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OPTIMAL INTERTEMPORAL ABATEMENT DECISIONS UNDER AMBIGUITY AVERSION

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In a two-period partial-equilibrium model, this article characterises ambiguity averse firms' optimal intertemporal abatement decisions in a cap-and-trade regime (ETS). Ambiguity aversion induces two effects that can be aligned or countervailing: (i) pessimism, which distorts firms' subjective beliefs by overweighting bad scenarios and (ii) ambiguity prudence, which corresponds to an increase in firms' discount factor. The direction and magnitude of these two effects depend on the degree of ambiguity aversion and initial allowance allocation, which is thus non neutral. Alternatively, firms are covered under a tax regime and only subject to ambiguity prudence. Under ambiguity aversion, both tax and ETS are not conducive to intertemporal cost-efficiency. We argue that ambiguity aversion on the part of firms can capture the influence of regulatory uncertainty on their abatement decisions and might contribute to observed recurrent patterns in existing ETSs: (1) there is a tendency towards allowance surplus formation when allowances are grandfathered, which is more pronounced under auctioning; (2) ambiguity prudence might support declining and low allowance prices.

Comments welcomed

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1 Introduction

According to Hasegawa & Salant (2014,[40]), «Permit markets may be subject to three kinds of uncertainty: (1) uncertainty about the aggregate demand for permits that will be resolved by an information disclosure at a fixed date in the future; (2) aggregate demand shocks in each period; and (3) regulatory uncertainty.» Point (1) is illustrated by Phase I of the European Union Emissions Trading System (EUETS) where European allowances (EUAs) suddenly lost two thirds of their value consecutive to the disclosure of verified emissions hinting at a demand for permits lower than expected[1]. Point (2) is reflected in the numerous price swings that have affected the EUETS since 2005, oft with no clear underlying causes[2]. In particular, Koch et al. (2014,[47]) show that no more than 10% of the EUA price variation can be explained by abatement-related fundamentals, the bulk of which stemming from economic conditions and growth of solar and wind powers. In a later study and regarding point (3), Koch et al. (2016,[48]) find the EUETS highly responsive to political events and announcements[3]. In addition, existing ETSs have proven to be over-allocated *ex ante*, the result of which being the formation of allowance surpluses along with declining and low prices. This is largely attributable to generous cap-setting in the first place in conjunction with pervasive uncertainty on economic growth, the reach of complementary policies[4], the use of offset credits, electricity imports and reshuffling, which all contribute to the erosion of the cap stringency[5] − see e.g. Borenstein et al. (2015,[12]), de Perthuis & Trotignon (2014,[60]) or Tvinnereim (2014,[76]). This has sparked ongoing design reforms in the form of *ex-post* allowance supply management in all existing ETSs, which generates regulatory uncertainty. Moreover, in the words of Heal & Millner (2014,[41]), «the issue of climate change is beset with uncertainties, many of which are only partially captured by our existing analytical tools», thereby adding to the level of regulatory uncertainty. However, firms' anticipations of the future environmental constraint, which fundamentally depend on both the credibility of the regulator's announcements and long-term targets, dictate their abatements and low-carbon investments through time. Due to continual interventions in existing ETSs,

[1]Further owing to the fact that Phase I vintage allowances were not bankable into Phase 2.

[2]For instance, EUAs nearly lost 50% of their value in early 2016, but opinions were mixed as to which factors, or combination thereof, generated the price drop − see e.g. `http://carbon-pulse.com/16460`.

[3]The backloading decision process is shown to have affected the EUA price negatively. In contrast, the price reacted positively to the announcement of the 2020 and 2030 policy packages, albeit to a smaller extent.

[4]Regarding policy overlap, Schmalensee & Stavins (2013,[70]) also underline the impact of railway deregulation in the US SO_2 trading program.

[5]This is due to significant uncertainty in baseline emissions, estimated to be «at least as large as uncertainty about the effect of abatement measures» (Borenstein et al. (2015,[12])). Coupled with little price elasticity, price volatility is likely to be high, with very low or high prices being the most likely outcomes.

ongoing concern about future regulatory action at an unknown time in the future should affect present allowance prices[6]. That regulatory uncertainty weighs on allowance prices is a point made by Salant (2015,[67]) for the EUETS[7].

It is apparent that uncertainty that prevails in existing ETSs is significant, stems from different sources and can hardly be deemed objectively predictable. Firms thus lack relevant information to properly assign a probability measure uniquely describing the stochastic nature of their decision problems, which corresponds to a situation of ambiguity[8]. There is ample evidence[9] that most individuals treat ambiguity differently than objective risk, i.e., they prefer gambles with known rather than unknown probabilities. Alternatives to the subjective expected utility (SEU) criterion of Savage (1954,[68]) have been proposed that differ in their treatment of objective and subjective probabilities[10]. A first generation of models has recourse to non-additive capacities where the weight of an outcome depends on its ranking among all possible outcomes, thereby representing non-additive beliefs − see e.g. Schmeidler (1989,[71]) and Chateauneuf et al. (2007,[18]). Because information is scarce, a second generation of models rather considers that agents have a set of multiple subjective priors. In this direction, Gilboa & Schmeidler (1989,[34]) provide behavioural foundations to the pioneering maxmin expected utility (MEU) criterion, later axiomatised by Ghirardato et al. (2004,[32]), which considers a combination of maximal and minimal expected utilities over a set of multiple priors. More recently, recursive expected utility (REU) models, as e.g. in Klibanoff et al. (2005,[44]), consider that agents have both a set of multiple first-order objective priors with a second-order subjective probability over them and that they are EU-maximisers over the two layers of uncertainty. In practice, faced with regulatory uncertainty, firms are confronted with different possible scenarios about the future regulatory framework and its related price and allowance demand forecasts. Such scenarios are often objectively known to all firms as they are provided by a group of experts[11]. Since firms have subjective

[6]Examples are many. E.g. the steady NZU price rise in early 2016 in the NZETS is attributable to the government's announcement that the 2:1 compliance rule should soon be abolished. Similarly, downward pressure on pilot prices in China results from regulatory uncertainty about the transition to a national market, especially regarding the carry-over provision for pilot allowances into the national market. RGA prices also increased when the 45% slash in the overall RGGI cap was discussed but before it was actually implemented.

[7]With references to the «peso problem» and the gold spot price in the 70's that conflict with the assumption of rational expectations under risk neutrality − cf. Salant & Henderson (1978,[66]) − Stephen Salant shows that regulatory uncertainty affects the EUA price, even for risk neutral agents. This suggests that such a regulatory risk cannot be entirely hedged against.

[8]In contrast, the notion of risk refers to situations where such probabilities are perfectly known.

[9]Initiated by Ellsberg (1961,[27]), who shows that rational decision-makers behave in ways incompatible with Savage's axioms, more precisely the sure-thing principle.

[10]See Etner et al. (2012,[28]) or Machina & Siniscalchi (2014,[50]) for a review.

[11]E.g. BNEF, Energy Aspects, ICIS-Tschach, Point Carbon, diverse academic fora or think tanks, etc.

beliefs over this set of scenarios/experts, a REU framework fits the situation well. Ambiguity aversion then corresponds to the additional aversion (w.r.t. risk aversion) to being unsure about the probabilities of outcomes and conduces agents to favour those acts that tend to reduce the level of ambiguity.

Assuming ambiguity on future firms' baseline emissions and allowance price, we develop a two-period abatement decision problem where firms display ambiguity aversion as a way to capture the impact of regulatory uncertainty, which cannot be hedged against. In particular, we investigate how ambiguity aversion alters optimal intertemporal abatement decisions, where ambiguity neutrality serves as our natural benchmark, and we analyse the influence of initial allowance allocation. The benchmark corresponds to a situation where the emission path is determined by the least-discounted cost solution, i.e., where intertemporal cost-efficiency obtains (in expectations), see e.g. Rubin (1996,[64]) and Schennach (2000,[69]). In addition, our framework is flexible enough as to nest both price and quantity regulations alternatively, which, as we will see, provides a crisp separation between the two effects induced by ambiguity aversion[12], namely pessimism and ambiguity prudence. Both regimes deteriorate in the presence of ambiguity and are not conducive to intertemporal cost-efficiency[13]. First, pessimism distorts firms' second-order subjective beliefs by over-weighting scenarios with low expected profits. It is conducive to an increase in allowance banking provided that those «bad scenarios» coincide with scenarios with high marginal benefit from banking. This ultimately relates to initial allocation, whose neutrality does not hold under ambiguity aversion[14]. If initial allocation is low (high) enough, and the firm expects to be net buyer (seller) of permits in the future, pessimism alone tends to raise (decrease) banking. In essence, pessimism accounts for the expected future position on the market in present abatement decisions, while these are solely based on future price expectation under ambiguity neutrality. Second, ambiguity prudence, which we define as decreasing absolute ambiguity aversion (DAAA) in the paper, amounts to an increase in firms' discount factor, whose intensity also depends on initial allocation. Controlling for pessimism, ambiguity prudence thus always leads to an increase in banking. Together, these two effects, which can be aligned or countervailing, determine the optimal abatement path under ambiguity aversion relative to ambiguity neutrality, but

[12]Since we merely compare the magnitude of the optimal abatement distortion induced by ambiguity aversion under the two regimes, our analysis differs from the «Prices vs Quantities» literature, i.e., a Weitzman-like welfare comparison − see Weitzman (1974,[78]).

[13]With our neat «Prices vs Quantities» framework, a quota regime is subject to pessimism and ambiguity prudence while a tax regime is only subject to the latter.

[14]The independence property of initial allocation in a cap-and-trade scheme does not hold as soon as one of the requiring assumptions sustaining the results of the seminal paper of Montgomery (1972,[54]) is relaxed − see e.g. Hahn & Stavins (2011,[38]).

comparative static results are hard to come by. In particular, an increase in the degree of ambiguity aversion does increase pessimism but its effect on ambiguity prudence is not clear. That is, a higher degree of ambiguity aversion is not necessarily conducive to a bigger adjustment in banking (in absolute terms). Using a simple parametrical example, numerical simulations illustrate the impact of initial allocation for different ambiguity aversion degrees. Our framework gives some theoretical foundations to observed patterns in existing ETSs. In short, we show that under ambiguity prudence, (i) there is a natural tendency towards surplus formation under grandfathering, which is even more pronounced under auctioning; (ii) firms apply a higher discount factor, which contradicts the standard banking rationale and might contribute to declining and persistent low prices.

1.1 Related Literature

In a similar vein, Baldursson & von der Fehr (2004,[3]) show that, under risk aversion on the part of firms, those who expect to be net short (long) tend to over-invest (under-invest) in abatement technology, as compared with risk neutrality. Ben-David et al. (2000,[5]) find similar results, which are also supported by laboratory experiments conducted by Betz & Gunnthorsdottir (2009,[11]). Under risk aversion, however, Baldursson & von der Fehr (2004,[3]) show that only the quota regime deteriorates, while the tax regime is cost-efficient. As noted by Ellerman et al. (2010,[26]), there is an asymmetry between net long and short entities since the former are under no compulsion to sell and can adopt a passive wait-and-see attitude as long as uncertainty is high and experience is being gained[15]. The literature comparing the incentive to make irreversible investment in tax *versus* ETS under uncertainty has mixed results. For instance, partial-equilibrium analyses tend to favour tax over ETS, with uncertainty on the allowance price (Xepapadeas, 2001,[82]) or aggregate demand (Chao & Wilson, 1993,[17]). However, using a general equilibrium model with uncertainty on abatement costs, Zhao (2003,[84]) finds that investment incentives decrease in the level of cost uncertainty, but more so under a tax than an ETS. Finally, Colla et al. (2012,[20]) show that the presence of speculators in ETSs, with whom risk averse firms can trade permits, augments the risk-bearing capacity of the market and tends to reduce price volatility.

Ambiguity aversion has been applied to a variety of fields in economics, such as finance (Gollier, 2011,[35]), self-insurance and self-protection (Alary et al., 2013,[1]; Berger, 2016,[8]), formation of precautionary savings (Gierlinger & Gollier, 2015,[33]; Berger, 2014,[7]) or health

[15] As in early Phase I of the EUETS, where industrial companies, acknowledged to be long, did not see a significant effect of the carbon price on their output cost as did power companies, acknowledged to be short.

(Treich, 2010,[75]; Berger et al., 2013,[6]), and can explain otherwise unaccounted for empirical facts such as the equity premium puzzle (Collard et al., 2011,[21]) or the negative correlation between asset prices and returns (Ju & Miao, 2012,[42]). Of more relevance to our problem is the emerging theory of the competitive firm under ambiguity aversion à la Sandmo (1971,[65]) as in Wong (2015a,[79]) or the effects of risk and model uncertainty aversions on optimal abatement decisions, applied to Integrated Assessment Models as in Millner et al. (2013,[53]) and Berger et al. (2016,[10]), that warrant higher mitigation objectives. There is also mounting evidence that individuals tend to display ambiguity aversion and especially DAAA (ambiguity prudence), see e.g. Berger & Bosetti (2016,[9]) and references therein. Our paper develops a two-period model to analyse what is fundamentally a fully dynamic problem. However, extending our model to more periods would be technically difficult[16] and face a lack of relevant data for calibration. The literature provides different ways to deal with such technicality. Millner et al. (2013,[53]) opt for two simple but polar exogenous learning scenarios: one where all ambiguity resolves after the first period, the other with persistent and unchanged ambiguity throughout. Guerdjikova & Sciubba (2015,[36]) consider two similar types of ambiguity structures: one where ambiguity vanishes at the first date, one where ambiguity changes but persists over time. Ju & Miao (2012,[42]) consider Markov economies[17] that exhibit persistent ambiguity and propose a generalized recursive smooth ambiguity model. Collard et al. (2011,[21]) assume ambiguity prudence away by imposing constant absolute ambiguity aversion so as to simplify Euler equations. An alternative is found in Gierlinger & Gollier (2015,[33]) and Traeger (2014,[74]) who use a one-step-ahead formulation, which is composed of nested sets of identical ambiguity structure. Our two-period model, however, already captures the essence of the effects of ambiguity aversion and, in a special case, is able to finely decompose the ambiguity prudence and pessimism effects.

The remainder is organised as follows. Section 2 sets out our modelling framework and underlying assumptions. Section 3.1 investigates the effects of ambiguity aversion on abatement decisions in a tax regime. Sections 3.2 and 3.3 analyse the allowance banking decisions in cap-and-trade regime under ambiguity aversion. Section 4.1 presents comparative static results, further illustrated by a parametrical example in section 4.2. Section 5 concludes.

[16] This begs the question of the way preferences and beliefs are updated as new information comes in. The recursive KMM formulation simultaneously satisfies dynamic consistency and learning under ambiguity operated via a prior-by-prior Bayesian updating of beliefs. However, tensions between dynamic consistency, Bayesian learning and the positive value of information remain. For further references and proposals to weaken dynamic consistency, see e.g. Galanis (2015,[31]).

[17] In particular, the ambiguity-averse agent has time-variant beliefs over consumption growth based on a binary hidden space (boom or recession) following a Markov-switching process.

2 The model

A set of polluting firms can abate emissions so as to comply with the environmental regulation they are liable under − either a tax or an ETS. There are two dates, 1 and 2, and firms' date-1 abatement decision is made in a context where ambiguity prevails at date 2, in a sense that will be defined below. At the beginning of date 2 ambiguity is resolved and firms' optimal date-2 abatement decision depends on their date-1 abatement level and the realised shock. The paper compares firms' optimal date-1 abatement decisions under ambiguity aversion relative to ambiguity neutrality, depending upon which type of regime is in place.

Firms. Let there be a continuum $\mathcal{S} = [0; S]$ of infinitesimally small, polluting, competitive firms indexed by $s \in \mathcal{S}$, where S is the mass of firms. When firm s does not make any abatement effort it emits its baseline emissions level $b(s)$. Firm s' abatement decisions at both dates are denoted $a_1(s)$ and $a_2(s)$, respectively. Let $\omega(s)$ alternatively denote firm s' initial endowment of allowances in an ETS or tax-threshold liability[18]. Regulation is effective at both dates and terminates at the end of date 2[19]. Under a tax, we assume that the date-1 tax rate is zero. This is without loss of generality and roughly captures that tax rates generally rise over time. In an ETS, we assume that date-1 compliance is effective and that all inter-firm trades opportunities are exhausted. However, firms may still undertake additional abatement in the perspective of more stringent date-2 requirements, i.e., a_1 corresponds to additional date-1 abatements that free up allowances that are banked into date 2. This ensures that the Rubin-Schennach banking condition is always satisfied and assumes corner solutions away. Therefore, after cleaning activities, firm s' date-2 emissions are $b(s) - a_1(s) - a_2(s)$. Abatement cost functions are identical for all firms and given by twice continuously differentiable functions, C_1 and C_2. Abatement is said to have long-term effect in the sense that C_2 also depends on the level of date-1 abatement, $C_2 \equiv C_2(a_1, a_2)$. Due to this effect the marginal cost of date-1 abatement is $\partial_{a_1}(C_1 + C_2)$. Abatement costs are assumed to be strictly increasing and strictly convex on $[0; \infty[$ with no fixed cost, that is $C_1' > 0$, $C_1'' > 0$, with $C_1(0) = 0$ and $C_1'(0) = 0$, and $\forall a_1 \geq 0$, $\partial_{a_2} C_2(a_1, \cdot) > 0$ and $\partial^2_{a_2 a_2} C_2(a_1, \cdot) > 0$ with $C_2(a_1, 0) = 0$. Firms are also assumed to face decreasing abatement

[18] That is, the tariff is charged only on the difference between emissions and the threshold; see e.g. Pezzey & Jotzo (2013,[61]).

[19] Alternatively, regulation is effective at date 2 only, in which case a_1 may correspond to (i) investment in abatement technology in anticipation of forthcoming regulation; or (ii) early reduction permits handed out to firms for early abatements. These three interpretations are equivalent provided that a given level of abatement or investment cuts down emissions by a corresponding amount, and that date-1 abatement or investment reduces both date-1 and date-2 emissions by a corresponding amount.

opportunities as in Bréchet & Jouvet (2008,[14]), i.e., the date-2 marginal cost of abatement is increasing in the level of date-1 abatement, $\forall a_1, a_2 \geq 0$, $\partial^2_{a_1 a_2} C_2(a_1, a_2) \geq 0$. This effect is compensated by a learning-by-doing effect as firms gain experience in the abatement technology by abating at date 1. As in Slechten (2013,[72]), this is captured by assuming that $\forall a_1, a_2 \geq 0$, $\partial^2_{a_1 a_1}(C_1(a_1) + C_2(a_1, a_2)) \geq \partial^2_{a_1 a_2} C_2(a_1, a_2)$ and $\partial^2_{a_2 a_2} C_2(a_1, a_2) \geq \partial^2_{a_1 a_2} C_2(a_1, a_2)$. To derive analytical results, we will assume that abatement cost functions are equipped with the following quadratic specification[20]

$$\forall a_1, a_2 \geq 0,\ C_1(a_1) = \frac{c_1}{2} a_1^2 \text{ and } C_2(a_1, a_2) = \frac{c_2}{2} a_2^2 + \gamma a_1 a_2,\ \text{with } c_1, c_2 > 0,\ c_2 > \gamma, \tag{1}$$

where $\frac{1}{c_i}$ measures each firm's date-i flexibility in abatement and γ denotes the long-term abatement effect coefficient. Note that specification (1) satisfies the above assumptions. For tractability reasons, we will sometimes need to assume that there is no long-term effect of abatement, i.e., $\partial_{a_1} C_2 \equiv 0$ or $\gamma = 0$. In essence, firms are identical but for their initial allocation $\omega(s)$ as this will be shown to be an essential driver of date-1 abatement adjustment under ambiguity aversion.

