# Does rivalry in R&D foster harmful innovations?

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**Abstract :**

In this paper, we consider that several firms are engaged in an innovation race and we determine the conditions under which an increase in rivalry leads them to hasten the R&D process at the risk of introducing a harmful innovation. We provide several methods of measurement for the degree of competition and we show that whenever there is some balance between the cost and the benefit induced by safety, then rivalry makes firms more inclined to overlook preventive/corrective actions and to break the sequentiality of R&D.

## 1 Introduction

In innovation races, rival firms compete to be the first to introduce a new product. Being first matters in the sense that the winner generally takes a larger share of the innovation's private value than the losers. Existing literature (e.g. Scherer, 1967; Loury, 1979; Lee & Wilde, 1980; Reinganum, 1981; Fudenberg & *al.*, 1983; Grossman & Shapiro, 1985) studies the relationship between the degree of rivalry and the amount of resources competitors devote to R&D. So as to keep models tractable, the characteristics of the innovation are generally assumed to be exogenous and known *ex ante*. Hence, firms are denied the ability to shape the innovation that they introduce and their sole responsability is to invest in R&D to make that process successful. Such framework is convenient in the sense that it allows R&D to be seen as a one-dimensional activity. However, critical aspects of innovations such as their quality, their impact on public health or the environment might actually result from decisions that are intentionally made by the firms at the time they are engaged in R&D.

In this paper, we depart from the traditional patent race framework by assuming that the innovation generates a damage whose magnitude depends on the winner's R&D strategy.[1] We further assume that the greater that damage, the smaller the private value of the innovation. This latter assumption can be justified as follows: first, the innovator might be directly and at least partially impacted by the damage. Also, consumers might be less willing to pay for a new product or process that induces a damage so its introduction is less profitable. Besides, when the public agency believes that an innovation is harmful, he might deny its introduction on the market *ex ante*, withdraw it *ex post* or impose financial penalties on the innovator. Finally, when a new product or process induces a damage, it might be vulnerable to future improvements that would render the initial innovation obsolete.

We shall successively examine three sets of R&D strategies (i.e. three types of actions) that can influence the size of the damage in order to determine whether our conclusions rely on a specific approach.

In the first set of R%D strategies, firms decide whether or not to devote a fraction of their - predetermined - per-period R&D resources to undertaking actions aimed at preventing the damage from arising. Such preventive actions may include the implementation of demanding safety protocols or the employment of a highly-skilled researchers. Firms have an interest in undertaking preventive actions because it enhances the profitability of introducing an innovation. However, taking such actions is expected to slow R&D down so it increases the cost of that

[1] Note that the innovation is assumed to fall under the scope of a single industrial property title whatever the size of the damage it generates.

process and it exposes firms to a higher risk of being defeated by one of their rivals.

In the second set of R&D strategies, firms choose whether or not to undertake corrective actions once they realize that they have made mistakes. Again, taking such actions is appealing in the sense that it might lead the firms to introduce an innovation whose profitability is greater. However, the implementation of corrective actions requires time, resources and exposes the firms to the risk of being defeated by one of their rivals. Also, such actions might transmit information to late competitors and strengthen rivalry.

Finally, in the third set of strategies, firms decide whether or not to overlap several of the steps R&D is made of. Overlapping steps is appealing in the sense that it saves time and it softens the burden of decreasing returns to scale. Yet, as noted by Scherer (1969), when firms break the sequentiality of R&D, poor decisions might be made because there are based of an insufficient knowledge. Overlapping steps may thus result in a wide range of consequences, among which the emergence of harmful innovations.

The main question we are seeking to answer is whether or not rivalry makes firms more inclined to implement unsafe R&D strategies that foster the emergence of harmful innovations.[2] From a methodological point of view, we must provide a suitable measurement for the degree of rivalry. In this paper, we mainly exploit the number of firms engaged in the race to get an idea of how intense rivalry is (see e.g Loury, 1979; Lee & Wilde, 1980). However, when studying the first set of R&D strategies, we shall consider alternative measurement methods in order to determine if our conclusions are driven by the way we interpret rivalry. In particular, if firms enter the race one after the other, then it becomes possible to measure the degree of rivalry through the size of the head start early entrants benefit from (see

[2] Note that we do not discuss the social optimality of firms' decisions.

e.g. Fudenberg & *al.*, 1983; Kamien & Schwartz, 1972). Moreover, when R&D is understood as a multi-stage process, we can measure the degree of rivalry through competitors' relative progress (see e.g. Grossman & Shapiro, 1985). Finally, if we introduce some flexibility in the division of the innovation's private value among competing firms, it then becomes possible to measure the degree of rivalry through the importance of the competitive threat (see e.g. Beath & *al.*, 1989).

