). It has profoundly modified the energy mix: shale gas is gradually replacing coal for electricity generation. Coal-fired power plants produced more than half of the total electricity supply in 1990, and natural gas-fired power plants 12%; in 2014, the figures were respectively 39% and 28%; in July 2015, the monthly natural gas share of total U.S. electricity generation (35%) surpassed the coal share (34.9%) (Energy Information Administration, 2015). CO_2 emissions have been reduced by 10% between 2007 and 2013. This reduction may be due to many other factors, but gas to coal substitution has certainly played a significant part[2]. This substitution is at the heart of the Obama's administration climate policy. Of course, in France, exploiting shale gas would not be appealing from the point of vue of climate change, because it would substitute to nuclear energy, not coal.

This paper does not pretend to examine all aspects of this complex problem. Our objective is to explore whether climate policy justifies developing more shale gas, and to address the question of a potential arbitrage between shale gas development and the transition to clean energy, when environmental damages, both local and global, are taken into account, and financial constraints as well. More precisely, we seek to answer two questions which relate to the role of shale gas as a "bridge fuel" between coal and renewables. The first question is whether a more ambitious

[2] According to the Economic Report of the President 2013, *"... actual 2012 carbon emissions are approximately 17 percent below the "business as usual" baseline. (...) of this reduction, 52 percent was due to the recession (...), 40 percent came from cleaner energy (fuel switching), and 8 percent came from accelerated improvement in energy efficiency (...)."* See also Feng *et al.* (2015).

climate policy should necessarily involve more shale gas extraction. The second one is whether authorizing fracking, compared to a moratorium, necessarily leads to postpone the development of clean renewables. This question is important, as one of the main arguments in favour of fracking is that it gives the world time to build renewable energy sources.

To answer these questions, we construct a Hotelling-like model where electricity may be produced by the means of three perfectly substitutable energy sources: an abundant dirty resource, coal, a non-renewable less polluting resource, shale gas, and an abundant clean resource, solar, provided that appropriate fixed costs are paid for. The three resources differ by their carbon contents and hence their potential danger for the climate (shale gas is less CO_2-emitting than coal), and the local damages their extraction causes (shale gas is more damaging, due to the fracking technology). The costs of electricity generation by the three resources also differ: shale gas is the cheapest resource, then coal, then solar. Exploration and development allow to build the shale gas reserves that will be extracted (Gaudet and Lasserre, 1988). Any quantity of shale gas can be developed, provided that the cost is paid for: physical scarcity is not an issue. A fixed R&D cost must be paid before solar production begins. It is decreasing in time due to technical progress. Following Chakravorty *et al.* (2006a, 2006b), climate policy takes the form of a ceiling under which atmospheric CO_2 concentration must be kept. Agents derive their utility from the consumption of electricity. The social planner seeks to maximize the intertemporal welfare, taking account of the climate constraint.

Chakravorty *et al.* (2008) explore thoroughly the question of the ordering of extraction of two fossil resources, differing by their unit extraction cost but also by their pollution content, in presence of an expensive clean backstop. Van der Ploeg and Withagen (2012) and Coulomb and Henriet (2014) also consider a three resources setting, and emphasize the role of the CO_2-emitting resource less polluting than coal. These three papers model neither exploration enabling to find fossil resources, nor R&D toward a clean substitute, whereas the arbitrage between these two types of investments is at the heart of our paper. Dasgupta *et al.* (1982) and Henriet (2012) introduce a fixed R&D cost prior to the use of the clean backstop, but the former does not consider climate policy whereas the latter incorporates a pollution constraint but only one fossil resource.

We show that, compared to a moratorium, authorizing shale gas extraction leads in most cases, and in particular when the local damage due to the fracking technology is large, and/or the price elasticity of electricity demand is low, to postpone the switch to clean renewables. However, when the local damage is low, authorizing shale gas extraction actually leads to bringing forward the transition to clean energy, provided that shale gas is polluting enough.

We then turn to the effects of strenghtening climate policy, when shale gas extraction is allowed. In most cases, and in particular for a large local damage and/or an inelastic electricity demand, tightening climate policy leads to increase the quantity of shale gas developed, at the expense of coal, and to extract it earlier. However, when the local damage is small, a more stringent climate policy may lead to reduce the quantity of shale gas developed, depending on the magnitude of the advantage of shale gas over coal in terms of CO_2 emissions. In all events, tightening climate policy makes the switch to solar happen earlier.

Finally, we suppose that the social planner faces a political constraint that compels him to meet the ceiling imposed by climate policy without increasing total energy expenditures, compared to their level absent this policy. The primary effect of this constraint is to increase the monetary costs associated to energy generation (production and investment costs), while the external cost (the local environmental damage) remains unchanged. Environmental matters becomes less important compared to costs, which is an incentive to develop more shale gas and extract it earlier. We show that when the price elasticity of electricity demand is low, a binding financial constraint leads to an over-investment in shale gas and postpones the switch to the clean backstop.

We calibrate the model for Europe, which makes sense because in 2014 coal still accounts for 26% of electricity generation, and perform simulations. In the most conservative case of a large local damage representing 75% of shale gas unit cost, we obtain that for a ceiling on atmospheric carbon concentration corresponding to a 3°C temperature increase, only 5.7% of total European shale gas resources should be extracted. A moratorium on shale gas development, together with the enforcement of the ceiling, entails an increase of 1.8% of energy expenditures and a decrease of 3.6% of intertemporal welfare compared to the reference scenario, and brings forward by 2 years the switch to solar energy. A financial constraint on energy expenditures leads to a massive over-investment in shale gas, as it leads to extract 3.5 times more shale gas than in the reference scenario, representing 20% of total European resources.

The remaining of the paper is as follows. Section 2 presents the model and the optimal solution. Section 3 studies the consequences of a moratorium on shale gas extraction. Section 4 shows how the optimal solution is modified when environmental policy becomes more stringent. Section 5 introduces the financial constraint. Section 6 presents illustrative simulations concerning electricity generation in Europe. Section 7 concludes.

2 The model

2.1 Assumptions

We consider an economy where electricity is initially produced by coal-fired power plants, and where two other energy sources, shale gas and solar, may be developed and used in electricity generation as well. Coal is supposed to be abundant but very polluting. Shale gas is non-renewable, and also polluting but to a lesser extent. Solar is abundant and clean. The three resources are perfect substitutes in electricity generation[3].

The label d for "dirty" stands for the dirty resource, namely coal. The pollution intensity of coal is θ_d: the extraction and use of one unit of coal leads to the emission of θ_d unit of CO_2 ("carbon" thereafter). The marginal long term production cost of electricity with coal is c_d. It is supposed to be constant. This cost includes the extraction cost of coal, but also capital costs and operating and maintenance costs[4]. The extraction rate of coal is $x_d(t)$. Coal is abundant: resources under the ground are so large that scarcity is not an issue (see Table 1).

[3] The assumption of perfect substitutability of the energy sources is reasonable as far as electricity generation is concerned. It is not the case at the moment in transport, which justifies our focus on electricity generation.

[4] This cost is in fact the levelized cost of electricity generated by coal-fired power plants. According to the US Energy Information Administration, "levelized cost of electricity (LCOE) is often cited as a convenient summary measure of the overall competitiveness of different generating technologies. It represents the per-kilowatt hour cost (in real dollars) of building and operating a generating plant over an assumed financial life and duty cycle. Key inputs to calculating LCOE include capital costs, fuel costs, fixed and variable operations and maintenance (O&M) costs, financing costs, and an assumed utilization rate for each plant type. The importance of the factors varies among the technologies. For technologies such as solar and wind generation that have no fuel costs and relatively small variable O&M costs, LCOE changes in rough proportion to the estimated capital cost of generation capacity. For technologies with significant fuel cost, both fuel cost and overnight cost estimates significantly affect LCOE." (EIA, 2014a).

	reserves		resources	
	EJ	GtC	EJ	GtC
conventional oil	4 900 – 7 610	98 – 152	4 170 – 6150	83 – 123
unconventional oil	3 750 – 5 600	75 – 112	11 280 – 14 800	226 – 297
conventional gas	5 000 – 7 100	76 – 108	7 200 – 8 900	110 – 136
unconventional gas	20 100 – 67 100	307 – 1026	40 200 – 121 900	614 – 1 863
coal	17 300 – 21 000	446 – 542	291 000 – 435 000	7 510 – 11 230
total	51 050 – 108 410	1002 – 1940	353 850 – 586 750	8 543 – 13 649

Reserves are those quantities able to be recovered under existing economic and operating conditions; resources are those whose economic extraction is potentially feasible.

Table 1: Estimates of fossil reserves and resources, and their carbon content. Source: IPCC WG III AR 5, 2014, Chapter 7 Table 7.2

The label e for "exhaustible" stands for shale gas. Its pollution intensity is θ_e, with $\theta_e \leq \theta_d$. Indeed, Heath *et al.* (2014), performing a meta-analysis of the literature to date, obtained that emissions from shale gas-generated electricity are approximately half that of coal-generated electricity, and that emissions from unconventional gas-generated electricity are roughly equivalent to those of conventional gas[5] (see Table 2). The most recent estimates by IPCC are consistent with these results (see Table 3). The long term marginal production cost of electricity using shale gas is c_e. As for coal, this includes the fuel extraction cost, other operating and maintenance costs and capital costs. We make the assumption that $c_e < c_d$ (see Energy Information Administration, 2014a and Table 4). The extraction of shale gas causes a local marginal damage d, supposed to be constant. This damage is due primarily to the technology employed to extract shale gas, namely hydraulic fracturing. It has been at the center of the discussions on shale gas development, around the world and in France in particular[6]. According to the review by Mason

[5]Notice that whereas the combustion of natural gas is without controverse less CO_2 emitting than the combustion of coal, methane leakage from the shale gas supply chain could be high enough to offset the benefits. Heath *et al.* (2014) do not take into account methane leakage in their analysis because of the wide variability of estimates (0.66–6.2% for unconventional gas, 0.53–4.7% for conventional gas).

[6]Coal extraction is also environmentally damaging (isses of land use, waste management, water pollution etc.). Besides, coal mining has been a very dangerous activity in the past, and still remains so in many developing

coal	shale	unconventional	conventional
980	470	460	450

Table 2: Median estimate of life cycle GHG emissions (g CO_2eq/kWh) from electricity generated using coal or different types of natural gas. Source: Heath *et al.*, 2014

	direct emissions	life-cycle emissions
	min / median / max	min / median / max
coal PC	670 / 760 / 870	740 / 820 / 910
gaz – combined cycle	350 / 370 / 490	410 / 490 / 650

Table 3: Emissions of selected electricity supply technologies (gCO_2eq/kWh). Source: IPCC WG III AR 5, 2014, Annex III Table A.III.2

et al. (2014), the literature to date offers very few empirical estimates of these negative externalities. Before beginning to extract shale gas, it is necessary to incur an upfront exploration cost. The total quantity of reserves X_e available after exploration and development is endogenous, and proportional to the exploration investment: $X_e = f(I)$, with $f'(.) > 0$ and $f''(.) < 0$. This can also be written $I = E(X_e)$, with $E'(X_e) > 0$ and $E''(X_e) > 0$, as in Gaudet and Lasserre (1988). We suppose that the exploration cost must be paid at the beginning of the planning horizon, even though the actual extraction of shale gas may be postponed to a later date[7]. The extraction rate of shale gas is $x_e(t)$.