Ambiguity aversion. Ambiguity originates from two sources: the date-2 allowance price and date-2 firm-level emission baselines[21]. Let the price risk $\tilde{\tau}$ be described by the objective cumulative distribution G^0, supported on $\mathrm{T} = [\underline{\tau}; \bar{\tau}]$, with $0 < \underline{\tau} < \bar{\tau} < \infty$. Let also the baseline risk $\tilde{b}$ be described by the objective cumulative distribution L^0, with support on $\mathrm{B} = \left[\underline{b}; \bar{b}\right]$ such that $0 < \underline{b} < \bar{b} < \infty$. These two risks are assumed to be independent, i.e., there is no connection between the prevailing market price and individual baselines[22]. Firms exhibit smooth ambiguity aversion in the sense of Klibanoff et al. (2005,[44]) and, as such, are uncertain about both G^0 and L^0. That is, they are confronted with a set of objective probability measures for $\tilde{\tau}$ and $\tilde{b}$, and are uncertain about which of those truly govern the two risks. For each realisation θ, called θ-scenario, of the random variable $\tilde{\theta}$, let $G(\cdot;\theta)$ and $L(\cdot;\theta)$ denote the objective probability measures for $\tilde{\tau}_\theta$ and $\tilde{b}_\theta$, the θ-scenario price and baseline risks, respectively. Ambiguity is represented by a second-order subjective

[20] This corresponds to a second-order Taylor expansion of abatement cost functions centred around baseline emissions. This assumption is standard, see e.g. Newell & Stavins (2003,[56]). The linear term is omitted for convenience − adding it would merely translate our results by a constant term.

[21] The case of ambiguity on abatement cost functions is available upon request from the author.

[22] A parallel assumption which is frequent in the literature on firms' decisions under uncertainty is that price and output be independent stochastic variables, see e.g. Viaene & Zilcha (1998,[77]) or Dalal & Alghalith (2009,[22]) and references therein.

probability distribution for $\tilde{\theta}$, F, supported on $\Theta = \left[\underline{\theta}; \bar{\theta}\right]$[23], which captures the firms' beliefs about which scenario they feel will materialise. For simplicity, let G and L be first-order independent given a θ-scenario, but second-order dependent across θ-scenarios. Attitudes toward ambiguity originate in the relaxation of the reduction of compound first and second order probabilities and firms' computations of their expected profits can be decomposed into three steps: first, in any given θ-scenario they compute their expected profits w.r.t. $G(\cdot;\theta)$ and $L(\cdot;\theta)$; second, each θ-scenario first-order expected profits is transformed by an increasing function ϕ; third, their second-order expected profits obtain by taking the expectation of the ϕ-transformed first-order expected profits w.r.t. F. Ambiguity aversion is characterised by a concave ϕ so that an ambiguity-averse agent dislikes any mean-preserving spread in the space of second-order expected profits, see e.g. (4). Firms are assumed to be risk neutral, i.e., they maximise raw profits so that the sole effects of ambiguity aversion are captured by the model[24]. Compared to other ambiguity aversion models, a KMM model of choice comes with both advantages in terms of tractability and nice properties (see also Gajdos et al. (2008,[30])) as it disentangles ambiguity (beliefs) from ambiguity aversion (tastes), has nice comparative static properties for which the decision-making under risk machinery readily applies, nests other ambiguity aversion models as special cases[25] and, as exposed below, can be embedded in a dynamic framework.

Intertemporal profits. Using the recursive smooth ambiguity model of Klibanoff et al. (2009,[45]), firms maximise their intertemporal profits w.r.t. date-1 abatement, that is

$$\max_{a_1 \geq 0} \pi_1(a_1) + \beta\phi^{-1}(\mathbb{E}_F\{\phi(\mathcal{V}(a_1;\tilde{\theta}))\}), \tag{2}$$

[23]Note that in Klibanoff et al. (2009,[45]) the scenario space Θ is finite but here we consider its continuous extension with a continuous subjective distribution F. Note also that the KMM axiomatisation is based on acts rather than probability distribution on T or B, the outcome spaces.

[24]In this sense, ϕ characterises aversion towards model uncertainty, cf. Marinacci (2015,[51]). In a KMM model of choice, ambiguity results from the combination of risk and model uncertainty and ambiguity aversion requires stronger aversion to model uncertainty than to risk. Thus, by assuming firms to be risk neutral, the effects of ambiguity aversion − in terms of magnitude − might be overestimated, see Berger & Bosetti (2016,[9]). If firms were risk averse and maximised the utility of their profits, the anticomonotonicity criterion to sign pessimism would have to be restated. Joint conditions on both risk and ambiguity aversion/prudence would emerge, see e.g. Gierlinger & Gollier (2015,[33],Prop3&4) and Wong (2015a,[79],Prop2). For comparative statics on risk aversion under ambiguity, see also Guetlein (2016,[37]). However, risk neutrality on the part of firms is a standard assumption, based on the grounds that they can diversify risk. They might still exhibit ambiguity aversion, empirical evidence of which exists for actuaries, see Cabantous (2007,[16]. Moreover, if we see a firm as a group of individuals making joint decisions, Brunette et al. (2015,[15]) show that individuals are less risk averse but more ambiguity averse in a group than alone.

[25]When ϕ displays CAAA with $\phi(x) = \frac{e^{-\alpha x}}{-\alpha}$, Klibanoff et al. (2005,[44]) show that, under some conditions, when the ambiguity aversion coefficient α tends to infinity, the KMM model approaches the MEU criterion.

where π_i is the date-i profits and for all $\theta \in \Theta$ and $a_1 \geq 0$, $\mathcal{V}(a_1;\theta) = \mathbb{E}_{G,L}\{\tilde{V}(a_1;\theta)|\theta\}$ is the θ-scenario date-2 expected profit with $\tilde{V}(a_1;\theta) = \max_{a_2} \pi_2(a_1, a_2; \tilde{\tau}_\theta, \tilde{b}_\theta)$. The second term in (2) thus corresponds to the discounted date-2 ϕ-certainty equivalent expected profit with $0 \leq \beta \leq 1$ the discount factor. $\mathbb{E}_F$ denotes the expectation parameter taken w.r.t. F conditional on all relevant information available to liable firms at date 1, and $\mathbb{E}_{G,L}\{\cdot|\theta\}$ denotes the expectation parameter taken w.r.t. $G(\cdot;\theta)$ and $L(\cdot;\theta)$ conditional on the true scenario being θ. Our partial-equilibrium model solely focuses on firms' abatements and ignores their output decisions[26]. That is, firms' net profits on the goods' markets are independent of the net emissions volume, and are denoted $\zeta_i > 0$ at date i. For any given couple (τ, b), date-1 and date-2 profits write, for all $a_1 \geq 0$,

$$\pi_1(a_1) = \zeta_1 - C_1(a_1) \text{ and } \pi_2(a_1, a_2; \tau, b) = \zeta_2 - C_2(a_1, a_2) - \tau(b - a_1 - a_2 - \omega), \quad (3)$$

where the last term in π_2 is the firm trading operations on the market for permits or compliance costs in a tax regime. Finally, let the ambiguity function ϕ be three times differentiable, increasing and concave, so that under ambiguity aversion the Jensen's inequality gives

$$\phi^{-1}(\mathbb{E}_F\{\phi(\mathcal{V}(a_1;\tilde{\theta})\}) \leq \mathbb{E}_F\{\mathcal{V}(a_1;\tilde{\theta})\}, \quad (4)$$

where the right-hand side corresponds to the ambiguity-neutral case (ϕ is linear), which, by reduction of compound lotteries, also corresponds to a Savagian expected date-2 profit taken w.r.t. $\bar{G} \equiv \mathbb{E}_F\{G(\cdot;\tilde{\theta})\}$ and $\bar{L} \equiv \mathbb{E}_F\{L(\cdot;\tilde{\theta})\}$. We further assume that there is no bias in ambiguity-neutral firms' beliefs, i.e., $\bar{G} \equiv G^0$ and $\bar{L} \equiv L^0$. This is based on the grounds that an ambiguity-neutral decision-maker should not be affected by the introduction of, or a shift in, ambiguity. Note finally that program (2) is well-defined provided that ambiguity tolerance $-\phi'/\phi''$ is concave; see e.g. Gierlinger & Gollier (2015,[33],Lemma 2). This condition is satisfied for standard ϕ functions, as those we use in section 4.2, and we therefore assume that this holds throughout the paper.

[26] This is a restrictive but usual assumption, see e.g. Zhao (2003,[84]) and Baldursson & von der Fehr (2004,[3]). It can be justified if firms produce different goods and/or belong to different industrial sectors. If so, this amounts to assuming that profits are always positive in the relevant range. While an interaction between the goods' market and environmental policy undoubtedly exists, there is uncertainty on its intensity. For instance, Martin et al. (2014,[52]) show that the UK carbon tax has reduced both energy use and intensity, but find no evidence of impacts on employment or production. Regarding the labour market, see the double dividend debate in Bovenberg & Goulder (1996,[13]). For an explicit treatment of the interaction of allowance trading with the output market, see Requate (1998,[62]) or Baldursson & von der Fehr (2012,[4]).

3 Optimal abatement decisions under ambiguity

3.1 Tax regime under ambiguity

Let us first consider that $G_\theta \equiv 0$. This corresponds to a situation where the regulator implements a proportional tax $t = \mathbb{E}_{G^0}\{\tilde{\tau}\}$ on emissions[27]. The s index is dropped as we analyse optimal date-1 abatement decisions for one representative ambiguity averse firm relative to ambiguity neutrality. The resolution proceeds in two steps, using backward induction. At date 2, for a given level of date-1 abatement $a_1 \geq 0$ and after having observed its baseline emission level b, the liable firm maximises its date-2 profit w.r.t. a_2, that is

$$\max_{a_2 \geq 0} \pi_2(a_1, a_2; t, b) = \zeta_2 - C_2(a_1, a_2) - t(b - a_1 - a_2 - \omega), \tag{5}$$

with the optimality condition $\partial_{a_2} C_2(a_1, a_2^*) = t$, so that cost-efficiency always obtains at date 2, where the optimal date-2 abatement is implicitly defined such that $a_2^* \equiv a_2^*(a_1; t)$. Note that this holds true irrespective of the firm's attitude towards ambiguity. Moving backward to date-1 abatement decisions, however, we need to distinguish between ambiguity neutrality and aversion. Indeed, the optimal date-1 abatement varies with the ambiguity level, as seen from date 1, in conjunction with the degree of ambiguity aversion. More precisely, at date 1, the firm's program is given by (2) where for all $\theta \in \Theta$ and $a_1 \geq 0$,

$$\mathcal{V}(a_1; \theta) = \zeta_2 - C_2\left(a_1, a_2^*(a_1; t)\right) - t(\bar{b}_\theta - a_1 - a_2^*(a_1; t) - \omega), \tag{6}$$

with $\bar{b}_\theta = \mathbb{E}_L\{\tilde{b}_\theta | \theta\}$. Using date-2 optimality, the θ-scenario expected marginal profitability from date-1 abatement satisfies $\mathcal{V}_{a_1}(a_1; \theta) = t - \partial_{a_1} C_2(a_1, a_2^*(a_1; t)) > 0$. Since both t and $\partial_{a_1} C_2$ are deterministic, $\mathcal{V}_{a_1}$ is deterministic and does not depend on the θ-scenario considered.

Ambiguity neutrality. With $\mathcal{V}_{a_1}$ deterministic and ϕ linear, the necessary first-order condition of program (2) defines the optimal date-1 abatement under ambiguity neutrality, $\bar{a}_1^t$, by $-C_1'(\bar{a}_1^t) + \beta \mathcal{V}_{a_1}(\bar{a}_1^t) = 0$. Combining optimality conditions at both dates then yields

$$C_1'(\bar{a}_1^t) + \beta \partial_{a_1} C_2(\bar{a}_1^t, a_2^*(\bar{a}_1^t, t)) = \beta \partial_{a_2} C_2(\bar{a}_1^t, a_2^*(\bar{a}_1^t; t)), \tag{7}$$

so that intertemporal cost-efficiency obtains in that the overall marginal date-1 abatement cost is equated to the marginal date-2 abatement cost. Note that with the quadratic abate-

[27] The tax rate t is exogenously given and certain. Whether it is optimal or not is of no relevance here.

ment cost specification (1), it comes that

$$\bar{a}_1^t = \frac{c_2 - \gamma}{c_1 c_2 - \beta\gamma^2}\beta t \geq 0 \quad \text{and} \quad a_2^*(\bar{a}_1^t; t) = \frac{t - \gamma \bar{a}_1^t}{c_2} = \frac{c_1 - \beta\gamma}{c_1 c_2 - \beta\gamma^2} t \geq 0. \tag{8}$$

With no long-term dependency, i.e. $\gamma = 0$, a_2^* is independent of a_1 and the overall abatement under ambiguity neutrality is such that $\bar{a}_1^t + a_2^*(t) = \frac{\beta t}{c}$, where $\frac{1}{c} = \frac{1}{c_1} + \frac{1}{\beta c_2}$ is the firm's aggregate flexibility in abatement over the two dates. The overall abatement volume is optimally apportioned between the two dates, that is, in proportion to each date flexibility in abatement: $\bar{a}_1^t = \frac{c}{c_1}\left(\frac{\beta t}{c}\right)$ and $a_2^*(t) = \frac{c}{\beta c_2}\left(\frac{\beta t}{c}\right)$. In particular, the ambiguity neutrality benchmark corresponds to a decision under risk − in our case for a risk neutral firm. Baldursson & von der Fehr (2004,[3]) show that intertemporal cost-efficiency continues to hold under risk aversion. As exposed below, however, this does not carry over to ambiguity aversion.

Ambiguity Aversion. With ϕ concave, the necessary first-order condition of program (2) defines the optimal date-1 abatement under ambiguity neutrality, $\hat{a}_1^t$, by

$$-C_1'(\hat{a}_1^t) + \beta \mathbb{E}_F \left\{ \frac{\phi'(\mathcal{V}(\hat{a}_1^t; \tilde{\theta}))\mathcal{V}_{a_1}(\hat{a}_1^t)}{\phi' \circ \phi^{-1}(\mathbb{E}_F\{\phi(\mathcal{V}(\hat{a}_1^t; \tilde{\theta}))\})} \right\} = 0, \tag{9}$$

which rewrites $-C_1'(\hat{a}_1^t) + \beta \mathcal{A}(\hat{a}_1^t)\mathcal{V}_{a_1}(\hat{a}_1^t) = 0$, where $\mathcal{A}$ is a function satisfying

$$\mathcal{A}(a_1) = \frac{\mathbb{E}_F\{\phi'(\mathcal{V}(a_1; \tilde{\theta}))\}}{\phi' \circ \phi^{-1}(\mathbb{E}_F\{\phi(\mathcal{V}(a_1; \tilde{\theta}))\})}. \tag{10}$$

Proposition 3.1 characterises the impact of ambiguity aversion on optimal date-1 abatement decisions as compared with ambiguity neutrality.

Proposition 3.1. *Ambiguity aversion is conducive to higher (lower) date-1 abatement than under ambiguity neutrality if, and only if, the liable firm displays Decreasing (Increasing) Absolute Ambiguity Aversion. Moreover, under Constant Absolute Ambiguity Aversion, the introduction of ambiguity aversion has no effect on date-1 abatement decision.*

Proof. Let ϕ be thrice differentiable. We first prove the following claim: ϕ is DAAA (IAAA) is equivalent to $\mathbb{E}\phi'(\cdot) \geq (\leq) \phi' \circ \phi^{-1}(\mathbb{E}\phi(\cdot))$. An agent is said to display Decreasing Absolute Ambiguity Aversion (DAAA) i.f.f. its Arrow-Pratt coefficient of absolute ambiguity aversion $-\frac{\phi''}{\phi'}$ is a non-increasing function. This is true i.f.f. $-\phi'''\phi' + \phi''^2 \leq 0$ and, upon rearranging, i.f.f. $\frac{-\phi'''}{\phi''} \geq \frac{-\phi''}{\phi'}$. This is equivalent to $-\phi'$ being more concave than ϕ, i.e., abolute prudence w.r.t. ambiguity exceeds absolute ambiguity aversion. In terms of certainty equivalent, this translates into $\phi^{-1}(\mathbb{E}\phi(\cdot)) \geq (-\phi')^{-1}(-\mathbb{E}\phi'(\cdot))$. Applying $-\phi'$ on both sides proves the claim. By concavity of the objective function, $\hat{a}_1^t \geq \bar{a}_1^t$ i.f.f. $-C_1'(\bar{a}_1^t) + \beta\mathcal{A}(\bar{a}_1^t)\mathcal{V}_{a_1}(\bar{a}_1^t) \geq 0$, which is equivalent to $\mathcal{A}(\bar{a}_1^t) \geq 1$. The proof then follows from the claim that $\mathcal{A}$ is bigger, lower or equal to 1 i.f.f. ϕ is DAAA, IAAA or CAAA, respectively. □

First, a tax regime is generally not conducive to intertemporal cost-efficiency under ambiguity aversion, except when the firm displays CAAA. This suggests that the relative merits of an emissions tax *vs* emissions trading as highlighted by Baldursson & von der Fehr (2004,[3]) under risk aversion would tend to fade away under ambiguity aversion. Second, Proposition 3.1 is in line with the literature on the formation of precautionary saving under ambiguity aversion, e.g. Osaki & Schlesinger (2014,[58]) and Gierlinger & Gollier (2015,[33]). Because the firm abates relatively more at date 1 than under ambiguity neutrality for sure when it displays DAAA, we follow Gierlinger & Gollier (2015,[33]) and Berger (2014,[7]) in identifying ambiguity prudence with DAAA[28]. Accordingly, we call $\mathcal{A}$ the ambiguity prudence coefficient, whose value is above one in case of ambiguity prudence. With this definition,

Corollary 3.2. *The firm forms precautionary date-1 abatement if, and only if, it displays prudence towards ambiguity.*

Under ambiguity aversion, the effective discount factor becomes $\beta\mathcal{A}$ so that the firm exhibiting DAAA (IAAA) applies a higher (lower) discount factor than under ambiguity neutrality. In this light, ambiguity prudence puts relatively more weight on date-2 profits than under ambiguity neutrality – lowering impatience, as it were – thereby leading to higher date-1 abatement levels, since this reduces date-1 profits to the benefits of date-2 profits, all else equal. A similar interpretation is that the DAAA property worsens the importance of any date-2 profit risk so that it can be assimilated to a «preference for an earlier resolution of uncertainty», as described by Strzalecki (2013,[73],Theorem 4).

[28] This definition, however, is presently not unique. Baillon (2016,[2]) defines ambiguity prudence with the less demanding condition that ϕ''' be positive (DAAA $\Rightarrow \phi''' > 0$), arguing that his definition is model-independent and follows directly from that for risk prudence. With the KMM certainty equivalent representation we use, however, $\phi''' > 0$ is not sufficient to guarantee the formation of precautionary banking, and only under DAAA is the ambiguity precautionary premium bigger than the ambiguity premium, see e.g. Osaki & Schlesinger (2014,[58]). Therefore, the DAAA property is the most natural definition for our analysis. Berger (2016,[8]) underlines the similarity between the KMM and Kreps-Porteus/Selden recursive formulations to pin down his definition of ambiguity prudence: just like the DARA property is required for precautionary saving under risk aversion in the K-P/S models, is DAAA required for precautionary saving under ambiguity aversion in the KMM framework. Finally note that Gierlinger & Gollier (2015,[33]) argue that DAAA should be a standard assumption, in parallel with the widely accepted DARA property in risk and that there is empirical evidence for DAAA, see e.g. Berger & Bosetti (2016,[9]).