When firms choose whether or not to undertake preventive actions, we show that the equilibrium R&D strategies are not affected by the degree of rivalry whenever implementing such actions is either very rewarding or, at the opposite, not quite worth it. In the former case, firms always undertake preventive actions and they never do in the latter. Only when there is some form of balance between the extra profitability associated with the introduction of a damage-free innovation and the extra cost and risk induced by a reduction in speed of R&D will the degree of rivalry be a determining factor. In that case, the more intense rivalry, the more difficult it is to sustain an equilibrium in which preventive actions are taken. We reach similar conclusions in the case for which firms decide whether or not to overlap several of the steps R&D is made of. Finally, in the case for which firms choose whether or not to undertake corrective actions once they realize they have made mistakes, we show that rivalry must be sufficiently weak for firms to find it optimal to take such actions.

Our conclusions support the idea, originally put forward by Gilbert & Shapiro (1990), that patents should be as narrow as possible. Indeed, the wider the patent, the more numerous the firms competing for the same industrial property title. Besides, our findings indicate that the public agency should not subsidy competing firms in the sense that feeding the rivalry could foster the emergence of harmful innovations. Also, our work mitigates the appeal of contests that deliberately create rivalry. Finally, it stresses the importance of rewarding safe R&D strategies in order to make firms immune to rivalry.

This paper is organized as follows: Section 2 explores the first set of R&D strategies in which firms decide whether or not to undertake actions aimed at preventing the damage from arising. In Section 3, we examine the second set of R&D strategies in which firms may undertake corrective actions once they realize that they have made mistakes. In Section 4, we analyze the third set of R&D strategies in which firms are given the opportunity to overlap several of the steps R&D is made of. Section 5 concludes. Section 6 is a short mathematical appendix.

## 2 Preventive actions

In this section, we explore the first set of R&D strategies in which rational, profit-maximizing and risk-neutral firms decide, at the time they are engaged in R&D, whether or not to undertake actions aimed at preventing the damage from arising. Let $x_i \in \{0, 1\}$ be firm $i$'s R&D strategy and $x_{-i}$ her competitors'. We set $x_i = 0$ when she does not undertake preventive actions and $x_i = 1$ when she does.[3] In the former case, her R&D strategy is said to be *unsafe* whereas it is said to be *safe* in the latter.

First, we measure the degree of rivalry through the number of competitors and we establish our main Proposition. Second, we use alternative measurements for the degree of rivalry in order to determine whether our previous conclusions hold. In particular, we shall use the importance of the competitive threat, the size of the head start early entrants benefit from, and competitors' relative progress within a multi-stage innovation race.

[3] We have chosen to focus on a discrete strategy set as for keeping the analysis simple.

## 2.1 The number of competitors

We consider $n \in \mathbb{N}$ symmetric firms who simultaneously enter a single-stage *winner-take-all* innovation race. Remaining in the race induces a per-period R&D cost equal to $c > 0$. This cost is assumed to be small enough so firms do not find it optimal to abandon R&D. Future amounts are discounted at rate $r \in ]0, 1[$. The time $\tau_i$ at which firm $i$ achieves innovation is a random variable that is assumed to follow an exponential distribution of parameter (*hazard rate*) $h(x_i) > 0$ so $\Pr(\tau_i = t) = h(x_i)e^{-h(x_i)t}$[4] and $\Pr(\tau_i \leq t) = 1 - e^{-h(x_i)t}$. The *hazard rate* indicates the instantaneous probability of achieving innovation (conditional upon previous failure). Note that R&D is memoryless under this specification. Let $\tau_{-i} = \min_{j\neq i} \{\tau_j\}$ the random variable that describes the time at which firm $i$ is defeated by one of her rival. $\tau_{-i}$ follows an exponential distribution of parameter $\sum_{j\neq i} h(x_j)$ so $\Pr(\tau_{-i} \leq t) = 1 - e^{-\sum_{j\neq i} h(x_j)t}$. We assume that $h(0) = h$ and $h(1) = ah$ with $a \in ]0, 1[$. Hence, undertaking preventive actions is costly in the sense that it diverts a fraction of the firm's resources from their primary purpose (i.e. achieving innovation) so it decreases the speed of R&D. In that case, the firm is more likely to be defeated by a rival and she incurs higher R&D costs.[5] Let $V(x_i)$ be the innovation's private value when firm $i$ has adopted the R&D strategy $x_i$ and wins the race. We assume that $V(0) = v > 0$ and that $V(1) = v(1 + b)$ where $b > 0$ is the *prize gap*. Hence, undertaking preventive actions is rewarding because it reduces the size of the damage and it makes - for the reasons outlined earlier - the introduction of the new product or process more profitable. When firm $i$ enters the race, her expected benefit is equal to