The label b for "clean backstop" stands for solar energy. The long term marginal production cost of electricity with solar is c_b. We make the assumption $c_b > \max(c_e + d, c_d)$. Solar power plants can be developed at a R&D cost $CF(t)$. It is supposed to be decreasing in time, because of technical progress: $CF'(t) < 0, CF''(t) > 0$ (Dasgupta *et al.*, 1982). The type of R&D we have in mind produces innovations that allow to rely on solar energy only for electricity generation. These innovations must solve the intermittency problem inherent to renewable energies (solar,

countries. However, the public attention is at the moment focused on local damages due to shale gas extraction. Moreover, d can be seen as a differential local damage, assumed to be positive.

[7]This assumption is technical. It allows to get rid of problems of concavity of the value function appearing when exploration and exploitation of shale gas reserves are performed at the same date.

	levelized capital cost	fixed O&M	variable O&M including fuel	transmission investment	total
conventional coal	60	4.2	30.3	1.2	95.6
natural gas-fired combined cycle	14.3	1.7	49.1	1.2	66.3
solar PV	114.5	11.4	0	4.1	130
solar thermal	195	42.1	0	6.0	243

Table 4: US average levelized cost of electricity (2012 \$/MWh). Source: EIA, 2014a

wind). They allow to develop for instance large scale electricity storage device and enhanced electric grid. The production rate of solar energy is $x_b(t)$.

The combustion of the two polluting resources generates carbon emissions that accumulate in the atmosphere. $Z(t)$ is the atmospheric concentration of carbon. Its change over time is given by:

$$\dot{Z}(t) = \theta_e x_e(t) + \theta_d x_d(t)$$

meaning that carbon concentration can only increase, as soon as fossil fuels are used for electricity generation. In other words, we suppose that there is no natural decay of carbon, which is an acceptable assumption, considering the large uncertainties surrounding the natural absorption process and its potential weakening as temperature increases.

Finally climate policy is modeled as a cap on the atmospheric carbon concentration $\overline{Z}$, following the strand of literature initiated by Chakravorty *et al.* (2006a, 2006b).

Electricity produced at date t is $x(t) = x_d(t)+x_e(t)+x_b(t)$. Agents derive their utility directly from the consumption of electricity. Let $u\left(x(t)\right)$ be the utility function at date t, with u twice continuously differentiable, strictly increasing and strictly concave, and ρ the social discount rate, assumed to be constant. The social planner chooses the extraction and production rates $x_d(t)$, $x_e(t)$, $x_b(t)$, the amount of shale gas developed X_e, and the date T_b at which the R&D investment for solar energy is made which maximize:

$$\int_0^\infty e^{-\rho t}\left[u\left(x_d(t)+x_e(t)+x_b(t)\right) - c_d x_d(t) - (c_e+d)x_e(t) - c_b x_b(t)\right]dt - E(X_e) - CF(T_b)e^{-\rho T_b}$$

under the constraints:

$$\int_0^\infty x_e(t)dt \leq X_e, \quad X_e(0) = X_e \text{ given} \tag{1}$$

$$\int_0^\infty (\theta_d x_d(t) + \theta_e x_e(t))dt \leq \overline{Z} - Z_0, \quad Z(0) = Z_0 \text{ given} \tag{2}$$

$$x_d(t) \geq 0,\ x_e(t) \geq 0,\ x_b(t) \geq 0 \tag{3}$$

In order to solve the general problem, we first assume that T_b and X_e are given, and we compute the constrained optimal price path. We obtain the value of the problem for each price path, and we maximize this value over T_b and X_e.

2.2 Ordering resource use

The current value Hamiltonain of the problem reads, with $\lambda(t)$ the scarcity rent associated to the stock of shale gas and $\mu(t)$ the carbon value:

$$\begin{aligned}\mathcal{H} = & \, u\left(x_d(t) + x_e(t) + x_b(t)\right) - c_d x_d(t) - (c_e + d)x_e(t) - c_b x_b(t) \\ & - \lambda(t)x_e(t) - \mu(t)\left(\theta_d x_d(t) + \theta_e x_e(t)\right)\end{aligned}$$

The first order necessary conditions of optimality are:

$$u'(x_d(t)) \leq c_d + \theta_d \mu(t) \tag{4}$$

$$u'(x_e(t)) \leq c_e + d + \lambda(t) + \theta_e \mu(t) \tag{5}$$

$$u'(x_b(t)) \leq c_b \tag{6}$$

with equality when the energy is actually used, and

$$\dot{\lambda}(t) = \rho\lambda(t) \tag{7}$$

$$\dot{\mu}(t) = \rho\mu(t) \text{ before the ceiling} \tag{8}$$

$$\lim_{t\to\infty} e^{-\rho t}\lambda(t)X_e(t) = 0 \tag{9}$$

$$\lim_{t\to\infty} e^{-\rho t}\mu(t)Z(t) = 0 \tag{10}$$

Following Chakravorty *et al.* (2006a, 2006b) and the subsequent literature, it is easy to see that at the optimum the three energy sources are used successively, the stock of shale gas developed, X_e, is exhausted, the ceiling is reached at the date of the switch to clean energy, T_b, and R&D costs, $CF(t)$, are paid when the clean backstop starts to be used, i.e. at date T_b (Dasgupta *et al.*, 1982).

We have supposed that the marginal cost of production of electricity with shale gas is lower than the one with coal: $c_e < c_d$. However, because of the existence of the local damage caused by shale gas extraction, the full marginal production cost for shale gas $c_e + d$ may be lower or higher than the marginal production cost for coal c_d. We successively study the two cases of a large and a small marginal local damage.

2.2.1 Large local damage

By large local damage we mean that the local damage more than compensates the gain in terms of production cost due to the use of shale gas instead of coal in electricity generation: $d > c_d - c_e$. Hence if the total marginal cost is taken into account, coal is cheaper than shale gas. However, shale gas has an advantage over coal as regards carbon emissions. We suppose that the local damage is not large enough to make solar cheaper than shale gas.

The price[8] path is potentially composed of three phases (see Chakravorty *et al.*, 2008, or Coulomb and Henriet, 2014).

In phase 1, coal is used in quantity $X_d = \frac{\bar{Z} - Z_0 - \theta_e X_e}{\theta_d}$, between dates 0 and T_e, at a price:

$$p_d(t) = c_d + \theta_d \mu_0 e^{\rho t} \tag{11}$$

with μ_0 such that: $\int_0^{T_e} x_d(t)dt = \int_0^{T_e} D\left(p_d(t)\right) dt = X_d$, where $D(.) = u'^{-1}(.)$ is the demand function.

In phase 2, shale gas is used in quantity X_e, between dates T_e and T_b, at a price:

$$p_e(t) = c_e + d + (\lambda_0 + \theta_e \mu_0) e^{\rho t} \tag{12}$$

with λ_0 such that: $\int_{T_e}^{T_b} x_e(t)dt = \int_{T_e}^{T_b} D\left(p_e(t)\right) dt = X_e$. T_e, the date of the switch from coal to shale gas, is endogenously determined by the continuity of the energy price at date T_e: $p_d(T_e) = p_e(T_e)$, i.e.

$$c_d + \theta_d \mu_0 e^{\rho T_e} = c_e + d + (\lambda_0 + \theta_e \mu_0) e^{\rho T_e} \tag{13}$$

[8] Of course, as we are considering a central planner problem, the term "price" is used simply but inaccurately to denote marginal utility.

In phase 3, the clean backstop is used at the constant price:

$$p_b(t) = c_b \tag{14}$$

from date T_b onwards.

One (or two) of these phases may not exist. For instance, in the absence of any constraint on the atmospheric carbon concentration (when $\overline{Z} \to \infty$), CO_2 emissions do not matter and, as coal is available in infinite amount and is the cheapest source of energy ($c_d < c_e + d < c_b$), it will be used alone forever. As soon as $\overline{Z}$ is finite however, there will be a switch to solar at some point. But is it useful to introduce shale gas as well? Clearly, if θ_e is close to θ_d, shale gas, which is more costly than coal, because of the local damage and the upfront development cost, and equally polluting, will never be used. On the other hand, if θ_e is close to zero and the ceiling constraint very tight, it may happen that shale gas is exploited from the beginning of the trajectory at the expense of coal.

To sum up, when the local damage due to shale gas extraction is large, shale gas does not replace coal immediately in electricity generation, unless its advantage in terms of carbon emissions is large and climate policy stringent enough to compensate its disadvantage in terms of local damage.

2.2.2 Small local damage

In this case, $d < c_d - c_e$. The advantage of shale gas in terms of production costs dominates. Shale gas is also less polluting than coal. It will be used immediately in electricity generation. But it may be the case that we return to coal, more costly and more polluting than shale gas, later on, because shale gas is scarce while coal is abundant.

Again, the price path is potentially composed of 3 phases.

In phase 1, shale gas is used in quantity X_e, between dates 0 and T_d. Its price is given by (12), with $(\lambda_0 + \theta_e \mu_0)$ such that: $\int_0^{T_d} x_e(t)dt = \int_0^{T_d} D\left(p_e(t)\right)dt = X_e$.

In phase 2, coal is used in quantity X_d, between dates T_d and T_b. Its price is given by (11), with μ_0 such that: $\int_{T_d}^{T_b} x_d(t)dt = \int_{T_d}^{T_b} D\left(p_d(t)\right)dt = X_d$. T_d, the date of the switch from shale gas to coal, is endogenously determined by $p_e(T_d) = p_d(T_d)$, i.e.

$$c_e + d + (\lambda_0 + \theta_e \mu_0)e^{\rho T_d} = c_d + \theta_d \mu_0 e^{\rho T_d} \tag{15}$$

In phase 3, the clean backstop is used at price c_b (see (14)) from date T_b onwards.

Here again, one of these phases may not exist. For instance, absent climate policy ($\overline{Z} \to \infty$) shale gas, the cheapest source of energy, is used first, then coal is used forever. Solar is never developed. As in the previous case, as soon as some climate policy is introduced, solar will be used at some point.

2.3 Optimal investments in shale gas and solar

We now find the optimal quantity of shale gas to be developed X_e and the optimal date of the switch to solar in electricity generation T_b.

2.3.1 Large local damage

When $d > c_d - c_e$, the optimal quantity of shale gas developed, X_e, and the optimal date of the switch from shale gas to solar, T_b, solve:

$$\lambda_0 = E'(X_e) \tag{16}$$

$$\left[u\left(x_e(T_b)\right) - (c_e + d + (\lambda_0 + \theta_e \mu_0)e^{\rho T_b})x_e(T_b)\right] - \left[u\left(x_b\right) - c_b x_b\right] = CF'(T_b) - \rho CF(T_b) \tag{17}$$

Equation (16) states that costs of exploration for finding shale gas reserves must be paid up to the point where the exploration cost of a marginal unit of reserve $E'(X_e)$ is equal to the value of this reserve under the ground, which is the initial scarcity rent λ_0. Equation (17) shows that at the optimal date of the switch from shale gas to solar the marginal benefice of the switch is equal to its marginal cost (Dasgupta *et al.*, 1982). It shows that the electricity price jumps downwards at the date of the switch, the size of the jump being proportional to the marginal cost of delaying R&D in the backstop technology.

Equations (1), (2), (13), (16) and (17) characterize the optimal solution when the sequence of energy use is coal (from 0 to T_e), shale gas (from T_e to T_b) and solar, i.e. when the three phases identified above exist.

We want now to check the conditions under which one of the two first phases does not exist, given that the last phase (solar) always exists as soon as some climate policy is introduced.