3.2 Cap-and-trade regime under pure price ambiguity

Now let $L_\theta \equiv 0$ and again consider the abatement decisions of one representative liable firm, hence dropping the s index. This corresponds to a cap-and-trade regime under pure price ambiguity − it is extrinsic to firms and transmitted via the allowance price only. Without loss of generality, further let the firm's date-2 baseline b be certain. Again, the firm solves program (2). In particular, at date 2, for any given baseline level b, allocated volume of quotas ω and date-1 abatement a_1, the firm chooses its optimal abatement level a_2^* such that, $\partial_{a_2} C_2(a_1, a_2^*) = \tau$, the observed allowance price. The optimal date-2 abatement is thus implicitly defined by $a_2^* \equiv a_2^*(a_1;\tau)$. By virtue of the Envelop Theorem applied to $\tilde{V}$, one has that $\tilde{V}_{a_1}(a_1;\theta) = \tilde{\tau}_\theta - \partial_{a_1} C_2(a_1, a_2^*(a_1;\tilde{\tau}_\theta))$, $\forall a_1 \geq 0$, $\forall \theta \in \Theta$.

Ambiguity neutrality. With ϕ linear, the necessary first-order condition of program (2) defines the optimal date-1 abatement level under ambiguity neutrality, $\bar{a}_1$, by

$$- C_1'(\bar{a}_1) + \beta \mathbb{E}_F\{\mathcal{V}_{a_1}(\bar{a}_1;\tilde{\theta})\} = 0, \tag{11}$$

so that, combined with date-2 optimality, one has that

$$C_1'(\bar{a}_1) + \beta \mathbb{E}_{\bar{G}} \left\{\partial_{a_1} C_2(\bar{a}_1, a_2^*(\bar{a}_1;\tilde{\tau}))\right\} = \beta \mathbb{E}_{\bar{G}} \left\{\tilde{\tau}\right\} = \beta \mathbb{E}_{\bar{G}} \left\{\partial_{a_2} C_2(\bar{a}_1, a_2^*(\bar{a}_1;\tilde{\tau}))\right\}. \tag{12}$$

Therefore, under ambiguity neutrality, intertemporal cost-efficiency obtains in expectations and $\bar{a}_1$ is independent of the initial allocation ω. Let us now state

Proposition 3.3. *Let there be no long-term effect of abatement, $\partial_{a_1} C_2 \equiv 0$. Then, in the face of an increase in uncertainty in the sense of a mean-preserving spread, the ambiguity neutral firm abates relatively more at date 1 if, and only if, $C_2''' > 0$.*

Proof. For a probability measure G^i, define the function O^i by

$$0 = -C_1'(\bar{a}_1^i) + \beta \mathbb{E}_{G^i} C_2'(a_2^*(\tilde{\tau})) \equiv O^i(\bar{a}_1^i),$$

where $\bar{a}_1^i$ is the date-1 optimal abatement when the price risk is distributed according to G^i and a_2^* does not depend on a_1 since we assume time separability. Let the measure G^j be a mean-preserving spread of G^i in the sense of Rothschild & Stiglitz (1971,[63]). Concavity of the objective function then yields

$$\bar{a}_1^j \geq \bar{a}_1^i \Leftrightarrow O^j\left(\bar{a}_1^i\right) \geq O^i\left(\bar{a}_1^i\right) = 0 \Leftrightarrow \mathbb{E}_{G^j} C_2'(a_2^*(\tilde{\tau})) \geq \mathbb{E}_{G^i} C_2'(a_2^*(\tilde{\tau})).$$

Using the Jensen's inequality, this holds true i.f.f. C_2' is convex. $\square$

Under ambiguity neutrality, the formation of precautionary date-1 abatement is hence conditional on the positivity of the third derivative of the abatement cost function − a condition reminiscent of the definition of risk prudence in the sense of Kimball (1990,[43]). Note that Chevallier et al. (2011,[19]) obtain a similar result with a risk on the future allocation of permits. In addition of providing analytical results, considering quadratic abatement cost functions ($C''' \simeq 0$) therefore guarantees that no precautionary date-1 abatement is formed under ambiguity neutrality, so that our model captures the sole effects of the introduction of ambiguity aversion. In particular, with the abatement cost specification (1), it comes that for all $a_1 \geq 0$ and θ in Θ,

$$a_2^*(a_1;\tilde{\tau}_\theta) = \frac{\tilde{\tau}_\theta - \gamma a_1}{c_2},\ \mathcal{V}_{a_1}(a_1;\theta) = \frac{c_2-\gamma}{c_2}\bar{\tau}_\theta + \frac{\gamma^2 a_1}{c_2},\ \text{and}\ \bar{a}_1 = \frac{c_2-\gamma}{c_1c_2-\beta\gamma^2}\beta\left\langle\tilde{\tau}\right\rangle, \tag{13}$$

where $\bar{\tau}_\theta = \mathbb{E}_G\left\{\tilde{\tau}_\theta|\theta\right\}$ is the θ-scenario average price and $\langle\tilde{\tau}\rangle = \mathbb{E}_{\bar{G}}\left\{\tilde{\tau}\right\}$ is the expected price under ambiguity neutrality. Note, in particular, that $\bar{a}_1$ is invariant to any MPS in $\tilde{\tau}$.

Ambiguity aversion. When ϕ is concave, the necessary first-order condition of program (2) defines the optimal date-1 abatement under ambiguity aversion, $\hat{a}_1$, by

$$-C_1'(\hat{a}_1) + \beta\frac{\mathbb{E}_F\{\phi'(\mathcal{V}(\hat{a}_1;\tilde{\theta}))\mathcal{V}_{a_1}(\hat{a}_1;\tilde{\theta})\}}{\phi'\circ\phi^{-1}(\mathbb{E}_F\{\phi(\mathcal{V}(\hat{a}_1;\tilde{\theta}))\})} = 0. \tag{14}$$

Normalising and decomposing (14) into two terms, yields

$$-C_1'(\hat{a}_1) + \beta\mathbb{E}_F\{\mathcal{D}(\hat{a}_1;\tilde{\theta})\mathcal{V}_{a_1}(\hat{a}_1;\tilde{\theta})\} = 0, \tag{15}$$

where the ambiguity prudence coefficient $\mathcal{A}$ is defined in (10) and $\mathcal{D}$ is a distortion function satisfying, for all $\theta \in \Theta$,

$$\mathcal{D}(a_1;\theta) = \frac{\phi'\left(\mathcal{V}(\bar{a}_1;\theta)\right)}{\mathbb{E}_F\{\phi'(\mathcal{V}(\bar{a}_1;\tilde{\theta}))\}}. \tag{16}$$

In addition to the $\mathcal{A}$-effect and relative to ambiguity neutrality, ambiguity aversion induces a second effect via $\mathcal{D}$, which distorts the second-order subjective prior F. By concavity of ϕ, the distortion function $\mathcal{D}$ overweights those scenarios inducing low-$\mathcal{V}$ values, which can be interpreted as a pessimism effect[29]. In particular, the pessimistically distorted second-order

[29] The distortion function is a Radon-Nikodym derivative and is akin to the martingale distortion occurring in robust control theory − see Hansen & Sargent (2001,[39]).

subjective measure, H, is given by

$$\forall \theta \in \Theta,\ H(\theta) = \int_{\underline{\theta}}^{\theta} \mathcal{D}(\bar{a}_1; X)\mathrm{d}F(X) = \frac{\mathbb{E}_F\{\phi'(\mathcal{V}(\bar{a}_1; \tilde{X}))|\tilde{X} \leq \theta\}}{\mathbb{E}_F\{\phi'(\mathcal{V}(\bar{a}_1; \tilde{\theta}))\}} F(\theta), \tag{17}$$

so that $H(\underline{\theta}) = 0$, $H(\bar{\theta}) = 1$ and $H' > 0$ on Θ. Therefore, by concavity of program (2), ambiguity aversion raises date-1 abatement relative to ambiguity neutrality i.f.f.

$$\mathcal{A}(\bar{a}_1)\mathbb{E}_H\{\mathcal{V}_{a_1}(\bar{a}_1; \tilde{\theta})\} \geq \mathbb{E}_F\{\mathcal{V}_{a_1}(\bar{a}_1; \tilde{\theta})\}, \tag{18}$$

that is, i.f.f. the future allowance price estimate under ambiguity aversion (LHS) is higher than under ambiguity neutrality (RHS). Controlling for the $\mathcal{A}$-effect, introducing ambiguity in the ambiguity-averse firm's decision is identical to a shift in the ambiguity-neutral firm's subjective beliefs from F to H. Due to ambiguity aversion, H (F) places relatively more weight on low-profit (high-profit) scenarios than F (H) does. Intuitively from (18), for pessimism to be conducive to over-abatement, those low-profit scenarios must coincide with high marginal profit ones, which is the subject of Proposition 3.4.

Proposition 3.4. *Under CAAA, pessimism raises date-1 abatement relative to ambiguity neutrality if, and only if, $(\mathcal{V}(\bar{a}_1; \theta))_\theta$ and $(\mathcal{V}_{a_1}(\bar{a}_1; \theta))_\theta$ are anticomonotone. In general, however, the two effects of ambiguity aversion can be aligned or countervailing: When ϕ displays DAAA (IAAA), ambiguity aversion is conducive to higher (lower) date-1 abatement than under ambiguity neutrality only if anticomonotonicity (comonotonicity) holds.*

Proof. By concavity of the objective function, $\hat{a}_1 \geq \bar{a}_1$ is equivalent to

$$\mathbb{E}_F\{\phi'(\mathcal{V}(\bar{a}_1; \tilde{\theta}))\mathcal{V}_{a_1}(\bar{a}_1; \tilde{\theta})\} \geq \phi' \circ \phi^{-1}(\mathbb{E}_F\{\phi(\mathcal{V}(\bar{a}_1; \tilde{\theta}))\})\mathbb{E}_F\{\mathcal{V}_{a_1}(\bar{a}_1; \tilde{\theta})\}.$$

With ϕ DAAA, note that a sufficient condition for the above to hold is

$$\mathbb{E}_F\{\phi'(\mathcal{V}(\bar{a}_1; \tilde{\theta}))\mathcal{V}_{a_1}(\bar{a}_1; \tilde{\theta})\} \geq \mathbb{E}_F\{\phi'(\mathcal{V}(\bar{a}_1; \tilde{\theta}))\}\mathbb{E}_F\{\mathcal{V}_{a_1}(\bar{a}_1; \tilde{\theta})\},$$

which is exactly $\mathrm{Cov}_F\{\phi'(\mathcal{V}(\bar{a}_1; \tilde{\theta})); \mathcal{V}_{a_1}(\bar{a}_1; \tilde{\theta})\} \geq 0$. Noting that ϕ' is non-increasing concludes. The above argument reverses when ϕ is IAAA. □

Absent the $\mathcal{A}$-effect, only when low-$\mathcal{V}$ scenarios coincide with high-$\mathcal{V}_{a_1}$ scenarios − a condition that holds under anticomonotonicity − does pessimism increase the H-weighted expected marginal benefit from abating at date-1 as compared to that under ambiguity neutrality, hence leading to over-abatement at date 1. The underlying intuition for anti-comonotonicity and pessimism is illustrated in Examples 3.5 and 3.6.

Example 3.5. *Let* $\Theta = \{\theta_1, \theta_2\}$ *and* $\mathcal{V}(a_1; \theta_i)$ *be increasing in* i*,* ϕ *be* CAAA *so that* $\mathcal{A} \equiv 1$*, and* $F = (q, \theta_1; 1-q, \theta_2)$ *with* $0 \leq q \leq 1$*. Pessimism overweights scenario* θ_1 *relative to* θ_2 *so that* $H = (\hat{q}, \theta_1; 1-\hat{q}, \theta_2)$ *with* $q \leq \hat{q} \leq 1$[30]*. Then, under ambiguity neutrality, date-1 abatement with prior* H*,* $\bar{a}_{1,H}$*, is higher than with* F*,* $\bar{a}_{1,F}$*, i.f.f.*

$$\hat{q}\mathcal{V}_{a_1}(\bar{a}_{1,F}; \theta_1) + (1-\hat{q})\mathcal{V}_{a_1}(\bar{a}_{1,F}, \theta_2) \geq q\mathcal{V}_{a_1}(\bar{a}_{1,F}; \theta_1) + (1-q)\mathcal{V}_{a_1}(\bar{a}_{1,F}, \theta_2),$$

which is true i.f.f. anticomonotonicity holds, since $\mathcal{V}_{a_1}(a_1; \theta_i)$ *would be decreasing in* i*.*

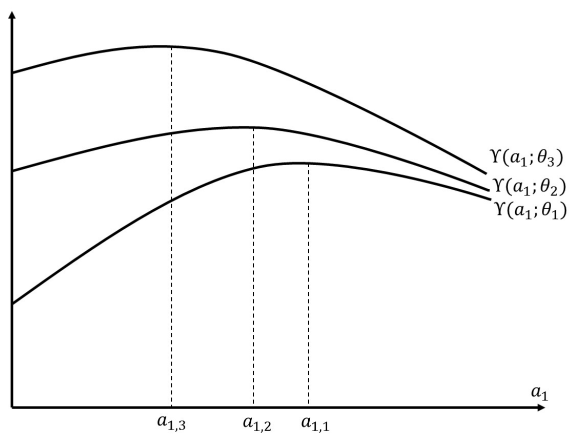


Figure 1: The effect of pessimism under anticomonotonicity.

Example 3.6. *Let* $\Theta = \{\theta_1, \theta_2, \theta_3\}$ *and* $\Upsilon(a_1; \theta_i)$ *denote the net intertemporal expected revenue from date-1 abatement in scenario* θ_i*, i.e.,* $\Upsilon(a_1; \theta_i) = \zeta_1 - C_1(a_1) + \beta\mathcal{V}(a_1; \theta_i)$*. W.l.o.g. let* Θ *be ordered such that* $\Upsilon(a_1; \theta_i)$ *is increasing in* i*. Suppose that anticomonotonicity holds, i.e.,* $\Upsilon_{a_1}(a_1; \theta_i)$ *is decreasing in* i*, as in Figure 1, where* $a_{1,i}$ *denotes the optimal banking level in scenario* θ_i*. Anticomonotonicity implies that* $a_{1,i}$ *is decreasing with* i *and that in moving towards higher banking levels, the spread in* $\Upsilon(a_1; \theta)$ *across* θ*-scenarios is reduced.*

Pessimism acts in line with the definition of ambiguity aversion that an ambiguity averse agent dislikes any MPS in the space of conditional second-order expected profit. Under CAAA with pessimism only, ambiguity aversion unconditionally adjusts date-1 abatement in the direction of reduced spread in Υ across scenarios: upwards if anticomonotonicity holds;

[30] With a MEU model of choice, $\hat{q} = 1$, from which it is clear that MEU is equivalent to KMM in the limiting case of infinite ambiguity aversion.

downwards if comonotonicity holds. Example 3.6 also illustrates that anticomonotonicity is quite demanding a requirement since $\Upsilon(a_1;\theta_i)$-lines cannot cross between scenarios. It is a strong requirement because one needs to sign the covariance for sure. Roughly speaking, however, it might be sufficient that the level of discrepancy in Υ across scenarios diminishes in a_1 for ambiguity aversion to raise date-1 abatement relative to neutrality, so that the anticomonotonicity could be relaxed somehow[31]. Note that the anticomonotonicity criterion is robust in the sense that it obtains under other representation theorems, see e.g. Appendix B for MEU preferences.

Present the $\mathcal{A}$-effect, one must account for the two ambiguity aversion-induced effects together. Assume for clarity that there is no long-term effect of abatement, i.e., $\partial_{a_1}C_2 \equiv 0$. By concavity of the objective function, $\hat{a}_1 \geq \bar{a}_1$ i.f.f.

$$\mathcal{A}(\bar{a}_1)\left(\langle\tilde{\tau}\rangle + \mathcal{P}(\bar{a}_1)\right) \geq \langle\tilde{\tau}\rangle\,, \tag{19}$$

with $\langle\tilde{\tau}\rangle = \mathbb{E}_{\bar{G}}\{\tilde{\tau}\}$, and $\mathcal{P}(\bar{a}_1)$ can be interpreted as a pessimism-only future allowance price distortion, evaluated at $a_1 = \bar{a}_1$, where

$$\mathcal{P}(a_1) = \frac{\mathrm{Cov}_F\{\phi'(\mathcal{V}(a_1;\tilde{\theta}));\mathcal{V}_{a_1}(a_1;\tilde{\theta})\}}{\mathbb{E}_F\{\phi'(\mathcal{V}(a_1;\tilde{\theta}))\}}. \tag{20}$$

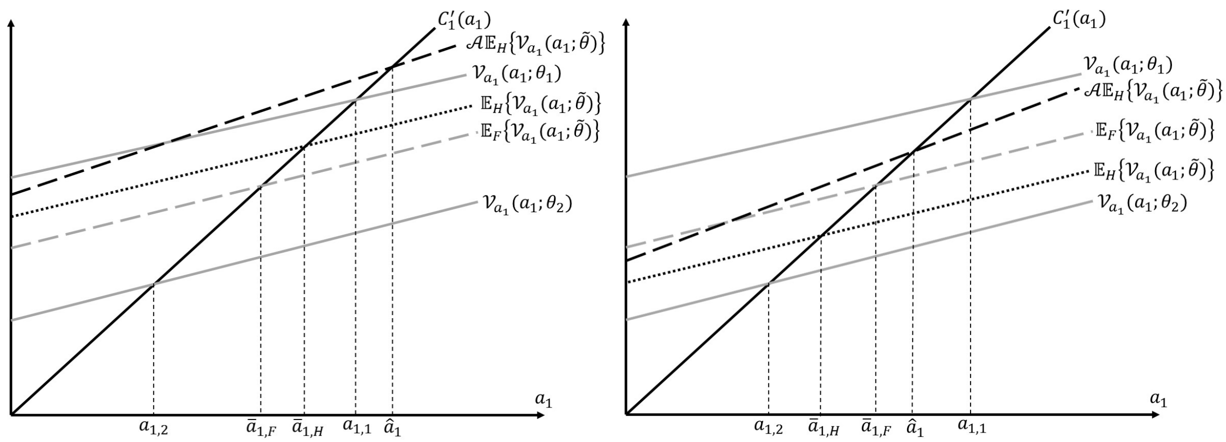
When ϕ is CAAA, i.e., $\mathcal{A}\equiv 1$, and anticomonotonicity holds, $\mathcal{P}$ is positive so that the date-2 pessimistically-distorted allowance price is higher than the ambiguity-neutral one, which leads to over-abatement at date 1 under ambiguity aversion. One must also account for the ambiguity prudence effect under DAAA or IAAA, as illustrated in Proposition 3.7.