$$B(x_i, x_{-i}) = \int_0^{+\infty} \Pr(\tau_i = t) \Pr(\tau_{-i} > t)e^{-rt}V(x_i)dt$$

[4] Pay attention to the fact that this expression is actually a probability density.

[5] Other consequences may include the depletion of the firm's resources before R&D succeeds or the introduction of an innovation once it has already become obsolete.

and her expected cost is equal to

$$C(x_i, x_{-i}) = \int_0^{+\infty} \Pr(\tau_i = t \text{ or } \tau_{-i} = t) \left\{ \int_0^t ce^{-rs} ds \right\} dt$$

Her expected profit $\Pi(x_i, x_{-i})$ is simply the difference between the expected benefit and cost. We study the symmetric game in which the $n$ firms simultaneously decide whether or not to undertake preventive actions as for maximizing their expected profit. Information is complete but imperfect. We search for pure-strategy Nash Equilibria (NE) such that all firms implement the same strategy. It is straightforward to show that all firms choosing not to undertake preventive actions is a NE whenever

$$b \leq b_0 = \frac{(1-a)(hv(n-1) + c + vr)}{av(r + nh)}$$

Likewise, all firms undertaking preventive actions if a NE so long as

$$b \geq b_1 = \frac{(1-a)(ahv(n-1) + vr + c)}{av(r + h + ah(n-1))}$$

Since, $b_0 \geq b_1$ the game has a single NE whenever $b \leq b_1$ or $b \geq b_0$. If $b \leq b_1$, then firms do not undertake preventive actions. If $b \geq b_0$, however, then all firms undertake those actions. If $b \in [b_1, b_0]$, then there are two symmetric NE. However, we show that the NE in which all firms undertake preventive actions Pareto dominates that in which they do not. We thus assume that firms coordinate on the Pareto dominant NE Therefore, the NE is such that all firms undertake preventive actions when $b \geq b_1$ and none of them take such actions otherwise. $b_1$ is continuous and increasing with $n$. When there is a single firm engaged in R&D (i.e. when $n = 1$), then $b_1$ is equal to $\underline{b} = \frac{(c+vr)(1-a)}{va(r+h)}$ and $b_1$ tends to $\overline{b} = \frac{1-a}{a}$ as the number of competitors tends to $+\infty$. Thus, if $b \leq \underline{b}$ or if $b \geq \overline{b}$, the number of firms engaged on the race has no impact on the equilibrium. When $b \in \left[\underline{b}, \overline{b}\right]$, however, firms undertake preventive actions so long as

$$n \leq \widehat{n} = 1 + \frac{a(vb(r+h) + vr + c) - c - vr}{ahv(1 - a(1+b))}$$

and they do not otherwise.

**Proposition 1** *If there is a balance between the prize gap and the hazard gap, then rivalry must be sufficiently weak for all firms undertaking preventive actions to be sustained as an equilibrium.*

Hence, the degree of rivalry does not affect the equilibrium R&D strategies whenever undertaking preventive actions is very rewarding or, at the opposite, not quite worth it. In the former case, competitors always undertake preventive actions whereas they never do in the latter. Only when the benefit of a larger value and the cost of undermined chances of success are similar will the degree of rivalry be a key determinant of the equilibrium R&D. In that case, rivalry does foster the emergence of harmful innovations.