If shale gas is used alone, and coal is left under the ground, then the values of λ_0, μ_0, T_b and

X_e must solve the system composed of equations (1), (16), (17) and

$$\theta_e X_e = \overline{Z} - Z_0 \tag{18}$$

which replaces (2). Moreover, to ensure that there exists no incentive to introduce coal at date 0, the initial price of shale gas $p_e(0)$ must be below the initial price of coal, $p_d(0)$, i.e. we must have

$$(\theta_d - \theta_e)\mu_0 \geq c_e + d - c_d + E'(X_e) \tag{19}$$

If the solution of the above system is such that this condition is satisfied, then shale gas is used alone to get to the ceiling. There exists a threshold value of the ceiling $\overline{Z}_1$ under which only shale gas is used. It is solution of the system composed of equations (1), (16), (17), (18) and (19), this last equation being taken as an equality.

If coal is used alone to get to the ceiling, then the values of μ_0 and T_b must solve the following system:

$$\theta_d \int_0^{T_b} x_d(t)dt = \overline{Z} - Z_0 \tag{20}$$

$$\left[u\left(x_d(T_b)\right) - (c_d + \theta_d \mu_0 e^{\rho T_b})x_d(T_b)\right] - \left[u\left(x_b\right) - c_b x_b\right] = CF'(T_b) - \rho CF(T_b) \tag{21}$$

where equation (20) is the combination of equations (1) and (2) for $X_e = 0$, and equation (21) is equation (17) in the case $X_e = 0$. Moreover, we must make sure that there is no incentive to extract shale gas: the final price of coal $p_d(T_b)$ must be lower than the price of the first unit of shale gas that could be extracted at date T_b, $c_e + d + \theta_e \mu_0 e^{\rho T_b}$. Hence we must have:

$$(\theta_d - \theta_e)\mu_0 e^{\rho T_b} \leq c_e + d - c_d \tag{22}$$

meaning that the marginal gain in terms of pollution of switching from coal to shale gas, evaluated at the carbon value at date T_b, is smaller than the marginal cost of the switch. If the solution of the above system is such that this condition is satisfied, then shale gas is never extracted. There exists a threshold value of the ceiling $\overline{Z}_2$, such that if $\overline{Z} \geq \overline{Z}_2$ shale gas is not developed. $\overline{Z}_2$ is solution of the system composed of equations (20), (21) and (22), this last equation being written as an equality.

For an intermediate ceiling $\overline{Z}$ such that $\overline{Z}_1 < \overline{Z} < \overline{Z}_2$, the three phases exist.

2.3.2 Small local damage

When $d < c_d - c_e$, the optimal quantity of shale gas developed, X_e, and the optimal date of the switch from coal to solar, T_b, solve:

$$\lambda_0 = E'(X_e) \tag{23}$$

$$\left[u\left(x_d(T_b)\right) - (c_d + \theta_d \mu_0 e^{\rho T_b}) x_d(T_b)\right] - \left[u\left(x_b\right) - c_b x_b\right] = CF'(T_b) - \rho CF(T_b) \tag{24}$$

The interpretation of these equations is similar to the one given in the case of a large local damage.

Equations (1), (2), (15), (23) and (24) characterize the optimal solution when the sequence of energy use is shale gas (from 0 to T_d), coal (from T_d to T_b) and solar (from T_b onwards).

As shale gas is cheaper and less polluting than coal, necessarily $c_e + d + \theta_e \mu_0 < c_d + \theta_d \mu_0$ $\forall \mu_0$. Hence $\exists \lambda_0 > 0$ s.t. $p_e(0) < p_d(0)$, meaning that there always exists scope for shale gas exploration and extraction.

Now, it is possible to switch directly from shale gas to solar, and leave coal forever in the ground? If shale is used, alone, to get to the ceiling, then λ_0, μ_0, T_b and X_e must solve the system composed of equations (1), (18), (23) and:

$$\left[u\left(x_e(T_b)\right) - (c_e + d + (\lambda_0 + \theta_e \mu_0) e^{\rho T_b}) x_e(T_b)\right] - \left[u\left(x_b\right) - c_b x_b\right] = CF'(T_b) - \rho CF(T_b) \tag{25}$$

Moreover, the final price of shale gas $p_e(T_b)$ must be lower than the price of the first unit of coal that could be extracted at date T_b, $p_d(T_b)$, i.e. we must have:

$$(\theta_d - \theta_e)\mu_0 e^{rT_b} > c_e + d - c_d + E'(X_e) e^{rT_b} \tag{26}$$

meaning that the cost in terms of pollution of switching to coal instead of going directly to solar is higher than the advantage in terms of production costs. It happens for values of the ceiling below $\bar{Z}_3$ defined by (1), (18), (23), (25) and (26) taken as an equality.

For $\overline{Z} > \overline{Z}_3$, the three resources are used.

To sum up, Fig. 1 represents the optimal succession of energy sources in electricity generation as a function of the stringency of climate policy. When the local damage is very large and climate policy lenient, coal is used alone to get to the ceiling. It is not optimal in this case to explore and develop shale gas. When environmental policy becomes more stringent, shale gas replaces coal at some point before the ceiling. For an even more stringent environmental policy, coal is completely evicted by shale gas. When the local damage is small shale gas is always developed, and its extraction begins immediately. If climate policy is lenient, shale gas is replaced by coal at some point before the ceiling, because it is abundant whereas shale gas is scarce and costly to develop. However, if climate policy is stringent, coal is completely phased out.

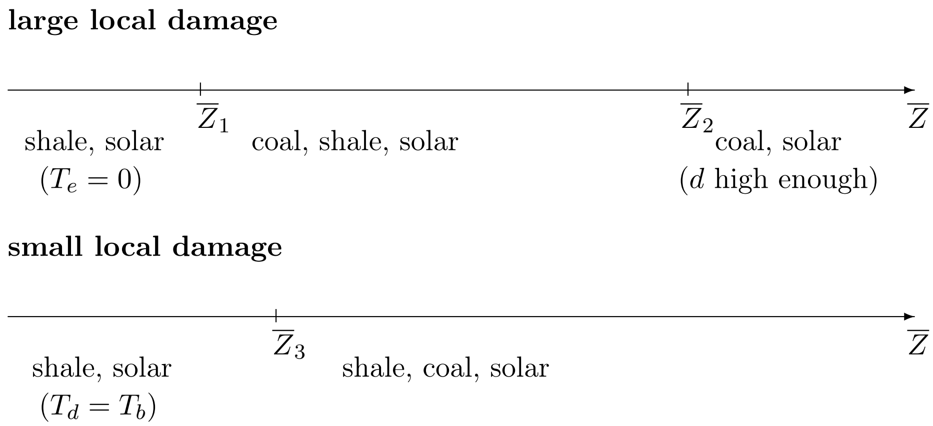

Figure 1: Optimal succession of energy sources as a function of the stringency of climate policy

3 A moratorium on shale gas extraction

Shale gas has been advocated as a bridge fuel to smooth the transition from polluting coal to emission-free renewable energy. One of the main question that arises is whether the extraction of shale gas should be used to buy time to make the more arduous shift to even cleaner forms of energy, or if its use does not justify postponing the transition to clean energy. To answer this question we compare the optimal energy transition analyzed above with an energy transition constrained by a moratorium on shale gas exploitation, for a given climate policy. Under the moratorium, the planner is left with two options for electricity generation: coal and solar energy. The solution obtained is of course sub-optimal. The moratorium imposes a cost on society in terms of intertemporal welfare[9].

[9]Note nevertheless that it leads to the optimal solution in the case where the development of shale gas is actually not optimal, that is when the local damage is large and climate policy lenient (more precisely, $\overline{Z} > \overline{Z}_2$;

3.1 Large local damage

We show that in this case, solar always arrives sooner with a moratorium, implying that shale gas is used to postpone the costly switch to clean renewables.

If there is a moratorium on shale gas extraction, then only coal is used until the ceiling is reached. The extraction path is the same as described in Dasgupta *et al.* (1982), with a fixed quantity $\overline{Z}/\theta_d$ of coal used.

Call $V(T_b)$ the value of the overall surplus when solar is available from date T_b, with a moratorium. From the enveloppe theorem, $V(.)$ is concave. Call T_b^* the date of the switch to solar at the optimum (no moratorium). We denote all the values of the variables at the optimum by a *, and all the values with a moratorium (at T_b given), by a $\tilde{}$.

We have that:

$$\left.\frac{\partial V(T_b)}{\partial T_b}\right|_{T_b^*} = \left[u\left(\tilde{x}_d(T_b^*)\right) - (c_d + \theta_d \tilde{\mu}_0 e^{\rho T_b^*})\tilde{x}_d(T_b^*)\right] - \left[u\left(x_b\right) - c_b x_b\right] - (CF'(T_b^*) - \rho CF(T_b^*))$$

i.e., using equation (17),

$$\left.\frac{\partial V(T_b)}{\partial T_b}\right|_{T_b^*} = \left[u\left(\tilde{x}_d(T_b^*)\right) - (c_d + \theta_d \tilde{\mu}_0 e^{\rho T_b^*})\tilde{x}_d(T_b^*)\right] - \left[u\left(x_e^*(T_b^*)\right) - (c_e + d + (\lambda_0^* + \theta_e \mu_0^*) e^{\rho T_b^*})x_e^*(T_b^*)\right]$$

The optimal date of arrival of solar precedes T_b^* if and only if $\left.\frac{\partial V(T_b)}{\partial T_b}\right|_{T_b^*} > 0$, i.e:

$$c_e + d + (\lambda_0^* + \theta_e \mu_0^*) e^{\rho T_b^*} > c_d + \theta_d \tilde{\mu}_0 e^{\rho T_b^*}$$

This inequality means that the price of energy at the optimal date of the switch to solar would be lower with a moratorium than without, which implies that the whole price path would be lower. This cannot be the case. Otherwise, extraction would be higher at each date with a moratorium than without, which contradicts the fact that the ceiling $\overline{Z}$ should not be violated in both cases.

3.2 Small local damage

We show that in this case, solar does not always arrive sooner with a moratorium, implying that shale gas should not necessarily be used to postpone the costly switch to clean renewables. In some cases, shale gas is only used to consume more energy at each date.

see Fig. 1). In this case, the moratorium is inconsequential.

If there is a moratorium on shale gas extraction, then only coal is used until the ceiling is reached, in fixed quantity $\overline{Z}/\theta_d$.

We now have that:

$$\left.\frac{\partial V(T_b)}{\partial T_b}\right|_{T_b^*} = \left[u\left(\tilde{x}_d(T_b^*)\right) - (c_d + \theta_d\tilde{\mu}_0 e^{\rho T_b^*})\tilde{x}_d(T_b^*)\right] - \left[u\left(x_b\right) - c_b x_b\right] - \left(CF'(T_b^*) - \rho CF(T_b^*)\right)$$
$$= \left[u\left(\tilde{x}_d(T_b^*)\right) - (c_d + \theta_d\tilde{\mu}_0 e^{\rho T_b^*})\tilde{x}_d(T_b^*)\right] - \left[u\left(x_d^*(T_b^*)\right) - (c_d + \theta_d\mu_0^* e^{\rho T_b^*})x_d^*(T_b^*)\right]$$

The optimal date of arrival of the clean technology precedes T_b^* if and only if $\left.\frac{\partial V(T_b)}{\partial T_b}\right|_{T_b^*} > 0$, i.e:

$$\mu_0^* > \tilde{\mu}_0$$

Let us look at extreme cases. If $\theta_e = 0$, then $\tilde{\mu}_0 > \mu_0^*$, otherwise more coal would be extracted between T_d^* and T_b^* and coal would also be extracted in the moratorium case between date 0 and T_d^* whereas a less polluting resource, shale gas, would be used in the optimum (no moratorium). This contradicts the fact that the ceiling $\overline{Z}$ should not be violated in both cases. If $\theta_e = \theta_d$, then $\tilde{\mu}_0 < \mu_0^*$, otherwise less coal would be extracted between T_d^* and T_b^* and coal consumption would also be lower, in the moratorium case, between date 0 and T_d^* than shale gas consumption at the optimum. This contradicts the fact that the ceiling $\overline{Z}$ should not be violated in both cases. If the price elasticity of demand is small enough[10], then $\tilde{\mu}_0 > \mu_0^*$. Otherwise, the overall demand would be higher on the moratorium extraction path and the energy mix would be more polluting. This contradicts the fact that the ceiling $\overline{Z}$ should not be violated in both cases.