Proposition 3.7. *Let $\partial_{a_1}C_2 \equiv 0$. Then, the following equivalence conditions obtain*
(i) When ϕ displays CAAA, *$\hat{a}_1 \geq \bar{a}_1$ if, and only if, $\mathcal{P}(\bar{a}_1) \geq 0$;*
(ii) When ϕ displays DAAA, *$\hat{a}_1 \geq \bar{a}_1$ if, and only if, $\mathcal{P}(\bar{a}_1) \geq \frac{1-\mathcal{A}(\bar{a}_1)}{\mathcal{A}(\bar{a}_1)}\langle\tilde{\tau}\rangle < 0$;*
(iii) When ϕ displays IAAA, *$\hat{a}_1 \leq \bar{a}_1$ if, and only if, $\mathcal{P}(\bar{a}_1) \leq \frac{1-\mathcal{A}(\bar{a}_1)}{\mathcal{A}(\bar{a}_1)}\langle\tilde{\tau}\rangle > 0$.*

Proposition 3.7 indicates the exact relation between the strengths of the pessimism and ambiguity prudence effects in determining the adjustment in date-1 abatement due to ambiguity aversion. In particular, the two effects can be countervailing, e.g. under ambiguity prudence, when $\mathcal{P}(\bar{a}_1) \in \left[\frac{1-\mathcal{A}(\bar{a}_1)}{\mathcal{A}(\bar{a}_1)}\langle\tilde{\tau}\rangle\,;0\right]$, the anticomonotonicity criterion does not hold but

[31] We could not get there analytically but this intuition is illustrated with numerical simulations in section 4.2. Note that Berger et al. (2016,[10]) transform the anticomonotonicity criterion into a convergence effect between scenarios. They are able to do so because they use a binary structure, i.e., good *vs* bad state, and ambiguity bears solely on the chances that these two states occur.

still, $\hat{a}_1 > \bar{a}_1$. That is, the ambiguity averse firm distorts its subjective prior by overemphasising low-$\mathcal{V}_{a_1}$ scenarios, hence implying under-abatement. However, this is more than compensated by the decrease in impatience due to ambiguity prudence, and overall, precautionary date-1 abatement is formed.



(a) Aligned effects. (b) Countervailing effects.

Figure 2: Joint effects of ambiguity prudence and pessimism.

Figure 2 graphically depicts the joint effects of pessimism and ambiguity prudence. Let $\Theta = \{\theta_1, \theta_2\}$, $\mathcal{V}_{a_1}(a_1;\theta_i)$ be decreasing in i and ϕ display DAAA. For clarity, consider that both H and $\mathcal{A}$ are constant − see Appendix C for H and $\mathcal{A}$ that vary with a_1. In particular, Figure 2 separates the pessimism effect ($\bar{a}_1 = \bar{a}_{1,F} \to \bar{a}_{1,H}$) from the ambiguity prudence effect ($\bar{a}_{1,H} \to \hat{a}_1$). Pessimism operates a vertical translation of the F-averaged expected marginal profitability from date-1 abatement within the $\mathcal{V}_{a_1}(a_1;\theta_2)$-$\mathcal{V}_{a_1}(a_1;\theta_1)$ band, directed towards the lower $\mathcal{V}$-value scenario[32] and ambiguity prudence then increases the slope of the H-averaged expected marginal profitability from date-1 abatement. In Figure 2a, anticomonotonicity holds so that H overweights θ_1 relative to θ_2 as compared to F, and the two effects are aligned. In Figure 2b, comonotonicity holds so that H overweights θ_2 relative to θ_1 as compared to F, and the two effects are countervailing − in this case, in terms of adjustment magnitude, ambiguity prudence dominates pessimism since overall, $\hat{a}_1 > \bar{a}_1$.

While Propositions 3.4 and 3.7 are intuitively appealing, the anticomonotonicity criterion is by no means informative in practice. Proposition 3.8 thus gives more tangible conditions under which this criterion holds. Abatement cost functions are equipped with the quadratic form (1), and we determine how anticomotonicity translates under this specification.

[32]With MEU preferences, the firm only considers the $\mathcal{V}_{a_1}(\bar{a}_1;\theta_2)$ line and ambiguity prudence is absent, for it is specific to the KMM criterion.

Proposition 3.8. *Ambiguity-prudent firms over-abate at date 1 only if*
(i) they expect to be in a net short position at date 2 under the abatement stream $(\bar{a}_1; a_2^*(\bar{a}_1; \tau_\theta^*))$ *in all* θ*-scenarios with* $\tau_\theta^* = (\mathbb{E}_{G_\theta} 1)^{-1} \mathbb{E}_{G_\theta} X$*;*
(ii) for a given allowance allocation ω*, they abate too little at date 1 under ambiguity neutrality* $\bar{a}_1 \leq \min_{\theta \in \Theta} a_{1,\theta} = b - \omega - a_2^*(\bar{a}_1; \tau_\theta^*)$*, or reciprocally,*
(iii) their allocation is relatively small $\omega \leq \omega^* \equiv \min_{\theta \in \Theta} \omega_\theta^* = b - \bar{a}_1 - a_2^*(\bar{a}_1; \tau_\theta^*)$*.*

Proof. The proof consists in signing the covariance. For all $\theta \in \Theta$, $\mathcal{V}_{a_1}(\bar{a}_1; \theta)$, $\bar{a}_1$, and a_2^* are given in (13). Differentiating $\mathcal{V}_{a_1}(\bar{a}_1; \theta)$ w.r.t. θ and then integrating by parts yields

$$\partial_\theta \mathcal{V}_{a_1}(\bar{a}_1; \theta) = \frac{c_2 - \gamma}{c_2} \int_{\mathrm{T}} x \partial_\theta g(x; \theta) \mathrm{d}x = \frac{\gamma - c_2}{c_2} \int_{\mathrm{T}} G_\theta(x; \theta) \mathrm{d}x,$$

where $G_\theta(\cdot; \theta) = \partial_\theta G(\cdot; \theta)$. Similarly, by the Envelop Theorem and differentiation w.r.t. θ,

$$\begin{aligned}
\partial_\theta \mathcal{V}(\bar{a}_1; \theta) &= -\int_{\mathrm{T}} C_2(\bar{a}_1, a_2^*(\bar{a}_1; x)) + x\left(b - \bar{a}_1 - a_2^*(\bar{a}_1; x) - \omega\right) \partial_\theta g(x; \theta) \mathrm{d}x \\
&= -\int_{\mathrm{T}} x\Big(b - \omega - \Big(1 - \frac{\gamma}{c_2}\Big)\bar{a}_1 - \frac{x}{2c_2}\Big) - \frac{\gamma^2 \bar{a}_1^2}{2c_2} \partial_\theta g(x; \theta) \mathrm{d}x \\
&= \int_{\mathrm{T}} \Big(b - \omega - \Big(1 - \frac{\gamma}{c_2}\Big)\bar{a}_1 - \frac{x}{c_2}\Big) G_\theta(x; \theta) \mathrm{d}x,
\end{aligned}$$

where the third equality obtains by integration by parts. For all $x \in \mathrm{T}$, let $k : x \mapsto b - \omega - \Big(1 - \frac{\gamma}{c_2}\Big)\bar{a}_1 - \frac{x}{c_2}$. In all generality, k changes sign over T. By continuity of k, let $\tau_0 \in \mathrm{T}$ be such that $k(\tau_0) = 0$, i.e., $\tau_0 = c_2(b - \omega) - (c_2 - \gamma)\bar{a}_1$[33]. For all $\theta \in \Theta$, let

$$\Gamma_\theta(\tau_0) = \frac{1}{c_2} \int_{\mathrm{T}} (\tau_0 - x) G_\theta(x; \theta) \mathrm{d}x,$$

so that, differentiating w.r.t. τ_0 yields $\Gamma'_\theta(\tau_0) = \frac{1}{c_2} \int_{\mathrm{T}} G_\theta(x; \theta) \mathrm{d}x$.
When $G \nearrow^\theta$, $\Gamma'_\theta > 0$ so that by definition, $\Gamma_\theta(\underline{\tau}) < 0$ and $\Gamma_\theta(\bar{\tau}) > 0$. Symmetrically, when $G \searrow_\theta$, $\Gamma'_\theta < 0$ so that by definition, $\Gamma_\theta(\underline{\tau}) > 0$ and $\Gamma_\theta(\bar{\tau}) < 0$. In both cases, by continuity of Γ_θ, $\forall \theta \in \Theta$, there exists $(\tau_\theta^*; a_{1,\theta})$ defined by $\tau_\theta^* = c_2(b - \omega) - (c_2 - \gamma)a_{1,\theta}$, such that $\Gamma_\theta(\tau_\theta^*) = 0$. By definition,

$$\int_{\mathrm{T}} (\tau_\theta^* - x) G_\theta(x; \theta) \mathrm{d}x = 0 \;\Rightarrow\; a_{1,\theta} = \frac{c_2}{c_2 - \gamma}\Big(b - \omega - \frac{1}{c_2} \frac{\int_{\mathrm{T}} x G_\theta(x; \theta) \mathrm{d}x}{\int_{\mathrm{T}} G_\theta(x; \theta) \mathrm{d}x}\Big).$$

In each θ-scenario, for a given ω, $a_{1,\theta}$ hence corresponds to the required date-1 abatement effort when the allowance price prevailing at date 2 is $\tau_\theta^* = \frac{\mathbb{E}_{G_\theta} X}{\mathbb{E}_{G_\theta} \mathbf{1}}$, i.e. when date-2 abatement

[33] This requires that $\underline{\tau} < c_2\big(b - \omega - \frac{c_2 - \gamma}{c_1 c_2 - \beta \gamma^2} \beta \langle \tilde{\tau} \rangle\big) < \bar{\tau}$, which we assume is the case.

is $a_2^*(\bar{a}_1;\tau_\theta^*)$. Two cases then arise depending on the monotonicity of G w.r.t. θ:
1. $G\nearrow^\theta$: $\forall\theta\in\Theta$, $\partial_\theta\mathcal{V}_{a_1}(\bar{a}_1;\theta)<0$ and $\partial_\theta\mathcal{V}(\bar{a}_1;\theta)>0$ i.f.f. $\frac{c_2}{2}\Big(b-\omega-\Big(1-\frac{\gamma}{c_2}\Big)\bar{a}_1\Big)>\tau_\theta^*$, i.e., i.f.f., $\bar{a}_1<a_{1,\theta}$;
2. $G\searrow_\theta$: $\forall\theta\in\Theta$, $\partial_\theta\mathcal{V}_{a_1}(\bar{a}_1;\theta)>0$ and $\partial_\theta\mathcal{V}(\bar{a}_1;\theta)<0$ i.f.f. $\frac{c_2}{2}\Big(b-\omega-\Big(1-\frac{\gamma}{c_2}\Big)\bar{a}_1\Big)>\tau_\theta^*$, i.e., i.f.f., $\bar{a}_1<a_{1,\theta}$.
In both cases, $\hat{a}_1>\bar{a}_1$ i.f.f. $\bar{a}_1<a_{1,\theta}$ $\forall\theta\in\Theta$, i.e., i.f.f. $\bar{a}_1<\min_{\theta\in\Theta}a_{1,\theta}$, which proves (ii), and $(i-iii)$ follow straightforwardly. Note finally that when ϕ display IAAA, $\hat{a}_1<\bar{a}_1$ i.f.f. $\bar{a}_1>\max_{\theta\in\Theta}a_{1,\theta}$. □

Proposition 3.8 shows that firms may over-abate or under-abate at date 1 and that the sign of the pessimism effect ultimately relates to initial allocation, which is thus non-neutral under ambiguity aversion. In particular, over-abatement occurs only in unfavourable situations where firms expect to be net buyers of allowances under the abatement stream $(\bar{a}_1;a_2^*(\bar{a}_1;\tau_\theta^*))$, for all θ-scenarios. In these situations, the marginal benefits of date-1 abatement (a lowering of both the likelihood of effectively being net short and the volume of allowance purchases) outweigh the marginal cost of date-1 abatement for sure. Otherwise, as soon firms are net long when abating $(\bar{a}_1;a_2^*(\bar{a}_1;\tau_\theta^*))$ in at least one θ-scenario, one cannot conclude with certainty on the DAAA-related effects on date-1 abatement[34]. Even in the extreme opposite favourable situations where firms are net long in all θ-scenarios can one not sign the DAAA-related effects, because pessimism and ambiguity prudence are countervailing[35].
Anticomonotonicity translates into a threshold criterion on initial conditions $(\bar{a}_1)$, or, alternatively, on allocation. Berger (2016,[8]) also obtains threshold conditions in translating anticomonotonicity in the case of optimal self-insurance and self-protection decisions under ambiguity aversion, in the specific case where ambiguity is concentrated on on state[36]. In terms of initial conditions, ambiguity prudence is line with a one-sided precautionary principle[37]: Only when date-1 abatement under neutrality is lower than a given threshold $(\min_{\theta\in\Theta}a_{1,\theta})$ will ambiguity aversion adjust date-1 abatement upwards. Since $\bar{a}_1$ is independent of the future abatement effort and only driven by the $\bar{G}$-expected allowance price, ambiguity aversion accounts for firms' expected future positions on the allowance market

[34]Again, this suggests that anticomonotonicity might actually be too strong a criterion to sign pessimism.

[35]It is noteworthy that this asymmetric effect of ambiguity prudence is in line with a loss aversion rationale (if the firm increased date-1 abatement, it could increase profits on the allowance market) or an endowment effect (it could reduce costly date-1 abatement and cover its emissions with received permits).

[36]There is also an noticeable parallel between banking and both self-insurance and self-protection: banking is costly, but (i) reduces the likelihood of being in a net short position at date 2 (role of self-protection); (ii) for a given date-2 net position, it increases date-2 profits by either increasing sales or reducing purchases of allowances (role of self-insurance).

[37]Under CAAA, with pessimism only, a two-sided precautionary principle applies.

when abating at date 1, and adjusts a_1 accordingly. Therefore, the key determinant in signing pessimism is initial allocation. When it is sufficiently low for firms to expect a net short position in all scenarios ($\omega < \omega^*$), ambiguity aversion generates over-abatement. Otherwise, no general results obtain[38].

Proposition 3.8 also highlights that clear comparative statics results under ambiguity aversion are hard to come by, e.g. signing the covariance is a difficult exercise in general, hence imposing restrictive threshold conditions[39]. While the mechanics behind pessimism and ambiguity prudence are intuitive, how these practically transpose is not trivial. First, these two effects can be countervailing or reinforce one another. Second, the pessimistically-distorted prior H and the ambiguity prudence function $\mathcal{A}$ are endogenous to the optimisation problem, which ultimately hinge upon initial conditions, i.e., $\bar{a}_1$ or ω. Third, it depends on both the underlying modelling assumptions and the abatement cost functional forms.

3.3 Cap-and-trade regime under baseline ambiguity

This section abstracts from our uncertainty framework to explicitly account for the relationship between aggregate demand for permits (baselines) and the prevailing market price. That is, there is uncertainty on firms' baselines and this is the sole factor at the source of price uncertainty. In particular, all firms are subject to individual baseline uncertainty, denoted by $\tilde{b}_\theta$ in scenario θ, and since allowances are tradable, all firms face the same market-wide price uncertainty, endogenously emerging from the allowance market. To provide clear analytical results, let abatement cost function be time separable[40] and $\tilde{b}_\theta(s)$ be equipped with a specific structure such that for all $\theta \in \Theta$ and $s \in \mathcal{S}$, $\tilde{b}_\theta(s) = \bar{b}_\theta + \tilde{\epsilon}_\theta(s)$. That is, individual baselines comprise a first term $\bar{b}_\theta$ common to all firms, specific to any given θ-scenario, and an idiosyncratic term $\tilde{\epsilon}_\theta(s)$, such that, for all $\theta \in \Theta$, $(\tilde{\epsilon}_\theta(s); s \in \mathcal{S})$ are i.i.d. with $\mathbb{E}_L\left\{\tilde{\epsilon}_\theta(s)|\theta\right\} = 0$ and finite variance. In any θ-scenario, the aggregate baseline emission level is hence given by

$$\int_{\mathcal{S}} \tilde{b}_\theta(s)\mathrm{d}s = \int_{\mathcal{S}} \bar{b}_\theta \mathrm{d}s + \int_{\mathcal{S}} \tilde{\epsilon}_\theta(s)\mathrm{d}s = S\bar{b}_\theta, \tag{21}$$

[38]The cut-off allocation volume ω^* will be refined in section 3.3. In section 4.2 a parametrical example will show that under DAAA, under-abatement occurs when ω is high enough − this is so because the $\mathcal{A}$-effect has almost no impact relative to the $\mathcal{P}$-effect around ω^*.

[39]Appendix D shows that when price ambiguity is binary, the conditions to sign pessimism are milder.

[40]With long-term dependency, our results carry over if we suppose symmetric allocation of permits. This ensures that all firms abate the same at both dates. However, firms being identical along all relevant dimensions, no trade occurs in equilibrium.

which is deterministic in any θ-scenario and where the last inequality obtains by the Law of Large Numbers for a continuum of i.i.d. variables. Again, firms solve program (2). At date 2, for any given allocation plan $(\omega(s))_{s\in\mathcal{S}}$, firms choose how much to abate such that, $\forall s \in \mathcal{S}$, $C_2'(a_2(s)) = \tau$, the observed allowance price and upon observing their own baseline. In particular, all firms abate by the same amount at date 2, $a_2 \equiv a_2(s)$, $\forall s \in \mathcal{S}$, so that for any θ-scenario, market closure yields

$$\int_{\mathcal{S}} \tilde{b}_\theta(s) - a_1(s) - a_2(\theta) - \omega(s)\mathrm{d}s = 0 \Rightarrow a_2(\theta) = \bar{b}_\theta - \frac{A_1 + \Omega}{S}, \tag{22}$$

where Ω is the total cap and A_1 the overall date-1 abatement volume carried into date 2. The implied allowance price in scenario θ is thus $\tau_\theta = C_2'\left(\bar{b}_\theta - \frac{A_1+\Omega}{S}\right)$, which is deterministic in any θ-scenario, but not across scenarios[41]. Therefore, by the Envelope Theorem and noting that individual date-1 abatement decisions have no influence on the date-2 allowance price (i.e., $\partial_{a_1}\tau_\theta = 0$), one has that $\forall a_1 \geq 0$, $\forall \theta \in \Theta$, $\tilde{V}_{a_1}(a_1;\theta) = \mathcal{V}_{a_1}(a_1;\theta) = \tau_\theta$. The necessary first-order conditions under ambiguity neutrality and aversion are still given by (11-14), so that the same anticomonotonicity criterion applies for the formation of precautionary date-1 abatement under ambiguity prudence. As Proposition 3.9 shows, however, the difference lies in the characterisation of when anticomonotonicity holds.

Proposition 3.9. *Let firms be ambiguity prudent and $\partial_{a_1} C_2 \equiv 0$. Then, firm $s \in \mathcal{S}$ forms precautionary banking only if $\omega(s) \leq \min_{\theta\in\Theta} \omega_\theta^* > \frac{\Omega}{S}$, which always holds under symmetric allowance allocation.*

Proof. Under ambiguity neutrality, all ambiguity-neutral firms abate the same amount at date 1, $\bar{a}_1 = (C_1')^{-1}(\beta\langle\tau_\theta\rangle)$, with $\langle\tau_\theta\rangle = \mathbb{E}_F\{\tau_\theta\}$. Denoting $\langle b_\theta\rangle = \mathbb{E}_F\left\{\bar{b}_\theta\right\}$ and with specification (1), it comes[42]

$$\bar{a}_1 = \frac{c}{Sc_1}\left(S\langle b_\theta\rangle - \Omega\right) \text{ and } a_2(\theta) = \bar{b}_\theta - \frac{c}{S}\Big(\frac{S\langle b_\theta\rangle}{c_1} + \frac{\Omega}{\beta c_2}\Big). \tag{24}$$

Let us now sign $\mathrm{Cov}_F\{\mathcal{V}(\bar{a}_1;\tilde{\theta});\mathcal{V}_{a_1}(\bar{a}_1;\tilde{\theta})\}$. Because τ_θ and $a_2(\theta)$ are deterministic in any

[41]This requires that, for all $\theta \in \Theta$, $S\bar{b}_\theta - A_1 - \Omega > 0$. When $A_1 = \bar{A}_1$ this is always the case provided that

$$\frac{\Omega}{S} > \frac{c_1 \max_{\theta\in\Theta} \bar{b}_\theta - c\langle b_\theta\rangle}{c_1 - c}. \tag{23}$$

[42]By definition, $\bar{a}_1 = \frac{\beta c_2}{Sc_1}\left(S\langle b_\theta\rangle - \bar{A}_1 - \Omega\right)$, so that $\bar{A}_1 = S\bar{a}_1 = \frac{c}{c_1}\left(S\langle b_\theta\rangle - \Omega\right)$, which then gives (24). Note that, for all $\theta \in \Theta$, the overall cap is met, i.e., $\int_{\mathcal{S}} \tilde{b}_\theta(s) - \bar{a}_1 - \bar{a}_2(\theta)\mathrm{d}s = \Omega$.

given θ-scenario, $\mathcal{V}(\bar{a}_1;\theta) = \zeta_2 - C_2(a_2(\theta)) - \tau_\theta(\bar{b}_\theta - \bar{a}_1 - a_2(\theta) - \omega(s))$. It follows that

$$\partial_\theta \mathcal{V}_{a_1}(\bar{a}_1;\theta) = \partial_\theta \tau_\theta = C_2''(a_2(\theta))\partial_\theta a_2(\theta) = C_2''(a_2(\theta))\partial_\theta \bar{b}_\theta,$$

where the last equality follows from $\partial_\theta \bar{A}_1 = 0$ ($\bar{A}_1$ is decided ex-ante). Similarly, since $\partial_\theta \bar{a}_1 = 0$, it comes that

$$\partial_\theta \mathcal{V}(\bar{a}_1;\theta) = \left(C_2''(a_2(\theta))\Psi(s;\theta) - C_2'(a_2(\theta))\right)\partial_\theta \bar{b}_\theta,$$

where $\Psi(s;\theta) = \bar{a}_1 + a_2(\theta) + \omega(s) - \bar{b}_\theta$ is firm s' expected net position on the allowance market in scenario θ under ambiguity neutrality. In general, anticomonotonicity holds provided that, for all $\theta \in \Theta$, $\Psi(s;\theta) < \frac{C_2'(a_2(\theta))}{C_2''(a_2(\theta))}$. Note that this allows a positive (i.e., long) net position. In particular, with quadratic abatement cost functions, it comes from (24) that $\Psi(s;\theta) = \omega(s) - \frac{\Omega}{S}$, which is nil for a symmetric allocation allocation plan. Hence, when allowance allocation is symmetric, anticomonotonicity always holds, irrespective of the monotonicity of $\bar{b}_\theta$ w.r.t. θ. Moreover, assuming for simplicity that the ratio of abatement technology between the two dates is unitary, $c_1 = \beta c_2$, one has that

$$\Psi(s;\theta) < \frac{C_2'(a_2(\theta))}{C_2''(a_2(\theta))} \quad \Leftrightarrow \quad \omega(s) < \min_{\theta\in\Theta} \omega_\theta^*, \quad \text{with} \quad \omega_\theta^* = \frac{\Omega}{2S} + \bar{b}_\theta - \frac{\langle b_\theta \rangle}{2},$$

where it follows from (23) that $\omega_\theta^* > \frac{\Omega}{S}$ for all $\theta \in \Theta$. □

The anticomonotonicity criterion is somewhat laxer than under pure price ambiguity, since a net long position under the abatement stream $(\bar{a}_1; a_2(\theta))$ can be sufficient to conduce to over-abatement at date-1 (provided that the net positive position is not too big). Given that firms are identical[43], grandfathering corresponds to a symmetric allowance allocation, under which ambiguity-prudent firms always over-abate at date 1. This is suggestive of a natural tendency towards precautionary banking formation in ETSs.

[43] In particular, firms are supposed to be equally ambiguity averse (they have the same ϕ function) and to have the same subjective beliefs, F. It is difficult to account for heterogeneous tastes towards ambiguity and beliefs. Section 4.1 studies the effect of an increase in the concavity of ϕ and Appendix A considers a market populated by a mix of ambiguity averse and neutral firms. For more details when these assumptions are relaxed, refer to Danan et al. (2016,[23]). We conjecture, however, that our result holds under more general assumptions.

4 Comparative statics and numerical simulations

4.1 Comparative statics

This section addresses the comparative statics of the above analysis, namely, the sensibility of optimal date-1 abatement decisions under ambiguity aversion to the degree of ambiguity aversion and the volume of initial allocation of permits.

Increase in ambiguity aversion. In the sense of Klibanoff et al. (2005,[44]), firm 2 is said to be more ambiguity averse than firm 1 if the ambiguity function of the former writes as an increasing and concave transformation of the latter's, i.e., if there exists a function ψ such that $\phi_2 = \psi \circ \phi_1$ with $\psi' > 0$ and $\psi'' \leq 0$. Let $\hat{a}_i$ denote firm i's optimal date-1 abatement under ambiguity aversion with ϕ_i. Let us now state

Proposition 4.1. *Let there be two ambiguity-averse ambiguity-prudent firms 1 and 2, where firm 2 is more ambiguity averse than firm 1 such that $\phi_2 = \psi \circ \phi_1$, with ψ increasing, concave and almost quadratic, $\psi''' \simeq 0$. Assume $\mathcal{V}(a_1; \tilde{\theta})$ and $\mathcal{V}_{a_1}(a_1; \tilde{\theta})$ are anticomonotone, so that both firms form precautionary date-1 abatement. Then, firm 2 abates relatively more than firm 1 at date 1 provided that firm 1's ambiguity prudence is not too strong, i.e. $\frac{-\phi_1'''}{\phi_1''} \leq \frac{-3\phi_1''}{\phi_1'}$.*

Proof. By concavity of the objective function, $\hat{a}_2 \geq \hat{a}_1$ i.f.f.

$$\mathcal{A}_2(\hat{a}_1)\mathbb{E}_F\{\mathcal{D}_2(\hat{a}_1; \tilde{\theta})\mathcal{V}_{a_1}(\hat{a}_1; \tilde{\theta})\} \geq \mathcal{A}_1(\hat{a}_1)\mathbb{E}_F\{\mathcal{D}_1(\hat{a}_1; \tilde{\theta})\mathcal{V}_{a_1}(\hat{a}_1; \tilde{\theta})\},$$

with $\mathcal{A}_i$ and $\mathcal{D}_i$ denoting the ambiguity prudence coefficient and distortion function for firm i − replace ϕ by ϕ_i in (10) and (16). Note that, for all θ in Θ, one has that

$$\frac{\mathcal{D}_2(\hat{a}_1; \theta)}{\mathcal{D}_1(\hat{a}_1; \theta)} = \psi' \circ \phi_1(\mathcal{V}(\hat{a}_1; \theta)) \frac{\mathbb{E}_F\{\phi_1'(\mathcal{V}(\hat{a}_1; \tilde{\theta}))\}}{\mathbb{E}_F\{\phi_2'(\mathcal{V}(\hat{a}_1; \tilde{\theta}))\}} \propto \psi' \circ \phi_1(\mathcal{V}(\hat{a}_1; \theta)).$$

W.l.o.g. let $\mathcal{V}(\hat{a}_1; \theta)$ be non-decreasing in θ. By definition, $\psi' \circ \phi_1 (\mathcal{V}(\hat{a}_1; \theta))$ is non-increasing in θ. Since for all θ, $\frac{\mathcal{D}_2}{\mathcal{D}_1}$ is non-increasing in θ, firm 2 displays a stronger pessimism effect than firm 1 in the sense that it overemphasises low-$\mathcal{V}$ scenarios even further. By anticomonotonicity, $\mathcal{V}_{a_1}(\hat{a}_1; \theta)$ is non-increasing in θ, and it holds that

$$\mathbb{E}_F\{\mathcal{D}_2(\hat{a}_1; \tilde{\theta})\mathcal{V}_{a_1}(\hat{a}_1; \tilde{\theta})\} \geq \mathbb{E}_F\{\mathcal{D}_1(\hat{a}_1; \tilde{\theta})\mathcal{V}_{a_1}(\hat{a}_1; \tilde{\theta})\}.$$

Hence, provided that $\mathcal{A}_2(\hat{a}_1) \geq \mathcal{A}_1(\hat{a}_1)$, it is always true that $\hat{a}_2 \geq \hat{a}_1$. We now investigate when $\mathcal{A}_2 \geq \mathcal{A}_1$ holds in general. This is equivalent to firm 2 being more ambiguity prudent than firm 1, i.e., $\frac{-\phi_2'''}{\phi_2''} \geq \frac{-\phi_1'''}{\phi_1''}$. Assuming $\psi''' = 0$ then yields

$$\phi_2'' = (\psi'' \circ \phi_1)\, \phi_1'^2 + (\psi' \circ \phi_1)\, \phi_1'', \text{ and } \phi_2''' = 3\,(\psi'' \circ \phi_1)\, \phi_1' \phi_1'' + (\psi' \circ \phi_1)\, \phi_1'''.$$

That $\frac{-\phi_2'''}{\phi_2''} \geq \frac{-\phi_1'''}{\phi_1''}$ hence rewrites $\frac{-\phi_1'''}{\phi_1''} \leq \frac{-3\phi_1''}{\phi_1'}$. □

Proposition 4.1 separates out two effects consecutive to an increase in ambiguity aversion from ϕ_1 to ϕ_2. First, it leads to an unambiguous increase in pessimism in the sense of a monotone likelihood deterioration as in Gollier (2011,[35]): Being more concave, ϕ_2 places even more weight on those low-profit scenarios than ϕ_1. Second, it also induces a shift in ambiguity prudence. Controlling for this second effect by imposing that both firms display CAAA, i.e. $\mathcal{A}_i \equiv 1$, $i = 1, 2$, it is immediate from Proposition 4.1 that

Corollary 4.2. *Assuming* CAAA *on the part of firms and that anticomonotonicity holds, an increase in ambiguity aversion is always conducive to higher date-1 abatement.*

A similar result can also be found in Osaki & Schlesinger (2014,[58],Prop 3). For ambiguity prudence and pessimism to be aligned, i.e., for firm 2 to abate more than firm 1 at date 1 for sure, $\mathcal{A}_2$ must be at least as big as $\mathcal{A}_1$, that is, firm 2 must be more ambiguity prudent than firm 1. To be able to quantify the shift in ambiguity prudence, ψ must be equipped with an additional property and we impose the simplest one, namely $\psi''' = 0$. With this, an increase in ambiguity aversion via ψ increases ambiguity prudence provided that initial ambiguity prudence (for firm 1) is not too strong relative to ambiguity aversion, i.e., $\frac{-\phi_1''}{\phi_1'} \leq \frac{-\phi_1'''}{\phi_1''} \leq \frac{-3\phi_1''}{\phi_1'}$. In other words, when precautionary date-1 abatement for firm 1 is already substantial or when the ambiguity prudence effect for firm 1 is relatively strong, an increase in ambiguity aversion via ψ might not be conducive to an increase in date-1 abatement on the part of firm 2 for sure[44].

Dependence to initial allocation volume. For clarity, let ϕ display CAAA and let there

[44] A similar cut-off condition on the strength of ambiguity prudence is found by Guerdjikova & Sciubba (2015,[36]). In a market populated by both ambiguity neutral (i.e. EU-maximisers) and ambiguity averse individuals (thus forming wrong beliefs as compared with EU-maximisers and, accordingly, having a tendency to disappear with time), they show that only those ambiguity-averse agents displaying strong ambiguity prudence, $\frac{-\phi'''}{\phi''} > \frac{-2\phi''}{\phi'}$, will survive. As pointed out by Baillon (2016,[2]), this motivates further work in the direction of extending the notion of ambiguity prudence to higher orders, which is beyond the scope of this paper.

be no long-term effect of abatement. Under these assumptions, $\hat{a}_1$ is defined by

$$- C_1'(\hat{a}_1) + \beta \frac{\mathbb{E}_F\{\phi'(\mathcal{V}(\hat{a}_1;\tilde{\theta}))\mathcal{V}_{a_1}(\hat{a}_1;\tilde{\theta})\}}{\mathbb{E}_F\{\phi'(\mathcal{V}(\hat{a}_1;\tilde{\theta}))\}} = 0, \tag{25}$$

where $\mathcal{V}_{a_1}(\hat{a}_1;\theta) = \bar{\tau}_\theta = \mathbb{E}_G\left\{\tilde{\tau}_\theta|\theta\right\}$. Taking the total differential of (25) yields

$$\frac{\mathrm{d}\hat{a}_1}{\mathrm{d}\omega} = \frac{\beta\Phi(\hat{a}_1)}{C_1''(\hat{a}_1) - \beta\Phi(\hat{a}_1)}, \tag{26}$$

where, since $\mathcal{V}_\omega = \mathcal{V}_{a_1} = \bar{\tau}_\theta$, and omitting arguments so as to avoid cluttering,

$$\Phi(a_1) = \frac{\mathbb{E}_F\{\mathcal{V}_{a_1}^2\phi''(\mathcal{V})\}\mathbb{E}_F\left\{\phi'(\mathcal{V})\right\} - \mathbb{E}_F\left\{\mathcal{V}_{a_1}\phi'(\mathcal{V})\right\}\mathbb{E}_F\left\{\mathcal{V}_{a_1}\phi''(\mathcal{V})\right\}}{\mathbb{E}_F\left\{\phi'(\mathcal{V})\right\}^2}. \tag{27}$$

In particular, note that $\frac{\mathrm{d}\hat{a}_1}{\mathrm{d}\omega} \in]-1;0[$ i.f.f. $\Phi(\hat{a}_1) < 0$. One can show that

$$\begin{aligned} \Phi(\hat{a}_1) &\propto \mathrm{Cov}_F\left\{\mathcal{V}_{a_1};\mathcal{V}_{a_1}\phi''(\mathcal{V})\right\}\mathbb{E}_F\left\{\phi'(\mathcal{V})\right\} - \mathrm{Cov}_F\left\{\mathcal{V}_{a_1};\phi'(\mathcal{V})\right\}\mathrm{E}_F\left\{\mathcal{V}_{a_1}\phi''(\mathcal{V})\right\} \\ &\propto \mathcal{P}(\hat{a}_1) - \mathcal{P}_2(\hat{a}_1) = \frac{\mathrm{Cov}_F\left\{\mathcal{V}_{a_1};\phi'(\mathcal{V})\right\}}{\mathbb{E}_F\left\{\phi'(\mathcal{V})\right\}} - \frac{\mathrm{Cov}_F\left\{\mathcal{V}_{a_1};\mathcal{V}_{a_1}\phi''(\mathcal{V})\right\}}{\mathbb{E}_F\left\{\mathcal{V}_{a_1}\phi''(\mathcal{V})\right\}}, \end{aligned} \tag{28}$$

where $\mathcal{P}(\hat{a}_1)$ is the pessimism-only price distortion and $\mathcal{P}_2(\hat{a}_1)$ can be interpreted as a second-order pessimism-only price distortion, both evaluated at $a_1 = \hat{a}_1$. Note that when the anticomonotonicity criterion holds, the two distortions are positive and $\Phi(\hat{a}_1) \leq 0$ i.f.f. $\mathcal{P}_2(\hat{a}_1) \geq \mathcal{P}(\hat{a}_1)$. It is difficult to determine the variations of $\hat{a}_1$ w.r.t. ω because it is hard to sign $\mathcal{P}_2(\hat{a}_1) - \mathcal{P}(\hat{a}_1)$ in general. In section 4.2, numerical simulations show that, in line with intuition, the level of optimal date-1 abatement unambiguously decreases with the permit handout volume, with intensities depending on the degree of ambiguity aversion and the initial allocation volume itself[45].

4.2 Parametrical illustration

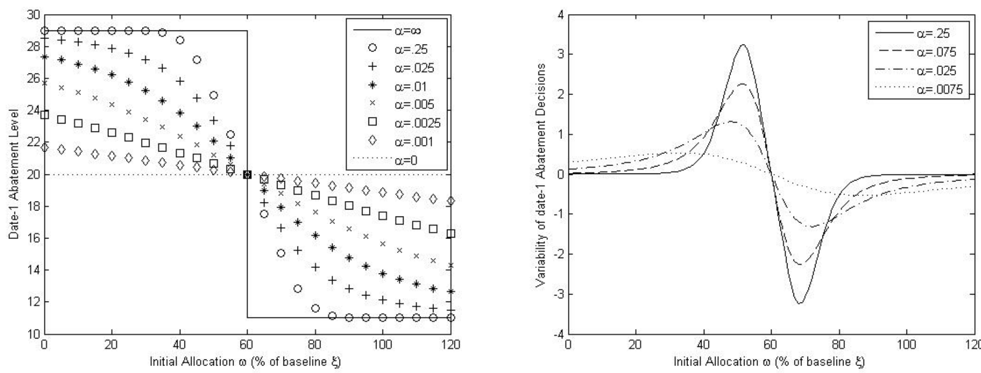
Without loss of generality, let there be no long-term effect of abatement $\gamma = 0$, along with $c_1 = c_2 = 1$ and $\beta = 1$. Let also $F \hookrightarrow \mathcal{U}(\Theta = [\![-\underline{\theta};\bar{\theta}]\!])$ and, for all scenario $\theta \in \Theta$, $G(\cdot;\theta) \hookrightarrow \mathcal{U}(\mathrm{T}_\theta = [\underline{\tau} + \theta; \bar{\tau} + \theta])$[46], where $0 < \bar{\theta} < \underline{\tau}$ and $\Delta\tau = \bar{\tau} - \underline{\tau} > 0$. Similarly, for all scenario $\theta \in \Theta$, $L(\cdot;\theta) \hookrightarrow \mathcal{U}(\mathrm{B}_\theta = \left[\underline{b} + \theta; \bar{b} + \theta\right])$ with $\Delta b = \bar{b} - \underline{b} > 0$. Finally, let $\underline{\tau} = 10$,

[45] This would suggest that $\mathcal{P}_2$ is bigger than $\mathcal{P}$, and, again, this calls for studying higher orders for ambiguity prudence.

[46] More generally, one could consider that G is uniform over $[\underline{\tau} - \varsigma\theta; \bar{\tau} + \theta]$ with ς a constant. This does not change the results and complicates computations.