The previous results are summarized in the following Proposition:

Proposition 1 *If the local damage is large, a moratorium on shale gas exploitation always brings forward the transition to clean energy, compared to the optimum. If the local damage is small, the moratorium only brings forward the transition to clean energy if shale gas is clean enough compared to coal, or if the price elasticity of electricity demand is low. Otherwise, the moratorium actually postpones the transition to clean energy.*

The intuition behind these results is the following. There are two reasons why one could want to extract shale gas. The first one is that it is cheaper than coal and the second one is that

[10]The empirical literature shows that this is actually the case. See Alberini *et al.* (2011), Table 1 pp. 871, for a survey of recent estimates of price elasticities of residential electricity consumption.

it is less polluting. In the case of a large local damage, shale gas is not actually cheaper than coal, so that its only advantage is that it is less polluting. The only reason to use shale gas is thus to buy time to decrease the cost of the switch to clean energy, by the combined effects of discounting and technical progress. It is actually optimal to do so. Hence, with a moratorium on shale gas, the switch to solar occurs sooner than without. Things are quite different when the local damage is small. Then, shale gas is cheaper than coal, and one may want to use it in order to consume more energy, even absent any climatic constraint. This incentive introduces a new effect that plays in the opposite direction, and is all the stronger since the price elasticity of demand is high. Then, if the cost of shale gas is small enough and the price elasticity of demand high enough, extracting shale gas leads to an increase in energy use and a ceiling reached more rapidly. The switch to clean energy happens sooner whitout a moratorium than with it.

4 A more stringent climate policy

We now perform exercises of comparative dynamics to see how the optimal solution is modified when environmental policy becomes more stringent. In particular, we wonder whether climate policy justifies developing more shale gas, and making the transition to solar earlier.

4.1 Large local damage

We show in Appendix A that in this case:

$$\frac{\partial T_e}{\partial \overline{Z}} > 0, \qquad \frac{\partial T_b}{\partial \overline{Z}} > 0, \qquad \frac{\partial X_e}{\partial \overline{Z}} < 0$$

When the marginal local damage of shale gas is large, with a lenient environmental policy few shale gas –if any– is extracted. Electricity is generated before the ceiling mainly by coal-fired power plants. However, as environmental policy becomes more stringent, the use of shale gas becomes more interesting because of its lower carbon content. This advantage on the climate point of view overcomes more and more the local damage drawback and the exploration cost that has to be paid prior to exploiting shale gas. It becomes therefore optimal to use shale gas earlier and to develop it in a greater amount.

A more severe climate policy also makes the switch to solar energy happen earlier. The reason is the same as for shale gas: the advantage of solar from the climate point of view overcomes more and more the fixed cost.

Clearly, in this case, the effect of a more stringent climate policy is to partially or even totally evict coal and replace it by more shale gas before the ceiling, and also to make the transition to clean energy happen sooner.

4.2 Small local damage

Likewise, a comparative dynamics exercise yields in the case of a small local damage (see Appendix B):

$$\frac{\partial T_d}{\partial \overline{Z}} < 0, \qquad \frac{\partial T_b}{\partial \overline{Z}} > 0$$

Remember that in this case it is optimal to develop shale gas first. Then, quite intuitively, when environmental policy becomes more stringent, the date of the switch to coal is postponed while the date of the switch to solar is brought forward. However, the effect of a more stringent climate policy on the amount of shale gas reserves developed depends on its relative carbon content. We show in Appendix B that the two polar cases where shale gas is not polluting at all and shale gas is as polluting as coal lead to very different outcomes:

$$\text{if } \theta_e = 0, \ \frac{\partial X_e}{\partial \overline{Z}} < 0$$
$$\text{if } \theta_e = \theta_d, \ \frac{\partial X_e}{\partial \overline{Z}} > 0$$

When shale gas is not polluting at all, the more stringent climate policy is, the more shale gas is developed. The total marginal variable cost of shale gas is smaller than the one of coal because the marginal local damage is small; furthermore, shale gas is not polluting. The only reason why coal is not completely evicted is the costly initial exploration investment needed to develop shale gas. However, when shale gas is as polluting as coal, imposing a climate policy does not favour shale gas: the more stringent climate policy is, the less shale gas is developed.

In the general case, when shale is polluting but less polluting than coal, we show in Appendix C that if the price elasticity of electricity demand is small enough, the more stringent climate policy, the more shale gas is extracted. The intuition is the following. A more severe climate policy obliges to emit less. There are two solutions to do so: decrease fossil energy consumption by making its price increase, or switch to a less emitting fuel. In the plausible case where the price elasticity of electricity demand is low, only the second option is left. The economy resorts to fuel switching, which means using more shale gas and switching to solar earlier. As we have already noticed, switching to solar is costly, and there are powerful incentives to postpone the switch as much as possible, namely technical progress and discounting. When demand is elastic, the first option is all the more interesting since the carbon content of shale gas is low and shale gas is cheap.

The previous results are summarized in the following Proposition:

Proposition 2 *Tightening climate policy always brings forward the transition to clean energy. When the local damage is large, it also leads to an increase of the quantity of shale gas developed, at the expense of coal. When the local damage is small, it may on the contrary lead to reduce the quantity of shale gas developed, if demand is elastic and the advantage of shale gas over coal in terms of carbon emissions is not large enough.*

5 A constraint on energy expenditures

In order to get more insights on the arbitrage between the development of the clean backstop, the development of shale gas and the cost of energy consumption, we add a constraint on total energy expenditures. The constraint says that energy expenditures relative to a given climate policy cannot exceed energy expenditures absent any climate policy. This constraint can be seen as a political constraint faced by the social planner. It is justified by the fact that the cost argument is prominent in the reluctance of many countries to tighten their climate policy, even if it is optimal from a welfare point of view.

Let A_0 be the present value of total energy expenditures:

$$A_0 = \int_0^\infty e^{-\rho t}\left[c_d x_d(t) + c_e x_e(t) + c_b x_b(t)\right] dt + E(X_e) + CF(T_b)e^{-\rho T_b} \tag{27}$$

The problem is the same as the original one except that we add the following constraint:

$$A_0 \leq A_0^{\text{ref}} \tag{28}$$

where A_0^{ref} is the present value of energy expenditures when there is no climate policy. The objective is to see whether the previous results are modified when we force climate policy to be costless.

It is worth stressing that energy expenditures are not necessarily higher with climate policy than without. Indeed, tightening climate policy increases the price of fossil energy and reduces demand, if it is sufficiently elastic. The overall effect may well compensate the higher investment costs for shale gas exploration and solar R&D, so that total energy expenditures decrease.

5.1 Solution

We have seen that the reference situation absent climate policy differs, depending on the value of the marginal local damage. If it is large, the reference path is a path where coal is used alone, from the origin onwards. Then $x_d(t) = D(c_d)$ and $A_0^{\text{ref}} = c_d D(c_d)/\rho$. If it is small, shale gas is used first (from 0 to T_d), then coal (from T_d onwards), and solar is never developed. Then:

$$A_0^{\text{ref}} = \int_0^{T_d} e^{-\rho t} c_e x_e(t) dt + \int_{T_d}^{\infty} e^{-\rho t} c_d x_d(t) dt - E(X_e)$$

with

$$\begin{aligned} x_e(t) &= D(c_e + d + \lambda_0 e^{\rho t}) \\ x_d(t) &= D(c_d) \end{aligned}$$

and where λ_0, X_e and T_d are solution of the following system:

$$\begin{aligned} \int_0^{T_d} x_e(t) dt &= X_e \\ \lambda_0 &= E'(X_e) \\ c_e + d + \lambda_0 e^{\rho T_d} &= c_d \end{aligned}$$

Let α be the Lagrange multiplier associated with constraint (28). The solutions are the same as the solutions without constraint, where c_e, c_d and c_b are replaced by $(1+\alpha)c_e$, $(1+\alpha)c_d$ and $(1+\alpha)c_b$, and $E(X_e)$ and $CF(T_b)$ are replaced by $(1+\alpha)E(X_e)$ and $(1+\alpha)CF(T_b)$.

When the financial constraint is binding, $\alpha > 0$. The primary effect of the constraint is to increase the monetary costs associated to electricity generation (extraction, investment and O&M costs), while the external cost d remains unchanged. Environmental matters become less important compared to costs. The declining importance of the local damage d is an incentive to develop more shale gas and extract it earlier. Note that, as the global damage must remain below the ceiling, μ_0 adjusts so that the importance of global damage does not decline. However, other effects can be playing in the other direction. To cancel demand effects, we explore in what follows the overall impact of the financial constraint in the case of an inelastic demand.

5.2 Effect on shale gas and clean backstop in the case of a low price elasticity of demand

We perform again exercises of comparative dynamics to explore whether the financial constraint modifies the arbitrage between shale gas and clean technology investments. We expect that the financial constraint leads to over-investment in shale gas extraction and under-investment in the clean backstop, compared to the optimal arbitrage. This is because the local damages become less important to the planner when he faces a financial constraint. We obtain non ambiguous analytical result in the case of a low price elasticity of demand (see Appendix C).

Proposition 3 *When the price elasticity of electricity demand is low enough, a binding financial constraint leads to more extraction of shale gas and postpones the date of the switch to the clean backstop.*

6 Simulations

We perform in this section illustrative simulations. We use standard functional forms: a quadratic utility function, a solar R&D cost decreasing at a constant rate due to exogenous technical progress, and a quadratic shale gas exploration cost:

$$\begin{aligned} u(x) &= ax - \frac{b}{2}x^2 \Longrightarrow D(p) = \frac{a-p}{b} \\ CF(t) &= CF_0 e^{-\gamma t} \\ E(X_e) &= \frac{\varepsilon}{2}X_e^2 \end{aligned}$$

We calibrate the model as far as possible to the European case, making the assumption that the unit costs of the three energy sources in electricity generation are equivalent in the US and in Europe, and that the marginal cost of shale gas exploration and development would be the same in Europe as in the US.

6.1 Calibration

Unit costs c_d, c_e and c_b are in \$/MWh, and are drawn from the US levelized cost of electricity from EIA (2014a), see Table 4.

Emission coefficients θ_d and θ_e are in tCO_2eq/kWh and come from Heath *et al.* (2014), see Table 2.

The exogenous rates of discounting and technical progress on the cost of R&D are arbitrarily[11] taken equal to $\rho = 0.02$ and $\gamma = 0.03$.

The initial carbon concentration in the atmosphere is $Z_0 = 400$ ppm, which amounts[12] to 3120 10^9 tCO_2. According to the IPCC SRES scenarii[13], around 50% of total emissions is projected to come from electricity generation. Around 11% of the greenhouse gases emitted worldwide in 2012 come from the European Union. Hence other things being equal, increasing total atmospheric carbon concentration by 150 ppm to reach 550 ppm CO_2 (i.e. reaching a 3°C target) corresponds to a European sectoral ceiling in electricity generation of $\overline{Z} = Z_0 + 150 * 0.5 * 0.11 = 408$ ppm $= 3183$ 10^9 tCO_2.