$\bar{\tau} = 30$, $\underline{b} = 50$, $\bar{b} = 150$, $\bar{\theta} = 9$, $\omega \in [0; 120]$ and denote $\langle \tau \rangle = \frac{\underline{\tau}+\bar{\tau}}{2}$ and $\langle b \rangle = \frac{\underline{b}+\bar{b}}{2}$. Under CAAA (DAAA), one takes $\phi(x) = \frac{e^{-\alpha x}}{-\alpha}$ ($\phi(x) = \frac{x^{1-\alpha}}{1-\alpha}$) with $\alpha > 0$ ($\alpha > 1$) the coefficient of absolute ambiguity aversion. Let $\hat{a}_1^\alpha$ be the optimal date-1 abatement under ambiguity aversion and by extension, let $\hat{a}_1^\infty$ denote the optimal date-1 abatement with the MEU representation theorem and $\hat{a}_1^0 = \bar{a}_1$ ($\hat{a}_1^1 = \bar{a}_1$) under CAAA (DAAA). We can now characterize how $\hat{a}_1^\alpha$ evolves with ω and α in the three following cases.

Quota regime under CAAA. In this case, the expected marginal benefit of banking is independent of a_1 and satisfies $\mathcal{V}_{a_1}(a_1;\theta) = \langle \tau \rangle + \theta$. It follows that anticomonotonicity holds provided that, for all $\theta \in \Theta$, $\partial_\theta \mathcal{V}(a_1;\theta) \leq 0 \;\Leftrightarrow\; \omega \leq \xi - a_1 - \langle \tau \rangle - \theta$. In particular, evaluated at $a_1 = \bar{a}_1 = \langle \tilde{\tau} \rangle = \mathbb{E}_F\{\mathcal{V}_{a_1}(a_1;\tilde{\theta})\} = \langle \tau \rangle$, anticomonotonicity holds i.f.f. $\omega \leq \xi - 2\langle \tau \rangle - \bar{\theta}$, which corresponds to the threshold ω^* given in Proposition 3.8[47]. With this, $\langle \tau \rangle = 20$ and $\omega^* = 51$. The optimal date-1 abatement $\hat{a}_1^\alpha$ satisfies $\hat{a}_1^\alpha = \langle \tau \rangle + \mathcal{P}(\hat{a}_1^\alpha)$ and its variations[48] w.r.t. α and ω are depicted in Figure 3a. With our specification, there are implicit upper



(a) $\hat{a}_1^\alpha = f(\omega)$ for different α's

(b) Optimal abatement variability

Figure 3: Cap-and-trade regime under CAAA.

and lower constraints on $\hat{a}_1^\alpha$ (29 and 11, respectively) since the maximal future price variation (date-1 abatement variation) is confined within a $-9; +9$ range around $\langle \tau \rangle = 20$. The dotted line represents the optimal date-1 abatement under ambiguity neutrality $\bar{a}_1$, which is independent of the initial allocation volume. The solid line characterises the optimal date-1 abatement level with the MEU representation theorem. It is a step function of initial allocation: if $\omega < \bar{\omega} = 60$, $\hat{a}_1^\infty$ equals the upper limit $\langle \tau \rangle + \bar{\theta}$; otherwise, it equals the lower limit

[47]Indeed, $\omega_\theta^* = \xi - \bar{a}_1 - a_2^*(\bar{a}_1;\tau_\theta^*)$, with $\bar{a}_1 = \langle \tilde{\tau} \rangle$ and $\tau_\theta^* = \left(\int_{\mathrm{T}_\theta} G_\theta(x;\theta)\mathrm{d}x\right)^{-1} \int_{\mathrm{T}_\theta} x G_\theta(x;\theta)\mathrm{d}x = \langle \tau \rangle + \theta$, since $G_\theta(x;\theta) = -\frac{1}{\Delta\tau}$ for $\underline{\tau} + \theta \leq x \leq \bar{\tau} + \theta$ and 0 otherwise.

[48]Note that this also corresponds to the date-2 allowance price distortion due to ambiguity aversion.

$\langle\tau\rangle - \bar{\theta}$. Other curves depict $\hat{a}_1^\alpha$ for various ambiguity aversion degrees α, and the KMM representation describes the continuum of optimal date-1 abatement levels between the two polar cases defined by ambiguity neutrality and MEU.

First note that $\hat{a}_1^\alpha$ unambiguously decreases with ω with a clear threshold at $\bar{\omega} = 60$ below (above) which over-(under-)abatement occurs, for all ambiguity aversion degrees. Note that this condition is laxer than that for anticomonotonicity to hold since $\omega^* < \bar{\omega}$[49]. Second, for any given initial allocation volume, the variation $|\hat{a}_1^\alpha - \bar{a}_1|$ increases with α. As in Corollary 4.2, when anticomonotonicity holds (in expectations), $\hat{a}_1^\alpha$-lines are ordered by increasing α and never cross, so that an increase in ambiguity aversion always leads to higher date-1 abatement. Note also that the bigger α, the more sensitive the variations in $\hat{a}_1^\alpha$ around $\bar{\omega}$ w.r.t. ω. In particular, for $\alpha = .25$, $\hat{a}_1^\alpha$ has already converged to its upper (resp. lower) limit when ω reaches 30 (resp. 90). Figure 3b depicts the variability of date-1 abatement adjustment under pessimism only w.r.t. allocation for different ambiguity aversion degrees[50]. The bigger α, the quicker $\hat{a}_1^\alpha$ adjusts to ω in a smaller $\bar{\omega}$-centred range. For lower α, the incentive to adjust date-1 abatement relative to ambiguity neutrality is smaller and more evenly spread over the entire allocation range.

Tax regime under DAAA. In this case, the expected marginal benefit of banking is independent of both a_1 and θ and satisfies $\mathcal{V}_{a_1}(a_1;\theta) = t = 20$. The optimal date-1 abatement $\hat{a}_1^\alpha$ satisfies $\hat{a}_1^\alpha = \mathcal{A}(\hat{a}_1^\alpha)t$ and its variations w.r.t. α and ω are depicted in Figure 4, which solely characterises the impact of ambiguity prudence. For all $\alpha > 1$, $\hat{a}_1^\alpha$ unambiguously decreases with ω and is always above $\bar{a}_1$. That is, the $\mathcal{A}$-effect is a decreasing function of allocation and has steeper variations for low α when ω is small. In particular, for a standard tax regime ($\omega = 0$), Figure 4 indicates that a higher degree of ambiguity aversion is not necessarily conducive to higher date-1 abatement levels. There exists a threshold $\bar{\alpha}$ such that $\hat{a}_1^\alpha$ increases (decreases) with α provided that α is below (above) $\bar{\alpha}$ – numerically, we find $\bar{\alpha} \simeq 11.55$. As apparent from Figure 4, one could also conjecture that for α high enough, $\hat{a}_1^\alpha \to \bar{a}_1$ (since the $\mathcal{A}$-effect is specific to the KMM representation, hence absent with the MEU criterion). Moving towards higher allocation levels, however, $\hat{a}_1^\alpha$ is ranked by increasing ambiguity aversion degrees. Note also that the ratio $\hat{a}_1^\alpha/\bar{a}_1 > 1$ is relatively smaller than for

[49] As mentioned earlier on, this suggests that anticomonotonicity might be too strong a requirement to sign pessimism. From the simulations, we infer that $\bar{\omega} = \mathbb{E}_F\{\omega_\theta^*\} = \xi - 2\langle\tau\rangle$, i.e., with our specification, the introduction of ambiguity aversion is conducive to an increase in date-1 abatement if, and only if, anticomonotonicity holds in expectations over Θ w.r.t. F.

[50] Figure 3b plots $\mathcal{P}(\bar{a}_1) - \mathcal{P}(\hat{a}_1^\alpha)$ as a function of ω. Using (19) and injecting the first-order condition for $\hat{a}_1^\alpha$, there is an incentive to increase date-1 abatement i.f.f. $\hat{a}_1^\alpha - \bar{a}_1 + \mathcal{P}(\bar{a}_1) - \mathcal{P}(\hat{a}_1^\alpha) > 0$ so that $\mathcal{P}(\bar{a}_1) - \mathcal{P}(\hat{a}_1^\alpha)$ can be interpreted as a proxy of the incentive to increase $\hat{a}_1^\alpha$ relative to $\bar{a}_1$.

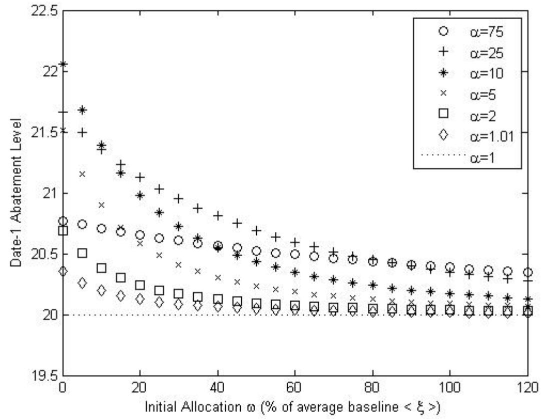
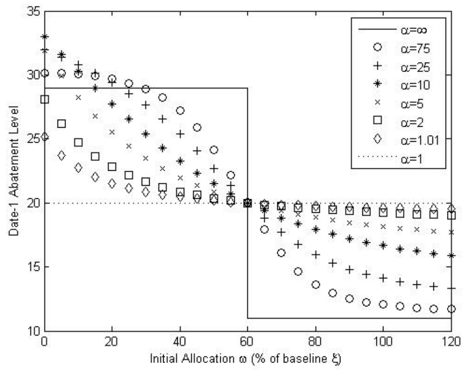


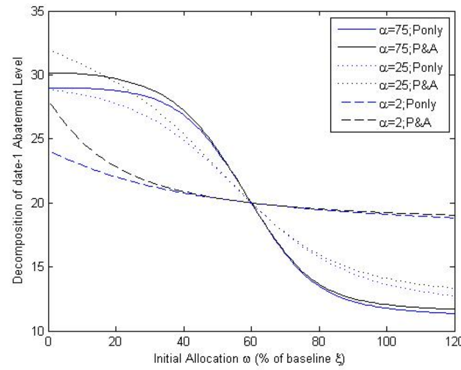
Figure 4: Tax regime under DAAA.

an ETS under CAAA, even when α is close to $\bar{\alpha}$. This suggests that the magnitude of the $\mathcal{A}$-effect is relatively small, as compared with the $\mathcal{P}$-effect.

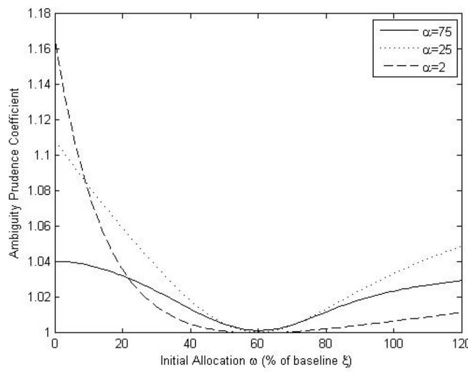
Quota regime under DAAA. This case is identical to that under CAAA, but further integrates ambiguity prudence. In particular, the optimal date-1 abatement $\hat{a}_1^\alpha$ satisfies $\hat{a}_1^\alpha = \mathcal{A}(\hat{a}_1^\alpha)\,(\langle\tau\rangle + \mathcal{P}(\hat{a}_1^\alpha))$, whose variations w.r.t. α and ω are depicted in Figure 5. Figure 5a is similar to Figure 3a save for small disruptions due to the ambiguity prudence effect. In particular, when $\omega > \bar{\omega}$, the $\mathcal{A}$-effect pushes $\hat{a}_1^\alpha$ up towards the $\bar{a}_1$-line, though without breaching it, and the lower limit $\langle\tau\rangle - \bar{\theta}$ is never reached. When $\omega < \bar{\omega}$, the $\mathcal{A}$-effect further adjusts banking upwards so that for relatively low allocation levels, the upper limit $\langle\tau\rangle + \bar{\theta}$ can be exceeded. As in the tax regime, for ω low enough, a higher ambiguity aversion degree is not necessarily conducive to higher date-1 abatement. More precisely, and as clear from Figure 5c, the magnitude of the $\mathcal{A}$-effect is more pronounced for low α when ω is small. Asymmetrically, when ω is big, it is relatively lower. Note that within the $[40; 80]$ band, the $\mathcal{A}$-effect is quasi-absent and ordered by increasing α around $\bar{\omega}$. That $\hat{a}_1^\alpha$-lines may cross when ω is low enough substantiates Proposition 4.1: An increase in ambiguity aversion is not necessarily conducive to higher date-1 abatement under DAAA as the ambiguity prudence effect might disrupts the pessimism-induced price distortion. In contrast, note that no such crossing exist when $\omega > \bar{\omega}$, i.e. there is an asymmetry in the $\mathcal{A}$-effect. The joint $\mathcal{A}$ and $\mathcal{P}$ effect is further illustrated by the decomposition in Figure 5b where it is clear that the $\mathcal{A}$-correction is more pronounced for low than big ω, and that it is almost nil within the $[40; 80]$ band. Except for low allocation levels, this further suggests that the $\mathcal{P}$-effect is the main driving factor behind the date-1 abatement adjustment, while the $\mathcal{A}$-effect often has a



(a) $\hat{a}_1^\alpha = f(\omega)$ for different α's



(b) Decomposition of the $\mathcal{P}$ and $\mathcal{A}$ effects



(c) Magnitude of the $\mathcal{A}$ effect

Figure 5: Cap-and-trade regime under DAAA.

much smaller impact in comparison.

5 Conclusion

The introduction of ambiguity aversion induces two effects, pessimism and ambiguity prudence, which can be aligned or countervailing. Defined as DAAA, ambiguity prudence corresponds on an increase in firms' discount factor, so that, controlling for pessimism, it is always conducive to over-abatement at date 1. Overweighting bad scenarios, pessimism leads to over-abatement provided that those bad scenarios with low expected profits coincide with scenarios having high expected marginal benefits from banking − the anticomonotonicity criterion. The magnitude of these two effects depends on the degree of ambiguity aversion and initial allocation, which makes clear general results on the impact of ambiguity aversion hard to come by. In particular, both the strength and direction of the pessimism-induced banking adjustment correspond to a precautionary principle whereby the expected future position on the allowance market is accounted for in present abatement decisions. This ultimately relates to initial allocation, whose independence property does not hold under ambiguity aversion. Only when liable firms are allocated too small a volume of allowances, and expect to in a net short future position, does ambiguity aversion raise date-1 abatement above the ambiguity-neutral optimal level. In a special case, numerical simulations suggest that pessimism is the main driving effect behind abatement adjustments, both in terms of direction and magnitude, while ambiguity prudence has a negligible impact, except when allocation is very low or very high, albeit to a lesser extent. In particular, especially when allocation is small, a higher degree of ambiguity aversion does not necessarily lead to higher over-abatement at date 1. Appendix A further shows that ambiguity aversion impedes allowance trading and that pessimism can be mitigated by the introduction of a market for forwards. In terms of comparability of instruments, both tax and ETS are not conducive to intertemporal cost-efficiency under ambiguity aversion. In our setup, an ETS is subject to both pessimism and ambiguity prudence effects, while a tax only to the latter.

The introduction of ambiguity aversion as a way to capture the influence of regulatory uncertainty on firms' decisions provides theoretical foundations for what could contribute to the general tendency towards allowance surplus formation and low prices. Extrapolating from Proposition 3.9 indicates that, when allowances are grandfathered, ambiguity averse firms would tend to over-abate in early phases. This is even more pronounced under auctioning, which in practice might correspond to firms from the power sector in the EUETS or in RGGI, as they must acquire all allowances through auctions. Ambiguity prudence on the part of market participants, hence using relatively higher discount factor, could contribute to the drop in prices observed in the EUETS in the last few years. This contradicts the standard rationale that banking adjusts so as to minimize the sum of discounted abatement costs and can be carried out at constant low rates (interest rates). This also complements the proposition of Neuhoff et al. (2012,[55]) that only speculators are willing to carry permits forward when the surplus exceeds the power sector's hedging demand (hence requiring higher appreciation rates) to explain the EUA price drop. Our results also corroborate Stephen Salant's claim that «correct diagnosis should precede treatment advice» (Salant, 2015,[67]) in the current context of market design revisions, notably in the form of *ex-post* allowance supply management (price-based cost-containment reserves in RGGI, California & Québec; quantity-based surplus-adjustment mechanism in the EUETS; *ex-post* allocation and cap adjustments in Chinese pilots). If allowance surplus is deemed excessive and source of market inefficiency (low price), *ex-post* allowance supply management can eliminate inefficiency. If regulatory uncertainty is excessive, however, *ex-post* allowance supply management is unlikely to correct inefficiency, and might even add to perceived regulatory risk (ambiguity level) and reinforce surplus formation and price decline, as highlighted in our paper.

Two ways to build on this paper can be identified. First, output decisions could be endogenised, i.e., in their banking decision, firms also account for the banking-induced future output price change. For instance, considering two types of firms (clean and dirty), Baldursson & von der Fehr (2012,[4]) find that this alters and sometime reverts their 2004 results[51]. One could also account for more inter-firm heterogeneity in terms of subjective beliefs or ambiguity attitudes, but the aggregation thereof is challenging − see e.g. Danan et al. (2016,[23]). Second, one could address the normative question of the most socially desirable way to allocate allowances through time and across firms. In our setup, the regulator should aim to allocate $\bar{\omega}$ allowances to the most ambiguity averse firms so as to minimise deviations from the ambiguity neutral benchmark. One could test different dynamic cap-adjustment procedures so as to limit the effects of ambiguity aversion through time, e.g. along the lines of Newell et al. (2005,[57]). In particular, one could define the optimal intertemporal trading ratio under ambiguity, as in Yates & Cronshaw (2001,[83]) or Feng & Zhao (2006,[29]) under risk. Another normative question is the effect of ambiguity aversion on the part the regulator. That is, knowing how firms react to a given tax rate or emissions cap, one could compare the effects of ambiguity aversion on setting the optimal cap or tax rate for the regulator. To give a flavour of the results one may expect, we can show that with ambiguous baselines and linear environmental damages[52] the introduction of ambiguity aversion has no effect on the socially optimal tax rate. In contrast, it is conducive to higher socially optimal emissions caps than under ambiguity neutrality, the higher the degree of ambiguity aversion on the part of the regulator. This contrasts with Millner et al. (2013,[53]) and Berger et al. (2016,[10]) who find that climate uncertainties warrant higher mitigation targets.

[51]Endogenising output decisions mitigates (exacerbates) risk exposure for dirty firms (clean firms or highly-allocated dirty firms); with small allocation, risk-averse clean and dirty firms alike (both on average and at the margin) reduce investment relative to the risk-neutral benchmark.

[52]With quadratic environmental damages, the optimal tax rate under ambiguity aversion is lower than under ambiguity neutrality. Note also that, when emissions generate of flow of damages at each period, the least-discounted abatement-cost emission path does not minimise welfare, defined as the discounted sum of damages plus firms' abatement costs − see Kling & Rubin (1997,[46]) and Leiby & Rubin (2001,[49]).

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Appendices – Supplemental Material

A Three brief extensions to the model

This appendix briefly develops one extension to the model by allowing for forwards trading and investigates the impacts of (i) ambiguity aversion on the equilibrium volume of trade; (ii) there being a mix of ambiguity averse and neutral agents on the allowance market.