The fixed cost of developing a clean technology at date 0, CF_0, is assumed to be the investment necessary to solve the intermittence problem inherent to renewable energy such as solar energy and wind power (for instance, large scale electricity storage device and enhanced electric grid). This investment is calibrated using the French Environment and Energy Management Agency report[14] (ADEME, 2015). This cost is the sum of the network capacity cost, the network fixed cost, the electricity storage system and pumped storage power stations costs. It amounts to 329 Million €/year. With $\rho = 2\%$, $CF_0 = 17\ 329/0.02 \simeq 866.45\ 10^9$ \$.

Demand is calibrated using the assumptions that:

- absent climate policy, electricity is produced by coal-fired power plants; hence $p = c_d = 95.6$ \$/MWh;
- the price elasticity of demand at this price is taken equal to 0.25 (see Alberini *et al.*, 2011).

[11] Sensitivity analysis around $\rho = 0.02$ and $\gamma = 0.03$ show that the results do not change significantly.

[12] Using the fact that 1 ppmv = 2.13 GtC = 2.13*3.664 $GtCO_2$ = 7.8 $GtCO_2$.

[13] http://www.ipcc.ch/ipccreports/sres/emission/index.php?idp=118#533

[14] www.ademe.fr/sites/assets/documents/rapport100enr_comite.pdf.

See Table 4 in the Appendix of the report.

Hence $a = \frac{1.25}{0.25} * 95.6 = 478$.

According to the World Development Indicators 2015, consumption per capita of electric power in the Euro area in 2011 is 6.5 MWh and the population of the Euro area in 2011 is 337 Million. This gives $b = 0.174\ 10^{-6}$. Note that the elasticity of demand is not constant, and is equal to -0.45 for a price of 150$, and -0.14 for a price of 60$.

To calibrate the marginal cost of shale gas exploration, we use data on US shale wells:

- The US shale gas production is given by the EIA Natural Gas Weekly Update[15]. We get monthly data from Jan. 2000 to Feb. 2015 for the major shale gas plays in billion cubic feet/day. We convert the data in MWh, take the average over the period Jan. 2008–Feb. 2015 and multiply by 365 to obtain an average annual production of the major plays in MWh. The four most productive plays are Marcellus (PA & WV), Haynesville (LA & TX), Fayetteville (AR) and Barnett (TX).

- We consider that the total cost of shale gas use in electricity generation is $E(X_e) + c_e X_e$. The corresponding marginal cost is then $E'(X_e) + c_e$ i.e., according to our specifications, $\varepsilon X_e + c_e$. We obtain this cost from Sandrea (2014), which gives the HH price[16] of US plays in $/Mcf. We sort the previous four shale gas plays by increasing HH price and cumulate the corresponding productions and obtain the parameters of the marginal cost function.

We obtain $\varepsilon = 0,051\ 10^{-9}$.

We check that the amount of shale extracted in a reference scenario (i.e. without any ceiling constraint) is consistent with data on shale gas reserves in urope. According to EIA, Europe is estimated to have 615 trillion cubic feet of technically recoverable resources of shale gas (see EIA, 2013) i.e. 180 10^9 MWh. With the previous calibration, for $d = 0$ (no local damage of shale gas) and $\overline{Z} \to \infty$ (no climate policy) we get $X_e = 160\ 10^9$ MWh. The order of magnitude is correct: absent environmental externalities, if the levelized cost of producing electricity with shale gas is lower than the one with coal, it is optimal to substitute shale gas to coal at the beginning of

[15] http://www.eia.gov/naturalgas/weekly/

[16] Fig. 1b p.4, "Basin Economics for various US plays (single well) shale gas" gives the current HH price for different plays (the Henry Hub price is the pricing point for natural gas futures contracts traded on the New York Mercantile Exchange and the OTC swaps traded on Intercontinental Exchange).

the horizon, and the quantity of shale gas that will be extracted is exactly equal to the stock available under the ground.

The parameters used for the simulations are given in Table 5.

c_d	c_e	c_b	CF_0	θ_d	θ_e	ρ	γ	ε	a	b	Z_0
95.6	66.3	130	866.45 10^9	0.98	0.47	0.02	0.03	0.051 10^{-9}	478	174 10^{-9}	3120 10^9

Table 5: Calibration parameters

6.2 Reference scenario

We suppose that the European sectoral ceiling in electricity generation is $\overline{Z} = 408$ ppm $= 3183$ 10^9 tCO_2 (see above).

In the case of a large marginal local damage, we make the assumption that this damage is equal to 3/4 of the unit cost of shale gas: $d = 66.3 * 3/4 = 26.52$ \$/MWh. It is then optimal to switch from coal to shale gas in $T_e = 30$ years, and from shale gas to solar in $T_b = 34$ years. Very few shale gas is extracted: we obtain $X_e = 7.8$ 10^9 MWh, whereas technically recoverable resources of shale gas in Europe are estimated to 138 10^9 MWh; hence only 5.7% of the total resources are developed. The price path is represented on Fig. 2.

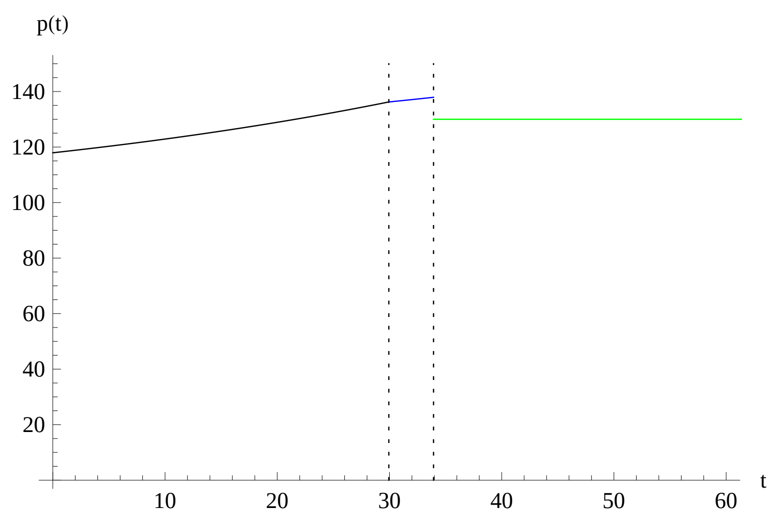

Figure 2: Price path in the reference scenario when the marginal local damage is large (black=coal, blue=shalegas, green=solar)

In the case of a small marginal local damage, we make by symmetry the assumption that this damage is equal to 1/4 of the unit cost of shale gas: $d = 66.3 * (1/4) = 16.575$ \$/MWh. For this level of damage coal is completely evicted by shale gas. To have an interior solution where the three energy sources are used, we then chose to take a local damage equal to 40% of the unit cost of shale gas: $d = 66.3 * 0.4 = 26.52$. It is then optimal to switch from shale gas to coal in $T_d = 60.7$ years, and from coal to solar in $T_b = 62.5$ years. Now, very few coal is extracted. The quantity of shale gas developed is $X_e = 126.4\ 10^9$ MWh, i.e. 92% of the total recoverable resources. The price path is represented on Fig. 3.

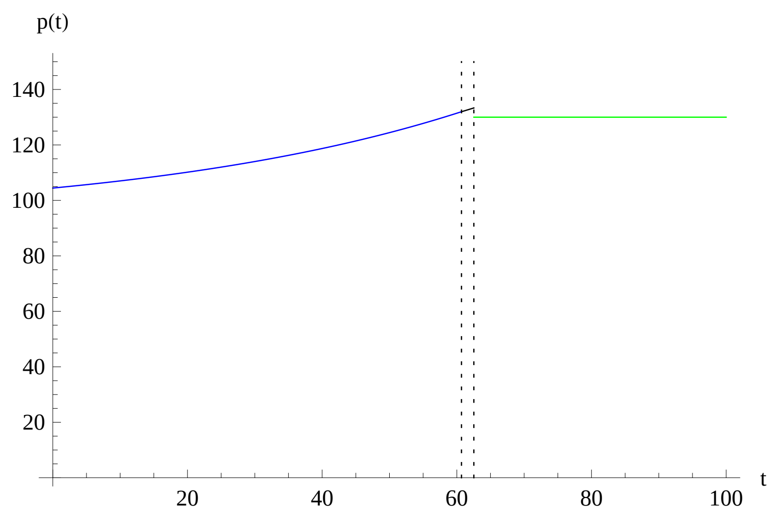

Figure 3: Price path in the reference scenario when the marginal local damage is small (black=coal, blue=shalegas, green=solar)

The solution is thus extremely sensitive to the magnitude of the marginal local damage. When the marginal damage is small, it is basically optimal to develop all European shale gas reserves, and to substitute shale gas to coal right now. The transition to solar energy will take place in about 60 years. In the most interesting case where the marginal local damage is high, the quantity of shale gas developed as well as the date of the switch to solar decrease rapidly when the damage increases.

6.3 The trade-off between local and global damages

Fig. 4 shows iso–X_e curves in the plane $(\overline{Z}, d)$. For the parameters given above, the local marginal damage is small if $d < c_d - c_e = 29.3$, large otherwise. Follow for instance the iso–X_e curve for $X_e = 100$ from the right to the left. First, the climate constraint is lenient and the local damage small. Shale gas is used first in electricity generation, then coal then solar. As we move to the left on Fig. 4, the same quantity of shale gas developed corresponds to a more and more stringent climate constraint and an increasing level of the local damage. The quantity of coal used is lower and lower and the switch to solar occurs earlier and earlier. Coal is progressively evicted by solar. When the local damage becomes larger than the threshold value of 29.3, materialized on Fig. 4 by the horizontal dotted line, coal becomes used first in electricity generation, now before shale gas. When the threshold $\overline{Z}_1$ is met, coal is completely evicted, and the economy switches directly from shale gas to solar.

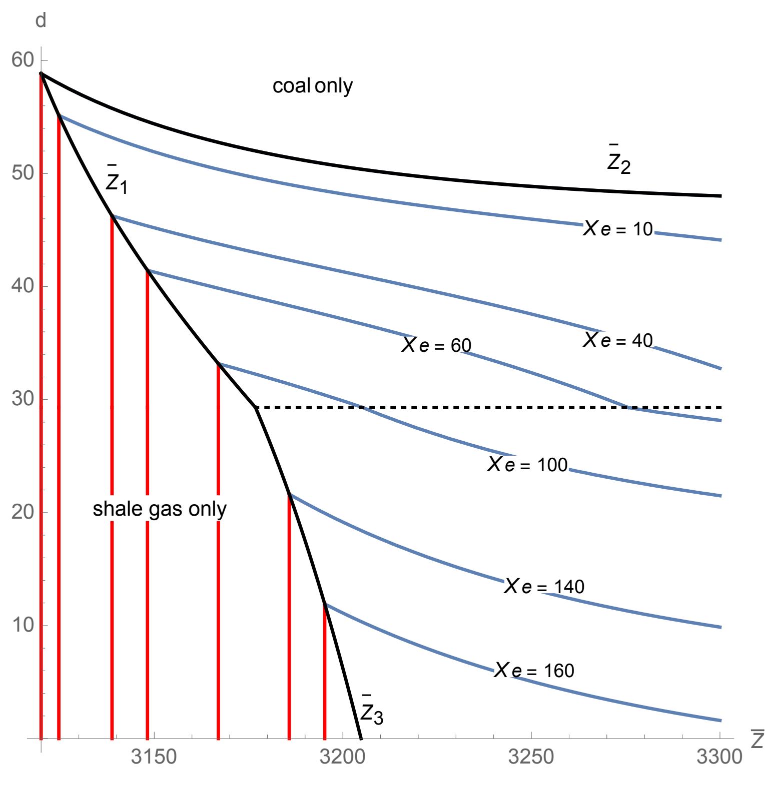

Figure 4: Iso-X_e lines

Fig. 5 shows the amount of shale gas developed as a function of the stringency of climate policy, for a given level of large local damage. It simply represents an horizontal section of Fig. 4, for $d > 29.3$[17]. The point is to emphasize the fact that this amount is a non-linear function of $\overline{Z}$. Starting with a very lenient climate policy, we see that X_e increases as $\overline{Z}$ decreases. The dominant effect is the substitution of shale gas to coal. After the threshold $\overline{Z}_1$, i.e. for a very stringent climate policy, X_e decreases linearly as $\overline{Z}$ decreases. The dominant effect is now the substitution of solar to shale gas.