Forward trading. It is natural to investigate whether the introduction of a market for forwards can diminish ambiguity and restore cost-efficiency. In practice, firms liable under cap-and-trade schemes have recourse to forward contracts for hedging purposes, e.g. power companies in the EUETS. Assume, therefore, that in addition to date-1 and date-2 abatement decisions, firms have the possibility to trade allowances in a forward market at date-1. Let a_f and p_f denote the volume of allowances contracted in the forward market and the forward price, respectively. Note that this does not change the optimal abatement decision at date-2. In particular, the firm's recursive program now writes

$$\max_{a_1 \geq 0, a_f} \zeta_1 - C_1(a_1) - p_f a_f + \beta\phi^{-1}(\mathbb{E}_F\{\phi(\mathcal{V}(a_1, a_f; \tilde{\theta}))\}), \tag{29}$$

where $\mathcal{V}(a_1, a_f; \theta) = \mathbb{E}_G\{\zeta_2 - C_2(a_1, a_2^*(a_1; \tilde{\tau}_\theta)) - \tilde{\tau}_\theta(\xi - a_1 - a_f - a_2^*(a_1; \tilde{\tau}_\theta) - \omega)|\theta\}$ for all $\theta \in \Theta$. The two necessary first-order conditions for the optimal date-1 abatement and contracted forward volumes under ambiguity aversion, $\hat{a}_1$ and $\hat{a}_f$, are given by

$$\begin{cases} -C_1'(\hat{a}_1) + \beta \dfrac{\mathbb{E}_F\{\phi'(\mathcal{V}(\hat{a}_1, \hat{a}_f; \tilde{\theta}))\mathcal{V}_{a_1}(\hat{a}_1, \hat{a}_f; \tilde{\theta})\}}{\phi' \circ \phi^{-1}(\mathbb{E}_F\{\phi(\mathcal{V}(\hat{a}_1, \hat{a}_f; \tilde{\theta}))\})} = 0, \\ -p_f + \beta \dfrac{\mathbb{E}_F\{\phi'(\mathcal{V}(\hat{a}_1, \hat{a}_f; \tilde{\theta}))\mathcal{V}_{a_f}(\hat{a}_1, \hat{a}_f; \tilde{\theta})\}}{\phi' \circ \phi^{-1}(\mathbb{E}_F\{\phi(\mathcal{V}(\hat{a}_1, \hat{a}_f; \tilde{\theta}))\})} = 0. \end{cases} \tag{30}$$

By the Envelop, $\mathcal{V}_{a_f}(\hat{a}_1, \hat{a}_f; \theta) = \tau_\theta \geq \mathcal{V}_{a_1}(\hat{a}_1, \hat{a}_f; \theta) = \tau_\theta - \mathbb{E}_G\{\partial_{a_1} C_2(\hat{a}_1, a_2^*(\hat{a}_1; \tilde{\tau}_\theta))|\theta\} > 0$, where $\tau_\theta = \mathbb{E}_G\{\tilde{\tau}_\theta|\theta\}$. Thus, without long-term effect of abatement, cost-efficiency in expectations is restored since $\beta\langle\tilde{\tau}\rangle = C_1'(\bar{a}_1) = p_f = C_1'(\hat{a}_1)$, as long as $p_f \in \mathrm{T}$ is predetermined, but irrespective of how p_f is priced. Otherwise, with long-term effect of abatement, combining the two first-order conditions in (30) gives

$$-C_1'(\hat{a}_1) - \beta\mathcal{A}(\hat{a}_1, \hat{a}_f)\mathbb{E}_F\{\mathcal{D}(\hat{a}_1, \hat{a}_f; \tilde{\theta})\mathbb{E}_G\{\partial_{a_1} C_2(\hat{a}_1, a_2^*(\hat{a}_1; \tilde{\tau}_\theta))|\theta\}\} + p_f = 0. \tag{31}$$

Assume also that the forward price is unbiased such that $p_f = \beta \langle \tilde{\tau} \rangle$, that is, forward contracts are fairly priced. By (30), for any $a_1 \geq 0$, the firm chooses its optimal forward contracted volume $a_f^*(a_1)$ by equating $\langle \tilde{\tau} \rangle$ to $\mathcal{A}(a_1, a_f^*(a_1))\mathbb{E}_F\{\mathcal{D}(a_1, a_f^*(a_1); \tilde{\theta})\tau_\theta\}$[53]. Therefore, by concavity of the objective function, $\hat{a}_1 \geq \bar{a}_1$ i.f.f.

$$\mathbb{E}_{\bar{G}}\{\partial_{a_1} C_2(\bar{a}_1, a_2^*(\bar{a}_1; \tilde{\tau}))\} \geq \mathcal{A}(\bar{a}_1, a_f^*(\bar{a}_1))\mathbb{E}_F\{\mathcal{D}(\bar{a}_1, a_f^*(\bar{a}_1); \tilde{\theta})\mathbb{E}_G\{\partial_{a_1} C_2(\bar{a}_1, a_2^*(\bar{a}_1; \tilde{\tau}_\theta))|\theta\}\}. \tag{32}$$

Using the quadratic specification, this is equivalent to

$$\langle \tilde{\tau} \rangle + \gamma(\mathcal{A}(\bar{a}_1, a_f^*(\bar{a}_1)) - 1)\bar{a}_1 \geq \mathcal{A}(\bar{a}_1, a_f^*(\bar{a}_1))\mathbb{E}_F\{\mathcal{D}(\bar{a}_1, a_f^*(\bar{a}_1); \tilde{\theta})\tau_\theta\}, \tag{33}$$

which, under the fair price assumption, is equivalent to $\mathcal{A}(\bar{a}_1, a_f^*(\bar{a}_1)) \geq 1$. Let us now state

Proposition A.1. *Consecutive to the introduction of forward contracts,*
(i) assuming there is no long-term effect of abatement and irrespective of how forward contracts are priced, cost-efficiency in expectations is restored;
(ii) accounting for long-term effect of abatement and assuming forward contracts are fairly priced, cost-efficiency in expectations obtains only under CAAA. In particular, under DAAA (IAAA), over-abatement (under-abatement) at date-1 persists.

Without long-term effect of abatement, the introduction of a forward market restores intertemporal cost-efficiency in expectations. That is, the optimal date-1 abatement level hinges neither upon the underlying ambiguity level nor upon the firm's attitude toward ambiguity. This contrasts with B&vdF who find that under risk aversion, inefficient date-1 over-abatement persists. However, our result is in line with recent extensions of the separation theorem under smooth ambiguity aversion, e.g. Wong (2015b,[80]), Wong (2015c,[81]) or Osaki et al. (2015,[59])[54]. With long-term effect of abatement, the introduction of a fairly-priced market for forward contracts only corrects for the pessimism effect, but the optimal date-1 abatement decision remains subject to the ambiguity prudence effect. In terms of date-1 abatement decisions, a cap-and-trade regime with forward contracts is hence akin to a tax regime. This contrasts with Wong (2015b,[80]), Wong (2015c,[81]) and Osaki et al. (2015,[59]) in that they use the static KMM formulation, hence absent any $\mathcal{A}$-effect.

[53] In particular, when ϕ is CAAA, a_f^* solves $\mathrm{Cov}_F\{\mathcal{V}(a_1, a_f^*(a_1); \tilde{\theta}); \mathcal{V}_{a_f}(a_1, a_f^*(a_1); \tilde{\theta})\} = 0$.

[54] In the presence of pure price ambiguity for a risk-averse ambiguity-averse competitive firm, see Wong (2015b,[80]). In the presence of price ambiguity and additive background risk for a risk-neutral and ambiguity-averse competitive firm, see Osaki et al. (2015,[59]). Finally, in the presence of price ambiguity and additive or multiplicative background risk for a risk-averse ambiguity-averse competitive firm, see Wong (2015c,[81]).

Equilibrium volume of trade. Let us now investigate the impact of ambiguity aversion on the part of firms on the overall volume of trade. In order to have clear variations of $\hat{a}_1$ w.r.t. $\bar{a}_1$ in both directions, let ϕ display CAAA. That is, under CAAA, when firm s (l) is allocated less (more) than $\min_{\theta\in\Theta}\omega^*_\theta$ ($\max_{\theta\in\Theta}\omega^*_\theta$) it expects to be net short (long) in all θ-scenarios under the abatement stream $(\bar{a}_1; a^*_2(\bar{a}_1;\tau^*_\theta))$ so that $\hat{a}^s_1 \geq \bar{a}_1 \geq \hat{a}^l_1$. At date 2, all firms equate their date-2 marginal abatement costs $\partial_{a_2} C_2(a_1; a^*_2)$ to the observed allowance price τ. In terms of total abatement for the three types of firms, one has that

$$a^*_2(\hat{a}^s_1;\tau) + \hat{a}^s_1 = \frac{\tau}{c_2} + \Big(1 - \frac{\gamma}{c_2}\Big)\hat{a}^s_1 \geq a^*_2(\bar{a}_1;\tau) + \bar{a}_1 \geq a^*_2(\hat{a}^l_1;\tau) + \hat{a}^l_1. \tag{34}$$

Since the net buying (selling) firm s (l) abates relatively more (less) and hence buys (sells) less allowances on the market than under ambiguity neutrality, the following holds

Proposition A.2. *Let allowances be non-symmetrically distributed such that some firms are endowed with* $\omega \notin [\min_{\theta\in\Theta}\omega^*_\theta; \max_{\theta\in\Theta}\omega^*_\theta]$. *Then, the equilibrium volume of trade is lower when firms are ambiguity averse than when they are ambiguity neutral.*

This is similar to B&vdF who find that risk aversion reduces the equilibrium volume of trade as compared with risk neutrality. Ambiguity and risk aversions might both provide another explanation for what Ellerman (2000,[25]) calls *autarkic compliance* in early phases of ETSs. Thin traded volumes are indeed observed in nascent schemes, e.g. presently in the SKETS or the Chinese pilots. Because covered entities are waiting for increased price discovery and due to high regulatory uncertainty, they tend to hold on to their quota allocation so that trades are scarce. During Phase I of the EUETS, the volume of trades (both in EUAs and futures) increased steadily over time as uncertainty gradually vanished, see e.g. Ellerman et al. (2010,[26],Chap.5). Our result shows that the presence of ambiguity aversion might provide a theoretical underpinning for such a prudent behaviour.

Different tastes for ambiguity. Let us now consider a mix of ambiguity averse and neutral firms in the market for allowances. Let there be no long-term effect of abatement and let $\varepsilon \in [0; 1]$ denote the share of ambiguity averse firms. For any $0 < \varepsilon < 1$, denote by $\hat{a}^\varepsilon_1$ and $\bar{a}^\varepsilon_1$ the optimal date-1 abatement levels for the ambiguity averse and neutral firms, respectively. Suppose also that ambiguity averse firms are allocated $\omega \leq \min_{\theta\in\Theta}\omega^*_\theta$ so that, in a market that contains either only ambiguity averse or ambiguity neutral firms, optimal date-1 abatement levels satisfy $\hat{a}^{\varepsilon=1}_1 = \hat{a}_1 \geq \bar{a}^{\varepsilon=0}_1 = \bar{a}_1$ and $\hat{A}_1 = S\hat{a}_1 \geq \bar{A}_1 = S\bar{a}_1$. For any mix ε, assume[55] that market closure at date 2 gives the allowance price in each scenario $\theta \in \Theta$ by $\tau_\theta^\varepsilon = c_2\Big(\bar{\xi}_\theta - \frac{\varepsilon \hat{A}_1 + (1-\varepsilon)\bar{A}_1 + \Omega}{S}\Big)$. Denoting by $\bar{\tau}_\theta$ and $\hat{\tau}_\theta$ the θ-scenario allowance price when $\varepsilon = 0$ and $\varepsilon = 1$, respectively, one has that $\hat{\tau}_\theta \leq \tau_\theta^\varepsilon \leq \bar{\tau}_\theta$. Symmetrically, when ambiguity averse firms receive a large allocation $\omega \geq \max_{\theta\in\Theta} \omega_\theta^*$, $\hat{a}_1 \leq \bar{a}_1$ and thus, $\bar{\tau}_\theta \leq \tau_\theta^\varepsilon \leq \hat{\tau}_\theta$. By comparing the necessary first-order conditions for $\bar{a}_1$ and $\bar{a}_1^\varepsilon$ on the one hand, and for $\hat{a}_1$ and $\hat{a}_1^\varepsilon$ on the other hand, the following holds

Proposition A.3. *Let $\varepsilon \in]0;1[$ denote the share of ambiguity averse firms. Then,*
(i) when they are allocated $\omega < \min_{\theta\in\Theta} \omega_\theta^*$, $\bar{a}_1^\varepsilon < \bar{a}_1 < \hat{a}_1 < \hat{a}_1^\varepsilon$;
(ii) when they are allocated $\omega > \max_{\theta\in\Theta} \omega_\theta^*$, $\bar{a}_1^\varepsilon > \bar{a}_1 > \hat{a}_1 > \hat{a}_1^\varepsilon$.

This shows that having a mix of ambiguity averse and neutral firms in the market where ambiguity averse firms are endowed with a relatively high or relatively low number of allowances brings the market further away from cost-efficiency. In particular, note that this also alters abatement decisions of ambiguity neutral agents.

B MEU preferences & anticomonotonicity

The anticomonotonicity condition is robust in the sense that it obtains with other ambiguity aversion representation theorems, here with the MEU criterion. Gilboa & Schmeidler (1989,[34]) put forth an behavioural foundation for the maxmin-EU decision rule − and with our interpretation that Θ represents the set of possible objective probability distributions, for the Wald's minimax decision criterion. With the α-maxmin decision criterion the firm grants a weight $0 \leq \alpha \leq 1$ to the worst scenario in Θ, and the complementary weight to the best scenario − this reduces to the Wald's criterion for $\alpha = 1$. Let us state

Proposition B.1. *With the MEU representation theorem, the introduction of ambiguity aversion is conducive to higher date-1 abatement levels than under ambiguity neutrality if, and only if, the sequences $(\mathcal{V}(\bar{a}_1;\theta))_\theta$ and $(\mathcal{V}_{a_1}(\bar{a}_1;\theta))_\theta$ are anticomonotone, where $\bar{a}_1$ denotes the optimal date 1-abatement under ambiguity neutrality.*

Proof. For the purpose of the proof, let Θ be a discrete finite set of cardinality $k = |\Theta|$, ordered such that $\theta_1 \leq \cdots \leq \theta_k$. Let $(q_i)_{i=1,\ldots,k}$ be the subjective prior such that q_i denotes the firm's subjective probability that the θ_i-scenario will materialize and $\sum_i q_i = 1$. W.l.o.g.

[55] This is a conservative assumption. As will be clear from Proposition A.3, defining τ_θ^ε with $\hat{A}_1^\varepsilon$ and $\bar{A}_1^\varepsilon$ instead of $\hat{A}_1$ and $\bar{A}_1$ would further amplify the deviation.

let the sequence $(\mathcal{V}(\bar{a}_1;\theta_i))_i$ be non-decreasing in i. Recall that $\bar{a}_1$ is defined by $-C_1'(\bar{a}_1) + \beta\sum_i q_i \mathcal{V}_{a_1}(\bar{a}_1;\theta_i) = 0$. The α-maxmin objective function, evaluated at $a_1 = \bar{a}_1$, reads

$$\begin{aligned}\Upsilon_\alpha(\bar{a}_1) &= \pi_1(\bar{a}_1) + \beta\Big(\alpha \min_{\theta\in\Theta} \mathcal{V}(\bar{a}_1;\theta) + (1-\alpha)\max_{\theta\in\Theta}\mathcal{V}(\bar{a}_1;\theta)\Big) \\ &= \pi_1(\bar{a}_1) + \beta\left(\alpha\mathcal{V}(\bar{a}_1;\theta_1) + (1-\alpha)\mathcal{V}(\bar{a}_1;\theta_k)\right),\end{aligned}$$

and let $\hat{a}_1^\alpha$ be the maximizer of Υ_α. By concavity of Υ_α, $\hat{a}_1^\alpha \geq \bar{a}_1$, i.f.f.

$$\alpha\mathcal{V}_{a_1}(\bar{a}_1;\theta_1) + (1-\alpha)\mathcal{V}_{a_1}(\bar{a}_1;\theta_k) \geq \sum_{i=1}^{k} q_i\mathcal{V}_{a_1}(\bar{a}_1;\theta_i).$$

Note also that, for all $a_1 \geq 0$, $\Upsilon_\alpha(a_1) \leq \Upsilon_{\mathrm{SEU}}(a_1)$ by virtue of ambiguity aversion. That is,

$$\alpha\mathcal{V}(a_1;\theta_1) + (1-\alpha)\mathcal{V}(a_1;\theta_k) \leq \sum_{i=1}^{k} q_i\mathcal{V}(a_1;\theta_i),$$

which, upon rearranging, yields

$$\begin{aligned}(\alpha - q_1)\mathcal{V}(a_1;\theta_1) &\leq \sum_{i=2}^{k-1} q_i\mathcal{V}(a_1;\theta_i) + (\alpha + q_k - 1)\mathcal{V}(a_1;\theta_k) \\ &\leq \Big(\alpha + \sum_{i=2}^{k} q_i - 1\Big)\mathcal{V}(a_1;\theta_k) = (\alpha - q_1)\mathcal{V}(a_1;\theta_k)\end{aligned}$$

since $(\mathcal{V}(a_1;\theta_i))_i$ is non-decreasing and $\sum_i q_i = 1$. Since $\mathcal{V}(a_1;\theta_k) \geq \mathcal{V}(a_1;\theta_1) > 0$, that $\alpha \geq q_1$ is a sufficient condition for $\Upsilon_\alpha(a_1) \leq \Upsilon_{\mathrm{SEU}}(a_1)$ to hold. With this, $\hat{a}_1^\alpha \geq \bar{a}_1$, i.f.f.

$$(\alpha - q_1)\mathcal{V}_{a_1}(\bar{a}_1;\theta_1) \geq \sum_{i=2}^{k-1} q_i\mathcal{V}_{a_1}(\bar{a}_1;\theta_i) + (\alpha + q_k - 1)\mathcal{V}_{a_1}(\bar{a}_1;\theta_k).$$

Note finally that it is sufficient for this to hold that $(\mathcal{V}_{a_1}(\bar{a}_1;\theta_i))_i$ be non-increasing in i since one would get

$$\sum_{i=2}^{k-1} q_i\mathcal{V}_{a_1}(\bar{a}_1;\theta_i) + (\alpha + q_k - 1)\mathcal{V}_{a_1}(\bar{a}_1;\theta_k) \geq \Big(\alpha + \sum_{i=2}^{k} q_i - 1\Big)\mathcal{V}_{a_1}(\bar{a}_1;\theta_2) = (\alpha - q_1)\mathcal{V}_{a_1}(\bar{a}_1;\theta_2),$$

which concludes the proof. $\square$

When $\alpha = 1$, note that an increase in the cardinality of Θ (say, from $|\Theta|$ to $|\Theta'|$), i.e., an increase in the ambiguity level, also corresponds to an increase in the degree of ambiguity aversion ($\min_{\theta\in\Theta'}\mathcal{V}(\cdot;\theta) \leq \min_{\theta\in\Theta}\mathcal{V}(\cdot;\theta)$ provided that $|\Theta'| \geq |\Theta|$), so that beliefs and tastes are not disentangled (here, due to the min operator). By linearity of the objective function, Proposition B.1 also applies to the ϵ-contamination model of Eichberger & Kelsey (1999,[24]), which corresponds to a convex combination of the SEU model, with weight $0 \leq \epsilon \leq 1$, and the Wald's criterion, with weight $1-\epsilon$. The agent therefore grants a degree of confidence ϵ to the SEU model being the correct one. With probability $1-\epsilon$, the agent recognizes that other criteria are possible, and takes the worst-case scenario to account for this. The reader is also referred to Gierlinger & Gollier (2015,[33]) for the case of multiplier preferences using robust control theory.

C The two effects of ambiguity aversion

With numerical simulations, this section clarifies the decomposition of the two effects of ambiguity aversion, namely, the subjective prior pessimistic distortion $F \to H$ and the ambiguity prudence effect $\mathcal{A}$. As in Figure 2, let there be only two scenarios $\Theta = \{\theta_1 = +5, \theta_2 = -5\}$ and let ϕ displays DAAA. Because there is no long-term effect of abatement decision, it follows that for all $\theta \in \Theta$ and admissible $a_1 \geq 0$, $\mathcal{V}_{a_1}(a_1;\theta) = \langle\tau\rangle + \theta$, where $\langle\tau\rangle = 20$. As compared with Figure 2, this results in having flat $\mathcal{V}_{a_1}$ curves. let also H and $\mathcal{A}$ vary with a_1. Let (q_1, q_2) denote the subjective relevance of the two possible scenarios according to F, i.e., $q_1 = q_2 = \frac{1}{2}$. Then, the two effects of ambiguity aversion are functions of the date-1 abatement level such that, for all $a_1 \geq 0$,

$$H(a_1) = \begin{cases} \hat{q}_1(a_1) = & q_1 \dfrac{\phi'\left(\mathcal{V}(a_1;\theta_1)\right)}{q_1\phi'\left(\mathcal{V}(a_1;\theta_1)\right) + q_2\phi'\left(\mathcal{V}(a_1;\theta_2)\right)} \\ \hat{q}_2(a_1) = & q_2 \dfrac{\phi'\left(\mathcal{V}(a_1;\theta_2)\right)}{q_1\phi'\left(\mathcal{V}(a_1;\theta_1)\right) + q_2\phi'\left(\mathcal{V}(a_1;\theta_2)\right)}, \end{cases} \tag{35}$$

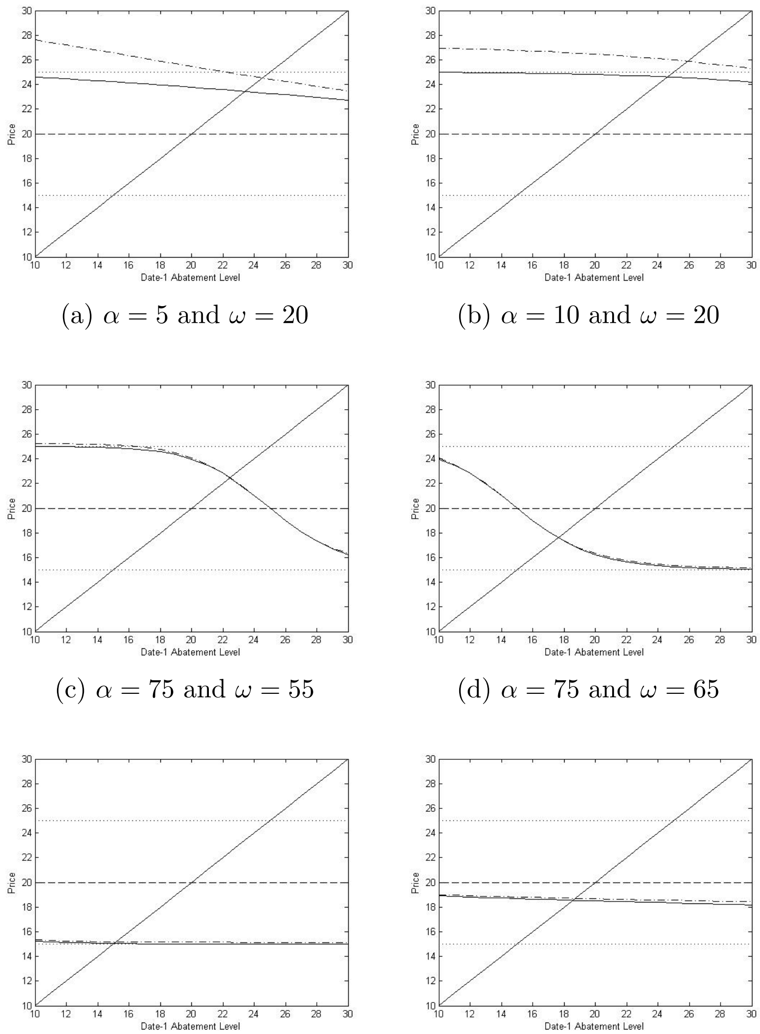
and,

$$\mathcal{A}(a_1) = \frac{q_1\phi'\left(\mathcal{V}(a_1;\theta_1)\right) + q_2\phi'\left(\mathcal{V}(a_1;\theta_2)\right)}{\phi' \circ \phi^{-1}\left(q_1\phi\left(\mathcal{V}(a_1;\theta_1)\right) + q_2\phi\left(\mathcal{V}(a_1;\theta_2)\right)\right)}. \tag{36}$$

With this, the necessary-first order condition for $\hat{a}_1$ writes

$$-C_1'(\hat{a}_1) + \beta\mathcal{A}(\hat{a}_1)\left(\langle\tau\rangle + \hat{q}_1(\hat{a}_1)\theta_1 + \hat{q}_2(\hat{a}_1)\theta_2\right) = 0, \tag{37}$$

and is graphically depicted in Figure 6 for different combinations of α and ω. The slanted solid line is C_1', the two dotted lines represent $\mathcal{V}_{a_1}(a_1;\theta_i)$, the dashed line is $\mathbb{E}_F\{\mathcal{V}_{a_1}(a_1;\tilde{\theta})\}$, the other solid line is $\mathbb{E}_F\{\mathcal{V}_{a_1}(a_1;\tilde{\theta})\}$ and the dash-dotted line is $\mathcal{A}(a_1)\mathbb{E}_F\{\mathcal{V}_{a_1}(a_1;\tilde{\theta})\}$. Figure 6 illustrates that the bulk of the variation in date-1 abatement level under ambiguity aversion



(a) $\alpha = 5$ and $\omega = 20$

(b) $\alpha = 10$ and $\omega = 20$

(c) $\alpha = 75$ and $\omega = 55$

(d) $\alpha = 75$ and $\omega = 65$

(e) $\alpha = 75$ and $\omega = 80$

(f) $\alpha = 5$ and $\omega = 90$

Figure 6: The two effects of ambiguity aversion.

is driven by pessimism and that the relatively weaker ambiguity prudence effect has more influence for lower α (Figures 6a and 6b). Figures 6c and 6d highlight the high sensibility of date-1 abatement around the threshold $\bar{\omega} = 60$ for relatively high α. Figures 6e and 6f compare the prior distortion for different level of ambiguity aversion, indicating that the distortion is more pronounced for higher α. Finally, Figures 6b and 6e underline that when ω is outside of the $[40 - 80]$ band, and for relatively high α, pessimism redistributes almost all the weight to the worst scenario.

D ETS under binary ambiguity

This appendix considers the case of binary price ambiguity, i.e., in all θ-scenarios, $\tilde{\tau}_\theta$ either takes the value $\underline{\tau} > 0$ with probability $0 \leq p(\theta) \leq 1$ or $\bar{\tau}$ with complementary probability, and $\Delta\tau = \bar{\tau} - \underline{\tau} > 0$. W.l.o.g. assume for clarity that abatement cost functions are time separable. Let the underlying objective allowance price lottery be $(p, \underline{\tau}; 1-p, \bar{\tau})$. Then, the no-ambiguity bias requires that $p = \mathbb{E}_F\{p(\tilde{\theta})\}$. The future price estimate of the ambiguity neutral firm is thus $\langle\tilde{\tau}\rangle = p\underline{\tau} + (1-p)\bar{\tau}$. $\Upsilon(\cdot;\theta)$ denotes the θ-scenario expected net intertemporal revenue from date-1 abatement, and satisfies, for all $a_1 \geq 0$ and $\theta \in \Theta$,

$$
\begin{aligned}
\Upsilon(a_1;\theta) &= \zeta_1 - C_1(a_1) + \beta\mathcal{V}(a_1;\theta) \\
&= \zeta - C_1(a_1) - \beta p(\theta)(C_2(a_2^*(\underline{\tau})) + \underline{\tau}(b - a_1 - a_2^*(\underline{\tau}) - \omega)) \\
&\quad - \beta(1-p(\theta))(C_2(a_2^*(\bar{\tau})) + \bar{\tau}(b - a_1 - a_2^*(\bar{\tau}) - \omega)),
\end{aligned}
\tag{38}
$$

where $\zeta = \zeta_1 + \beta\zeta_2$. With quadratic abatement cost functions, it follows that

$$
\Upsilon(a_1;\theta) = \zeta - C_1(a_1) + \beta\Big(p(\theta)\Delta\tau\Big(b - a_1 - \omega - \frac{\langle\tau\rangle}{c_2}\Big) - \bar{\tau}\Big(b - a_1 - \omega - \frac{\bar{\tau}}{2c_2}\Big)\Big), \tag{39}
$$

where $\langle\tau\rangle$ denotes the date-2 average price when $p = \frac{1}{2}$, i.e., $\langle\tau\rangle = \frac{\underline{\tau}+\bar{\tau}}{2}$. Similarly, the θ-scenario expected net marginal revenue from date-1 abatement, evaluated at $a_1 = \bar{a}_1$, writes

$$
\Upsilon_{a_1}(\bar{a}_1;\theta) = -C_1'(\bar{a}_1) + \beta\mathcal{V}_{a_1}(\bar{a}_1;\theta) = -C_1'(\bar{a}_1) + \beta(\bar{\tau} - p(\theta)\Delta\tau), \tag{40}
$$

which is decreasing in θ i.f.f. $p(\theta)$ is increasing in θ and, by optimality under ambiguity neutrality, nil when $p(\theta) = p$. It follows that $\Upsilon_{a_1}(\bar{a}_1;\theta)$ changes sign from positive to negative at $p(\theta) = p$. From (39), one sees that when the liable firm expects to be net buyer of allowances under the abatement stream $\Big(\bar{a}_1; \frac{\langle\tau\rangle}{c_2}\Big)$, $\Upsilon(\bar{a}_1;\theta)$ is relatively high (low) when $p(\theta)$ is relatively big (small). Therefore, for those θ-scenarios satisfying $p(\theta) < p$ where $\Upsilon(\bar{a}_1;\theta)$ is relatively low, $\Upsilon_{a_1}(\bar{a}_1;\theta) > 0$ so that increasing a_1 will increase $\Upsilon(a_1;\theta)$. Conversely, for those θ-scenarios satisfying $p(\theta) > p$ where $\Upsilon(\bar{a}_1;\theta)$ is relatively high, $\Upsilon_{a_1}(\bar{a}_1;\theta) < 0$ so that increasing a_1 will decrease $\Upsilon(a_1;\theta)$. Combining the two cases results in a reduced spread of expected profits across θ-scenarios. This substantiates the intuition behind pessimism (and anticomonotonicity). More formally, let us now state

Proposition D.1. *Let allowance price ambiguity be binary and abatement cost functions be quadratic and time separable. Assuming ambiguity prudence, the prevalence of ambiguity*

aversion increases date-1 abatement relative to ambiguity neutrality
(*i*) *only if the firm expects to be net buyer of allowances under the abatement stream* $(\bar{a}_1; \bar{a}_2)$, *with* $\bar{a}_1 = \frac{\beta\langle\tilde{\tau}\rangle}{c_1}$ *and* $\bar{a}_2 = \frac{\langle\tau\rangle}{c_2}$*; or equivalently,*
(*ii*) *only if* ω *is below the threshold* $\bar{\omega} = b - \bar{a}_1 - \bar{a}_2$*; or equivalently,*
(*iii*) *only if* p *is above the threshold* $\bar{p} = \frac{1}{\beta c_2 \Delta\tau}\left(\beta c_2 \overline{\tau} + c_1 \langle\tau\rangle - c_1 c_2 (b-\omega)\right) \in [0;1]$.

Proof. Again, the proof consists in signing the covariance. By differentiation w.r.t. θ, one has that, $\partial_\theta \mathcal{V}_{a_1}(\bar{a}_1;\theta) = -p'(\theta)\Delta\tau$, $\forall\theta \in \Theta$. Similarly, using (39),

$$\partial_\theta \mathcal{V}(\bar{a}_1;\theta) = p'(\theta)\Delta\tau\Big(b - \bar{a}_1 - \omega - \frac{\langle\tau\rangle}{c_2}\Big) = p'(\theta)\Delta\tau\Big(b - \omega - \frac{\beta\langle\tilde{\tau}\rangle}{c_1} - \frac{\langle\tau\rangle}{c_2}\Big).$$

Therefore, anticomonotonicity holds i.f.f. $b - \omega - \frac{\beta\langle\tilde{\tau}\rangle}{c_1} - \frac{\langle\tau\rangle}{c_2} > 0$, i.e., the firm is a net buyer of allowances when it abates $(\bar{a}_1; \bar{a}_2)$. Note that by definition, $\langle\tilde{\tau}\rangle = \overline{\tau} - p\Delta\tau$, which is decreasing with p. Anticomonotonicity thus holds i.f.f.

$$2\beta c_2\left(\overline{\tau} - p\Delta\tau\right) + c_1\left(\overline{\tau} + \underline{\tau}\right) < 2c_1c_2\left(b - \omega\right), \tag{41}$$

that is, i.f.f., $p > \bar{p}$. For $\bar{p}$ to be admissible, one needs[56] $\beta\underline{\tau} \leq c\left(b-\omega\right) \leq \beta\overline{\tau}$. □

Initial allocation continues to dictate how the optimal date-1 abatement decision under ambiguity aversion compares with that under ambiguity neutrality − but the condition for signing pessimism, i.e., for anticomonotonicity to hold is milder[57] as compared with Proposition 3.8. The ambiguity averse firm must be net short under only one given abatement stream $(\bar{a}_1; \bar{a}_2)$ − not across all θ-scenarios. This can be likened to a situation where the firm has no idea about the future allowance price at all and thus considers the equiprobable price scenario − under ambiguity neutrality, the liable firm is not affected by ambiguity. This also translates into an upper-threshold (lower-threshold) condition on initial allocation (p). An explicit $\bar{p}$-threshold allows us to characterize the effects of an increase in the ambiguity level, here proxied by the price range $\Delta\tau$, for given degree of ambiguity aversion. To do so, we analyse how $\bar{p}$ varies consecutive to an increase in $\Delta\tau$.
In response to an infinitesimal positive shift in $\Delta\tau$ from $\Delta\tau$ to $\Delta\tau + \delta\tau$, δp denotes the shift in $\bar{p}$ around equilibrium (41) in the two polar cases where $\bar{\tau}$ increases by $\delta\tau$ with $\underline{\tau}$ fixed, or

[56]When the date-2 allowance price is $c\,(b-\omega)$, the overall abatement effort $b-\omega$ has been optimally apportioned between the two dates, i.e., in proportion to the flexibility in abatement at the two dates. With this in mind, it makes sense to have a possible price range such that $\beta\underline{\tau} < c\,(c-\omega) < \beta\bar{\tau}$.

[57]With a similar binary structure, Alary et al. (2013, [1]) and Wong (2015a,[79]) show that anticomonotonicity is always satisfied so that the impact of ambiguity aversion is clear.

symmetrically, where $\underline{\tau}$ decreases by $\delta\tau$ with $\bar{\tau}$ fixed. For an upward shift in $\Delta\tau$, $\bar{p}$ reacts such that

$$2\beta c_2\left(\delta\tau - \bar{p}\delta\tau - \bar{\tau}\delta p - \delta p\delta\tau + \underline{\tau}\delta p\right) + c_1\delta\tau = 0, \quad \text{i.e.,} \quad R^{\uparrow} = \frac{\delta p}{\delta\tau} = \frac{2\beta c_2(1-\bar{p}) + c_1}{2\beta c_2\Delta\tau} > 0, \tag{42}$$

where $\delta p\delta\tau \simeq 0$ in the first order and $R^{\uparrow}$ denotes the rate of increase in $\bar{p}$ consecutive to an increase in $\bar{\tau}$ by $\delta\tau$. Similarly, for a downward shift in $\Delta\tau$, $\bar{p}$ reacts such that

$$2\beta c_2\left(\bar{p}\delta\tau + \bar{\tau}\delta p + \delta p\delta\tau - \underline{\tau}\delta p\right) + c_1\delta\tau = 0, \quad \text{i.e.,} \quad R^{\downarrow} = -\frac{\delta p}{\delta\tau} = \frac{2\beta c_2\bar{p} + c_1}{2\beta c_2\Delta\tau} > 0, \tag{43}$$

where $\delta p\delta\tau \simeq 0$ again and $R^{\downarrow}$ denotes the rate of decrease in $\bar{p}$ consecutive to a decrease in $\underline{\tau}$ by $\delta\tau$, in absolute terms. It follows that

$$R^{\uparrow} - R^{\downarrow} = \frac{1-2\bar{p}}{\Delta\tau} > 0 \text{ i.f.f. } \bar{p} < \frac{1}{2}. \tag{44}$$

For a symmetric increase in $\Delta\tau$, for which $\langle\tau\rangle$ is unchanged, from $\Delta\tau$ to $\Delta\tau+2\delta\tau$, $\langle\tilde{\tau}\rangle_{\Delta\tau+2\delta\tau} = \langle\tilde{\tau}\rangle_{\Delta\tau} + \delta\tau(1-2p)$ so that $\langle\tilde{\tau}\rangle_{\Delta\tau+2\delta\tau} \geq \langle\tilde{\tau}\rangle_{\Delta\tau}$ i.f.f. $p \leq \frac{1}{2}$ i.f.f. $\langle\tilde{\tau}\rangle_{\Delta\tau} \leq \langle\tau\rangle$. An increase in the ambiguity range hence always brings $\langle\tilde{\tau}\rangle$ closer to $\langle\tau\rangle$, which is the central price scenario in determining how ambiguity aversion adjusts date-1 abatement. An increase in $\Delta\tau$ always brings $\bar{p}$ closer to $\frac{1}{2}$, which is in line with a precautionary principle. More precisely,

- when $\bar{p} > \frac{1}{2}$, the firm abates more at date 1 under ambiguity aversion than under neutrality only if $p \geq \bar{p} > \frac{1}{2}$, that is, only if $\langle\tilde{\tau}\rangle$ is below $\langle\tau\rangle$: it foresees a price below that under the $\langle\tau\rangle$-scenario (and does not abate enough relative to this scenario) and ambiguity aversion corrects this by increasing a_1. In increasing $\Delta\tau$ symmetrically, both $\bar{p}$ and $\langle\tilde{\tau}\rangle$ decrease overall, which renders the criterion for ambiguity aversion to increase date-1 abatement relative to neutrality laxer;

- when $\bar{p} < \frac{1}{2}$, the firm abates more at date 1 under ambiguity aversion than under neutrality even in the case where $p \in \left[\bar{p}; \frac{1}{2}\right]$ so that $\langle\tilde{\tau}\rangle > \langle\tau\rangle$ and the ambiguity-neutral firm abates more at date 1 than under the $\langle\tau\rangle$-scenario. In increasing $\Delta\tau$ symmetrically, both $\bar{p}$ and $\langle\tilde{\tau}\rangle$ increase overall, which renders the criterion for ambiguity aversion to increase date-1 abatement relative to neutrality stricter.

In other words, when the condition for ambiguity aversion to raise date-1 abatement relative to neutrality is relatively demanding (lax), an increase in the ambiguity range $\Delta\tau$ makes it laxer (more demanding), which is in line with a precautionary principle.

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