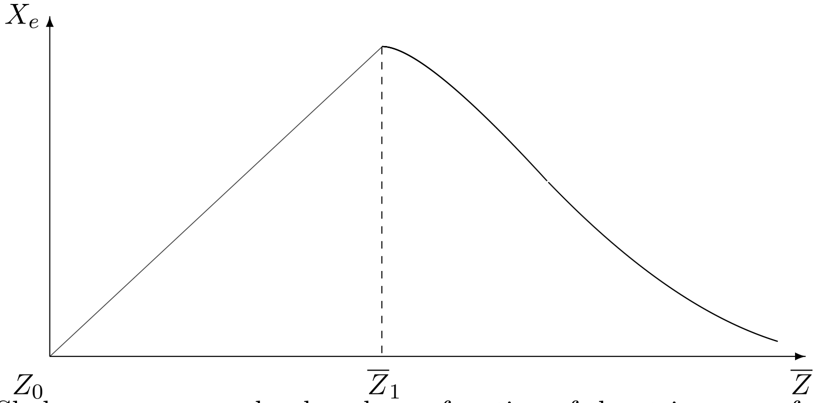

Figure 5: Shale gas resources developed as a function of the stringency of climate policy

6.4 A moratorium on shale gas development

Simulations show us that for a given climate policy, the moratorium brings forward the date of the switch to solar energy and increases energy expenditures. It actually makes the transition to the clean backstop happen sooner, but the compliance to climate policy is more costly.

For a large local damage $d = 66.3 * (3/4)$, we obtain that the switch to solar occurs 2 years earlier, energy expenditures increase by 1.8% and intertemporal welfare decrease by 3.6%. As the quantity of shale gas optimally developed for this level of the damage is very small, the effect of the moratorium is very moderate.

For a small local damage $d = 66.3 * 0.4$, we obtain that the switch to solar occurs 30 years earlier, energy expenditures increase by 26.7% and intertemporal welfare decrease by 33.5%. Now the negative effect of the moratorium is massive.

[17] The function has the same shape in the case of a small local damage, except that X_e does not tend to zero for a very lenient claimte policy.

6.5 The consequences of a financial constraint

We now compare the results of simulations performed with and without the constraint on energy expenditures, in order to see which of the previous effects dominates and in what circumstances. We focus on the case of a large local damage, which is the more interesting.

Fig. 6 represents how X_e changes with $\overline{Z}$, in the reference case (solid line) and the constrained case (dotted line). The quantity of shale gas extracted is larger in the constrained case than in the reference case, which is coherent with Proposition 3. Indeed, we have chosen for the calibration a low price elasticity of electricity demand.

Fig. 7 shows that date T_b of development of the clean backstop is postponed compared to the reference scenario, while date T_e of the switch from coal to shale gas is brought forward. The results on date T_b is robust but the result on date T_e is not. As T_b is postponed, if the quantity of shale was unchanged, T_e would be postponed as well, however, another effect is playing in the other direction: the quantity of shale extracted is increased, so that the total duration of shale gas use is lengthened.

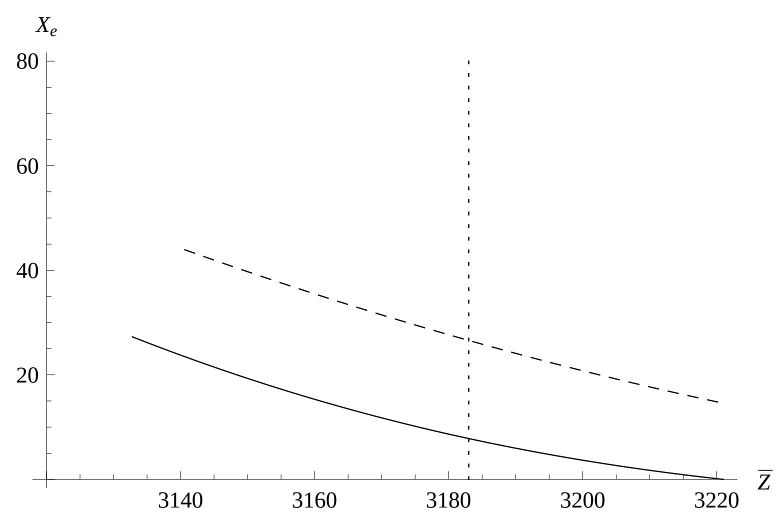

Figure 6: Quantity of shale gas developed as a function of the value of the ceiling in the reference case (solid line) and the constrained case (dotted line) when the marginal local damage is large

The constraint on energy expenditures actually modifies the arbitrage between the different energy sources. When the price elasticity of demand is low, which is true for electricity demand, the development of the clean backstop is always postponed and the quantity of shale gas devel-

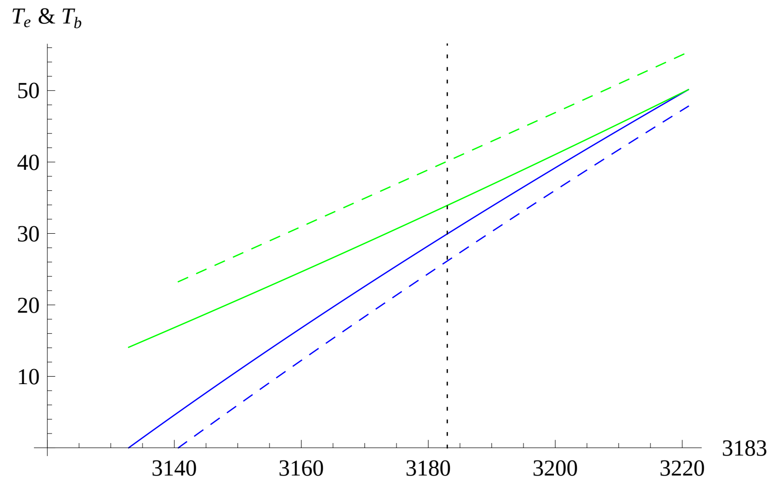

Figure 7: Switching dates T_e (blue) and T_b (green) as functions of the value of the ceiling in the reference case (solid line) and the constrained case (dotted line) when the marginal local damage is large

oped always increased. The main reason of this over-investment in shale gas due to the financial constraint is that as local damages are not monetary costs, the relative total variable cost of shale gas decreases compared to those of coal and solar.

7 Conclusion

This paper has explored one particular aspect of the complex problem posed by unconventional gas: does climate policy justify developing more shale gas, and what is the consequence for the switch to clean energy? We have developed a model whose assumptions are appropriate to study this question, but do not allow us to address other aspects, among which two seem particularly important.

First, the economy we consider here is a closed economy, which makes it impossible to study the potential leakage effect of an asymmetric climate policy. In a companion paper (Daubanes *et al.,* 2016), we consider an open economy with two zones, one producing coal and shale gas and implementing a carbon ceiling constraint, the other one producing coal only and having no climate policy. Coal production of the first zone may be exported to the other one. We address the following questions. (1) Faced with a more stringent climate constraint, should the shale gas producing economy increase its gas production? (2) Does this strategy decrease or increase global emissions?

Second, our model is a partial equilibrium of the electricity sector. However, shale gas supporters in the US put forward that it has allowed to create jobs, relocate some manufacturing activities, lower the vulnerability to oil shocks, and impact positively the external balance (IMF, 2014). Hence, the general equilibrium effects of shale gas exploitation should be analyzed.

Some other aspects of the shale gas question are worth studying, among which, in no particular order: the reasons why in France, not only the *exploitation* of shale gas is banned, but also the *exploration* of potential reserves; the impact of the subsoil property rights regimes on the decision to develop shale gas; the NIMBY effects of shale gas extraction in densely populated areas; etc. The question of the value of local damages associated with extraction should also receive attention, as this value may not be exogenous but instead depend on the investment in technology to reduce local damages. These aspects are left for future research.

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Appendix

A Large local damage

In this case, equations (1) and (2) may be written as:

$$\int_{T_e}^{T_b} x_e(t)dt = X_e \tag{29}$$

$$\int_0^{T_e} \theta_d x_d(t)dt + \int_{T_e}^{T_b} \theta_e x_e(t)dt = \overline{Z} - Z_0$$

Using (29), this last equation reads:

$$\int_0^{T_e} x_d(t)dt = \frac{1}{\theta_d}\left(\overline{Z} - Z_0 - \theta_e X_e\right) \tag{30}$$

Totally differentiating system (29), (30), (13), (17) and (16) yields:

$$x_e(T_b)dT_b - x_e(T_e)dT_e + \int_{T_e}^{T_b} dx_e(t)dt = dX_e$$

$$x_d(T_e)dT_e + \int_0^{T_e} dx_d(t)dt = \frac{1}{\theta_d}\left(d\overline{Z} - \theta_e dX_e\right)$$

$$\left[\theta_d\mu_0 - (\lambda_0 + \theta_e\mu_0)\right]\rho dT_e + (\theta_d - \theta_e)d\mu_0 - d\lambda_0 = 0$$

$$\left[u'\left(x_e(T_b)\right)dx_e(T_b) - (c_e + d + (\lambda_0 + \theta_e\mu_0)e^{\rho T_b})dx_e(T_b) - ((d\lambda_0 + \theta_e d\mu_0) + (\lambda_0 + \theta_e\mu_0)\rho dT_b)e^{\rho T_b}x_e(T_b)\right]$$
$$= \left(CF''(T_b) - \rho CF'(T_b)\right)dT_b$$

$$d\lambda_0 = E''(X_e)dX_e$$

As

$$x_d(t) = D(p_d(t)) \Rightarrow dx_d(t) = D'(p_d(t))dp_d(t) = D'(p_d(t))\theta_d e^{\rho t}d\mu_0$$
$$x_e(t) = D(p_e(t)) \Rightarrow dx_e(t) = D'(p_e(t))dp_e(t) = D'(p_e(t))e^{\rho t}\left(d\lambda_0 + \theta_e d\mu_0\right)$$

the first 2 equations read equivalently:

$$x_e(T_b)dT_b - x_e(T_e)dT_e + \left[\int_{T_e}^{T_b} D'(p_e(t))e^{\rho t}dt\right]\left(d\lambda_0 + \theta_e d\mu_0\right) = dX_e$$

$$x_d(T_e)dT_e + \left[\int_0^{T_e} D'(p_d(t))e^{\rho t}dt\right]\theta_d d\mu_0 = \frac{1}{\theta_d}\left(d\overline{Z} - \theta_e dX_e\right)$$

Besides,

$$\dot{D}(p_d(t)) = D'(p_d(t))\dot{p}_d(t) = D'(p_d(t))\theta_d\mu_0\rho e^{\rho t}$$

$$\Rightarrow \int_0^{T_e} D'(p_d(t))e^{\rho t}dt = \frac{1}{\theta_d\mu_0\rho}\int_0^{T_e}\dot{D}(p_d(t)dt = \frac{1}{\theta_d\mu_0\rho}\left[D(p_d(T_e)) - D(p_d(0)\right] = \frac{x_d(T_e) - x_d(0)}{\theta_d\mu_0\rho}$$

and

$$\int_{T_e}^{T_b} D'(p_e(t))e^{\rho t}dt = \frac{x_e(T_b) - x_e(T_e)}{(\lambda_0 + \theta_e\mu_0)\rho}$$

Hence the first 2 equations read:

$$-x_e(T_e)dT_e + x_e(T_b)dT_b - dX_e + \frac{x_e(T_b) - x_e(T_e)}{(\lambda_0 + \theta_e\mu_0)\rho}\left(d\lambda_0 + \theta_e d\mu_0\right) = 0$$

$$x_d(T_e)dT_e + \frac{\theta_e}{\theta_d}dX_e + \frac{x_d(T_e) - x_d(0)}{\mu_0\rho}d\mu_0 = \frac{1}{\theta_d}d\bar{Z}$$

Using the equality between marginal utilities, the fourth equation simplifies, and we obtain easily:

$$A \times \begin{pmatrix} dT_e \\ dT_b \\ dX_e \\ d\lambda_0 \\ d\mu_0 \end{pmatrix} = \begin{pmatrix} 0 \\ \frac{1}{\theta_d} \\ 0 \\ 0 \\ 0 \end{pmatrix} d\overline{Z}$$

with

$$A = \begin{pmatrix} -x_e(T_e) & x_e(T_b) & -1 & \frac{x_e(T_b)-x_e(T_e)}{(\lambda_0+\theta_e\mu_0)\rho} & \theta_e\frac{x_e(T_b)-x_e(T_e)}{(\lambda_0+\theta_e\mu_0)\rho} \\ x_e(T_e) & 0 & \frac{\theta_e}{\theta_d} & 0 & \frac{x_e(T_e)-x_d(0)}{\mu_0\rho} \\ \left[\lambda_0 + (\theta_e - \theta_d)\mu_0\right]\rho & 0 & 0 & 1 & \theta_e - \theta_d \\ 0 & (\lambda_0 + \theta_e\mu_0)\rho x_e(T_b) + z_1 & 0 & x_e(T_b) & \theta_e x_e(T_b) \\ 0 & 0 & -z_2 & 1 & 0 \end{pmatrix}$$

where

$$z_1 = \left(CF''(T_b) - \rho CF'(T_b)\right)e^{-\rho T_b} > 0$$

$$z_2 = E''(X_e) > 0$$

Hence:

$$\begin{aligned}
&\rho\theta_d\mu_0(\lambda_0+\theta_e\mu_0)\det A \\
&= \theta_d\left[\underbrace{(x_e(T_e)-x_e(T_b))}_{>0} x_d(0)\theta_d\mu_0 + \underbrace{(x_d(0)-x_e(T_e))}_{>0} x_e(T_b)\left(\lambda_0+\theta_e\mu_0\right)\right] z_1 z_2 \\
&+ \rho\left\{\left[\underbrace{(\theta_e x_e(T_b)-\theta_d x_d(0))}_{<0}\theta_e\mu_0 - x_d(0)\theta_d\lambda_0\right]\underbrace{(\lambda_0+(\theta_e-\theta_d)\mu_0)}_{<0} + x_e(T_e)\theta_d\lambda_0^2\right\} z_1 \\
&+ \rho\theta_d x_d(0)x_e(T_e)x_e(T_b)\theta_d\mu_0(\lambda_0+\theta_e\mu_0)z_2 \\
&+ \rho^2\theta_d(\lambda_0+\theta_e\mu_0)x_e(T_b)\left[x_e(T_e)\lambda_0^2 - x_d(0)(\lambda_0+\theta_e\mu_0)\underbrace{(\lambda_0+(\theta_e-\theta_d)\mu_0)}_{<0}\right]
\end{aligned}$$

i.e. $\det A > 0$.

$$A^{-1}\times\begin{pmatrix} 0 \\ \frac{1}{\theta_d} \\ 0 \\ 0 \\ 0 \end{pmatrix} = \frac{1}{\rho\theta_d\mu_0(\lambda_0+\theta_e\mu_0)\det A}\times$$

$$\begin{pmatrix}
\mu_0\left(\lambda_0+\theta_e\mu_0\right)\left[\frac{\theta_d}{\lambda_0+\theta_e\mu_0}\left(x_e(T_e)-x_e(T_b)\right)z_1z_2 + \rho z_1(\theta_d-\theta_e) + \rho x_e(T_b)\left(x_e(T_e)z_2\theta_d + \rho(\theta_d-\theta_e)\left(\lambda_0+\theta_e\mu_0\right)\right)\right] \\
-\rho x_e(T_b)\mu_0(\lambda_0+\theta_e\mu_0)\left[-x_e(T_e)\theta_d z_2 + \rho\theta_e\underbrace{(\lambda_0+(\theta_e-\theta_d)\mu_0)}_{<0}\right] \\
-\rho\mu_0\left[-x_e(T_b)z_1\theta_e\underbrace{(\lambda_0+(\theta_e-\theta_d)\mu_0)}_{<0} + x_e(T_e)\theta_d\lambda_0\left(z_1+\rho x_e(T_b)(\lambda_0+\theta_e\mu_0)\right)\right] \\
-z_2\rho\mu_0\left[-x_e(T_b)z_1\theta_e\left(\lambda_0+(\theta_e-\theta_d)\mu_0\right) + x_e(T_e)\theta_d\lambda_0\left(z_1+\rho x_e(T_b)(\lambda_0+\theta_e\mu_0)\right)\right] \\
-\rho\mu_0\left(\lambda_0+\theta_e\mu_0\right)\left[\begin{array}{c} \frac{\theta_d\mu_0}{\lambda_0+\theta_e\mu_0}(x_e(T_e)-x_e(T_b))z_1z_2 + x_e(T_b)z_1z_2 - \rho z_1\left(\lambda_0+(\theta_e-\theta_d)\mu_0\right) \\ -\rho x_e(T_b)\left[-x_e(T_e)\theta_d\mu_0 z_2 + \rho(\lambda_0+\theta_e\mu_0)\left(\lambda_0+(\theta_e-\theta_d)\mu_0\right)\right] \end{array}\right]
\end{pmatrix}$$

As $\det A > 0$, we deduce:

$$\frac{\partial T_e}{\partial \overline{Z}} > 0, \quad \frac{\partial T_b}{\partial \overline{Z}} > 0, \quad \frac{\partial X_e}{\partial \overline{Z}} < 0, \quad \frac{\partial \lambda_0}{\partial \overline{Z}} < 0, \quad \frac{\partial \mu_0}{\partial \overline{Z}} < 0$$

B Small local damage

In this case, equations (1) and (2) may be written as:

$$\int_0^{T_d} x_e(t)dt = X_e \tag{31}$$

$$\int_{T_d}^{T_b} x_d(t)dt = \frac{1}{\theta_d}\left(\overline{Z} - Z_0 - \theta_e X_e\right) \tag{32}$$

Totally differentiating system (31), (32), (15), (17) and (16) yields:

$$x_e(T_d)dT_d + \frac{x_e(T_d) - x_e(0)}{(\lambda_0 + \theta_e\mu_0)\rho} = dX_e$$

$$x_d(T_b)dT_b - x_d(T_d)dT_d + \frac{x_d(T_b) - x_d(T_d)}{\theta_d\mu_0\rho} = \frac{1}{\theta_d}\left(d\overline{Z} - \theta_e dX_e\right)$$

$$-((d\lambda_0 + \theta_e d\mu_0) + (\lambda_0 + \theta_e\mu_0)\rho dT_d)e^{\rho T_d}x_e(T_d) + \theta_d(d\mu_0 + \mu_0\rho dT_d)e^{\rho T_d}x_d(T_d) = 0$$

$$-\theta_d(d\mu_0 + \rho dT_b)e^{\rho T_b}x_d(T_b) = \left(CF''(T_b) - \rho CF'(T_b)\right)dT_b$$

$$d\lambda_0 = E''(X_e)dX_e$$

Using $x_e(T_d) = x_d(T_d)$, we obtain:

$$A \times \begin{pmatrix} dT_d \\ dT_b \\ dX_e \\ d\lambda_0 \\ d\mu_0 \end{pmatrix} = \begin{pmatrix} 0 \\ \frac{1}{\theta_d} \\ 0 \\ 0 \\ 0 \end{pmatrix} d\overline{Z}$$

with

$$A = \begin{pmatrix} x_d(T_d) & 0 & -1 & \frac{x_d(T_d)-x_e(0)}{(\lambda_0+\theta_e\mu_0)\rho} & \theta_e\frac{x_d(T_d)-x_e(0)}{(\lambda_0+\theta_e\mu_0)\rho} \\ -x_d(T_d) & x_d(T_b) & \frac{\theta_e}{\theta_d} & 0 & \frac{x_d(T_b)-x_d(T_d)}{\mu_0\rho} \\ \left[-\theta_d\mu_0 + (\lambda_0+\theta_e\mu_0)\right]\rho & 0 & 0 & 1 & -(\theta_d - \theta_e) \\ 0 & y_1 & 0 & 0 & \theta_d x_d(T_b) \\ 0 & 0 & -E''(X_e) & 1 & 0 \end{pmatrix}$$

where

$$y_1 = \left(CF''(T_b) - \rho CF'(T_b)\right)e^{-\rho T_b} + \rho x_d(T_b)\theta_d\mu_0 > 0$$

Let's denote

$$y_2 = E''(X_e)\left[x_d(T_d)\theta_d\mu_0 + x_e(0)\left(\lambda_0 + (\theta_e - \theta_d)\,\mu_0\right)\right]$$

According to (15), we have:

$$\lambda_0 + (\theta_e - \theta_d)\,\mu_0 = \left(c_d - (c_e + d)\right)e^{-\rho T_d} > 0$$

which implies that y_2 is also positive.

We have

$$\begin{aligned}
&-\rho\theta_d\mu_0(\lambda_0 + \theta_e\mu_0)\det A \\
&= \rho x_d(T_b)^2\theta_d^2\mu_0\left\{\rho(\lambda_0 + \theta_e\mu_0)(\lambda_0 + (\theta_e - \theta_d)\mu_0) + E^{''}(X_e)\left[x_d(T_d)\theta_d\mu_0 + x_e(0)(\lambda_0 + (\theta_e - \theta_d)\mu_0)\right]\right\} \\
&+ y_1\rho\left\{x_d(T_d)\theta_d\lambda_0^2 + x_e(0)\theta_e^2\mu_0(\lambda_0 + (\theta_e - \theta_d)\mu_0) - x_d(T_b)\theta_d(\lambda_0 + \theta_e\mu_0)(\lambda_0 + (\theta_e - \theta_d)\mu_0)\right\} \\
&+ y_1E^{''}(X_e)\theta_d\left\{x_e(0)(\lambda_0 + \theta_e\mu_0)\left(x_d(T_d) - x_d(T_b)\right) + x_d(T_b)\theta_d\mu_0(x_e(0) - x_d(T_d))\right\}
\end{aligned}$$

It is straightforward that the terms of the first and third lines are positive. Let look at the term of the second line:

$$y_1\rho\left\{x_d(T_d)\theta_d\lambda_0^2 + x_e(0)\theta_e^2\mu_0(\lambda_0 + (\theta_e - \theta_d)\mu_0) - x_d(T_b)\theta_d(\lambda_0 + \theta_e\mu_0)(\lambda_0 + (\theta_e - \theta_d)\mu_0)\right\}$$

Dividing by $y_1\rho > 0$, it has the sign of:

$$\begin{aligned}
&\lambda_0^2(\theta_d x_d(T_d) - \theta_d x_d(T_b)) \\
&+ \lambda_0\mu_0(\theta_e^2 x_e(0) + \theta_d^2 x_d(T_b) - 2\theta_e\theta_d x_d(T_b)) \\
&+ \mu_0^2\theta_e(\theta_e^2 x_e(0) + \theta_d^2 x_d(T_b) - \theta_e\theta_d x_d(T_b) - \theta_e\theta_d x_e(0))
\end{aligned}$$

It is straightforward that $\lambda_0^2(\theta_d x_d(T_d) - \theta_d x_d(T_b)) > 0$. Moreover

$$\lambda_0\mu_0(\theta_e^2 x_e(0) + \theta_d^2 x_d(T_b) - 2\theta_e\theta_d x_d(T_b)) = \lambda_0\mu_0 x_d(T_b)(\theta_d - \theta_e)^2 + \lambda_0\mu_0\theta_e^2(x_e(0) - x_d(T_b)) \quad (33)$$

and

$$\mu_0^2\theta_e(\theta_e^2 x_e(0) + \theta_d^2 x_d(T_b) - \theta_e\theta_d x_d(T_b) - \theta_e\theta_d x_e(0)) = \mu_0^2\theta_e(\theta_d - \theta_e)(\theta_d x_d(T_b) - \theta_e x_e(0)) \quad (34)$$

so that regrouping the last two terms (33) and (34), one gets :

$$\lambda_0\mu_0\left(\theta_e^2 x_e(0)+\theta_d^2 x_d(T_b)-2\theta_e\theta_d x_d(T_b)\right)+\mu_0^2\theta_e\left(\theta_e^2 x_e(0)+\theta_d^2 x_d(T_b)-\theta_e\theta_d x_d(T_b)-\theta_e\theta_d x_e(0)\right)$$
$$=\lambda_0\mu_0 x_d(T_b)(\theta_d-\theta_e)^2+\lambda_0\mu_0\theta_e^2(x_e(0)-x_d(T_b))+\mu_0^2\theta_e(\theta_d-\theta_e)(\theta_d x_d(T_b)-\theta_e x_e(0))$$
$$=\lambda_0\mu_0 x_d(T_b)(\theta_d-\theta_e)^2+\lambda_0\mu_0\theta_e^2(x_e(0)-x_d(T_b))+\mu_0^2\theta_e(\theta_d-\theta_e)((\theta_d-\theta_e)x_d(T_b)-\theta_e(x_e(0)-x_d(T_b)))$$
$$=\lambda_0\mu_0 x_d(T_b)(\theta_d-\theta_e)^2+\lambda_0\mu_0\theta_e^2(x_e(0)-x_d(T_b))+\mu_0^2\theta_e(\theta_d-\theta_e)^2 x_d(T_b)-\mu_0^2\theta_e^2(\theta_d-\theta_e)(x_e(0)-x_d(T_b))$$
$$=\mu_0 x_d(T_b)(\theta_d-\theta_e)^2(\lambda_0+\theta_e\mu_0)+\mu_0\theta_e^2(x_e(0)-x_d(T_b))(\lambda_0+\mu_0(\theta_e-\theta_d))$$

which is positive. As a result:

$$\det A<0$$

We also obtain:

$$A^{-1}\times\begin{pmatrix}0\\ \frac{1}{\theta_d}\\ 0\\ 0\\ 0\end{pmatrix}=\frac{1}{\theta_d(\lambda_0+\theta_e\mu_0)\det A}\begin{pmatrix} y_1\left[E''(X_e)(x_e(0)-x_d(T_d))\theta_d+\rho\left(\theta_d-\theta_e\right)\left(\lambda_0+\theta_e\mu_0\right)\right]/\rho\\ -x_d(T_b)\theta_d\left[\rho(\lambda_0+\theta_e\mu_0)\left(\lambda_0+\left(\theta_e-\theta_d\right)\mu_0\right)+y_2\right]\\ y_1\left[x_d(T_d)\theta_d\lambda_0-x_e(0)\theta_e\left(\lambda_0+\left(\theta_e-\theta_d\right)\mu_0\right)\right]\\ y_1E''(X_e)\left[x_d(T_d)\theta_d\lambda_0-x_e(0)\theta_e\left(\lambda_0+\left(\theta_e-\theta_d\right)\mu_0\right)\right]\\ y_1\left[\rho(\lambda_0+\theta_e\mu_0)\left(\lambda_0+\left(\theta_e-\theta_d\right)\mu_0\right)+y_2\right]\end{pmatrix}$$

As $\det A<0$, we deduce:

$$\frac{\partial T_d}{\partial\overline{Z}}<0,\quad\frac{\partial T_b}{\partial\overline{Z}}>0,\quad\frac{\partial X_e}{\partial\overline{Z}}\text{ ambiguous},\quad\frac{\partial\lambda_0}{\partial\overline{Z}}\text{ ambiguous},\quad\frac{\partial\mu_0}{\partial\overline{Z}}<0$$

$\frac{\partial X_e}{\partial\overline{Z}}$ and $\frac{\partial\lambda_0}{\partial\overline{Z}}$ have the same sign as $x_e(0)\theta_e\left(\lambda_0+\left(\theta_e-\theta_d\right)\mu_0\right)-x_d(T_d)\theta_d\lambda_0$. It is negative when $\theta_e=0$, and positive when $\theta_e=\theta_d$.

C Low price elasticity of demand

Step 1. Expenditure $px(p)$ is continuous and increasing with p. From Lagrange theorem, denoting $p_{T_b}\equiv p(T_b)$ and $x_{T_b}=x(p(T_b))$ there exists a price $p_i\in]c_b,p_{T_b}[$ such that:

$$p_{T_b}x_{T_b}=c_bx_b+(x(p_i)+p_ix'(p_i))(p_{T_b}-c_b)$$

The elasticity of demand at price p_i is $\epsilon_i=-\frac{p_ix'(p_i)}{x(p_i)}$ so that the above equation can be rewritten:

$$\frac{x_{T_b}}{x(p_i)}=\frac{c_bx_b}{p_{T_b}x(p_i)}+(1-\epsilon_i)(1-\frac{c_b}{p_{T_b}})$$

or

$$\frac{x_{T_b}}{x(p_i)} - 1 = \frac{c_b}{p_{T_b}}(\frac{x_b}{x(p_i)} - 1) - \epsilon_i(1 - \frac{c_b}{p_{T_b}})$$

As $\frac{x_{T_b}}{x(p_i)} - 1 < 0$ and $\frac{c_b}{p_{T_b}}\left(\frac{x_b}{x(p_i)} - 1\right) > 0$, denoting $\epsilon = \max_i(\epsilon_i)$, it comes that

$$\frac{x_{T_b}}{x(p_i)} - 1 = O(\epsilon) \tag{35}$$

$$\frac{x_b}{x(p_i)} - 1 = O(\epsilon)\frac{p_{T_b}}{c_b} \tag{36}$$

Similarly, using Lagrange theorem between prices c_e and c_b, one gets, with $p_j \in]c_e, c_b[$:

$$\frac{x_b}{x(p_j)} - 1 = O(\epsilon) \tag{37}$$

$$\frac{x_{c_e}}{x(p_j)} - 1 = O(\epsilon)\frac{c_b}{c_e} \tag{38}$$

So that, if the price elasticity of demand is such that $\epsilon\frac{cb}{c_e} = O(\zeta)$, then $\frac{x_b}{x(c_e)} - 1 = O(\zeta)$.

Step 2. Recall that:

$$(u(x_b) - c_b x_b) - (u(x_{T_b}) - p_{T_b}x_{T_b}) = -(CF'(T_b) - \rho CF(T_b)) \tag{39}$$

But $-(CF'(T_b) - \rho CF(T_b))$ is decreasing with T_b (as $CF'' > 0$) so that $-(CF'(T_b) - \rho CF(T_b)) < -\left(CF'(\frac{\bar{Z}}{\theta_d x_{c_e}}) - \rho CF(\frac{\bar{Z}}{\theta_d x_{c_e}})\right)$ and using equation (38), it comes that $\forall c_b, c_e$, there exists ϵ such that $-(CF'(T_b) - \rho CF(T_b)) \leq -\left(CF'(\frac{\bar{Z}}{\theta_d x_b}) - \rho CF(\frac{\bar{Z}}{\theta_d x_b})\right)$. Equation (39) thus implies that:

$$(u(x_b) - c_b x_b) - (u(x_{T_b}) - p_{T_b}x_{T_b}) \leq -\left(CF'(\frac{\bar{Z}}{\theta_d x_b}) - \rho CF(\frac{\bar{Z}}{\theta_d x_b})\right)$$

so that

$$0 \leq p_{T_b}x_{T_b} - c_b x_b \leq -\left(CF'(\frac{\bar{Z}}{\theta_d x_b}) - \rho CF(\frac{\bar{Z}}{\theta_d x_b})\right)$$

so that

$$0 \leq \frac{p_{T_b}x_{T_b}}{c_b x_b} - 1 \leq \frac{-\left(CF'(\frac{\bar{Z}}{\theta_d x_b}) - \rho CF(\frac{\bar{Z}}{\theta_d x_b})\right)}{c_b x_b}$$

and thus

$$1 \leq \frac{p_{T_b}}{c_b} \leq \left[1 + \frac{-\left(CF'(\frac{\bar{Z}}{\theta_d x_b}) - \rho CF(\frac{\bar{Z}}{\theta_d x_b})\right)}{c_b x_b}\right]\frac{x_b}{x_{T_b}}$$

Substituting the equation above in equation (36), it comes that:

$$\frac{x_b}{x(p_i)} - 1 \leq O(\epsilon)\left[1 + \frac{-\left(CF'(\frac{\bar{Z}}{\theta_d x_b}) - \rho CF(\frac{\bar{Z}}{\theta_d x_b})\right)}{c_b x_b}\right]\frac{x_b}{x_{T_b}}$$

which can be rewritten, multiplying both sides by $\frac{x_{T_b}}{x_b}$:

$$\frac{x_{T_b}}{x(p_i)} - \frac{x_{T_b}}{x_b} \leq O(\epsilon)\left[1 + \frac{-\left(CF'(\frac{\bar{Z}}{\theta_d x_b}) - \rho CF(\frac{\bar{Z}}{\theta_d x_b})\right)}{c_b x_b}\right]$$

For an arbitrarily small ζ, one can find ϵ such that $\epsilon\left[1 + \frac{-\left(CF'(\frac{\bar{Z}}{\theta_d x_b}) - \rho CF(\frac{\bar{Z}}{\theta_d x_b})\right)}{c_b x_b}\right] \leq \zeta$. As a result, if $\epsilon\left[1 + \frac{-\left(CF'(\frac{\bar{Z}}{\theta_d x_b}) - \rho CF(\frac{\bar{Z}}{\theta_d x_b})\right)}{c_b x_b}\right] = O(\zeta)$, then, using equation (35): $\frac{x_{T_b}}{x_b} = \frac{x_{T_b}}{x(p_i)} + O(\zeta) = 1 + O(\zeta)$.

So that, $\forall \zeta, c_e, c_b, x_b, \bar{Z}$, there exists ϵ such that, if the elasticity of demand is always below ϵ then $\forall p \in [c_e, p_{T_b}]$,:

$$\frac{x_p}{x_e} = 1 + O(\zeta)$$

For a small local damage, we have shown that $\frac{dX_e}{d\bar{Z}}$ has the sign of $x_e(0)\theta_e(\lambda_0 + \theta_e - \theta_d)\mu_0 - x_d(T_d)\theta_d\lambda_0$. Using that, for a sufficiently low elasticity of demand $x_e(0) = x_d(T_d) + O(x_e(0)\zeta)$, it comes that $\frac{dX_e}{d\bar{Z}}$ has the sign of $-x_e(0)((\theta_d - \theta_e)(\lambda_0 + \theta_e\mu_0) + O(\zeta\theta_d\lambda_0)) < 0$.