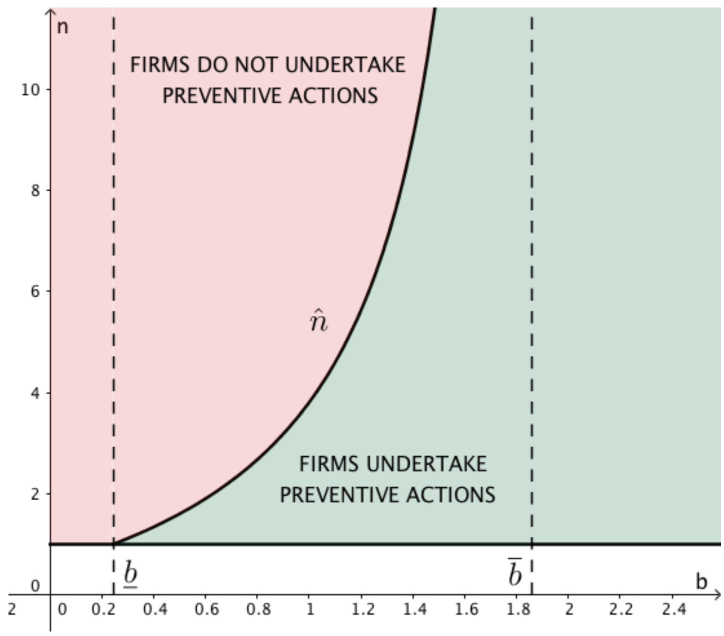


Figure 1: Equilibrium R&D strategies when rivalry is measured through the number of competitors

As depicted in Figure 1, if the prize gap is sufficiently large (i.e. if $b \geq \overline{b}$), then all firms undertake preventive actions whatever the degree of rivalry. In that case, indeed, surrendering a (much) greater prize in order to increase chances of winning the race can never constitute an equilibrium. At the opposite, if the prize gap is small enough (i.e. if $b \leq \underline{b}$), then implementing a safe R&D strategy is not worth it so none of them undertake preventive actions whatever the degree of rivalry. When the equilibrium R&D strategies are affected by the number of competitors engaged in the race (i.e. if $b \in \left[\underline{b}, \overline{b}\right]$), we observe that the more intense rivalry, the more difficult it is to sustain an equilibrium in which all firms undertake preventive actions. Therefore, the entry of a new rival within the race - and thus the intensification of rivalry - might induce all incumbent firms to adapt their R&D strategy and to stop undertaking preventive actions.[6] The argument is reversible if one firm drops out of the race. We obtain this result because the more numerous the competitors, the more appealing it is for them to trade a smaller chance to get a larger prize against a larger chance to get a smaller prize. Indeed, the weaker rivalry, the more costly it is to surrender the larger prize because each firm expects to win the race with high probability. In particular, only when rivalry is sufficiently strong may that cost be offset by the benefit associated with greater chances of success and an increase in the speed of R&D.

Besides, we note that the larger the prize gap or the smaller the *hazard* gap (i.e. the larger $a$, $b$ or $v$ or the smaller $h$), the greater $\widehat{n}$, the rivalry threshold above which firms do not undertake preventive actions. Indeed, if the prize gap is large or if the hazard gap is small, implementing a safe R&D strategy is rewarding and it does not significantly slow R&D down so rivalry must be very intense to deter firms from undertaking preventive actions. At the opposite, the larger the discount rate or the per-period cost of R&D (i.e. the larger $r$ or $c$) the smaller $\widehat{n}$. When those parameters take large values, implementing an unsafe R&D strategy is appealing because it is expected to accelerate the introduction of the innovation so firms

[6] If firms were allowed to adapt their R&D strategy at the time an entry occurs.

incur R&D costs for a shorter period of time and they introduce an innovation whose discounted value is larger given that they win the race.

Now that we have described the influence of competition on firms' R&D strategies when using a specific measurement for the degree or rivalry, we turn to investigating whether or not our main result holds for a alternative measuring methods.

## 2.2 Alternative measurements

### 2.2.1 The importance of the competitive threat

In *winner-take-all races*, the winner collects the whole innovation's private value. Because of its tractability, this class of races is often adopted in the economic literature (see e.g. Loury, 1979; Lee & Wilde, 1980; Grossman & Shapiro, 1985; etc.). Yet, losers can get a slice of the pie for at least two reasons: first, they might be able to imitate the innovation; second, they might benefit from the winner's knowledge and be at the origin of a patentable improvement of the initial innovation. In this section, we relax the assumption according to which the race is *winner-take-all* and we measure the degree of rivalry through the share of the innovation's private value that goes to the winner. We borrow the term *competitive threat* from Beath & *al.* (1989) to characterize the difference between the winner's and the losers' prizes. The larger this difference, the bigger the competitive threat so the more intense rivalry. Indeed, when the winner takes the lion's share, the incentives to be first are strong so firms have an interest in throwing all their forces into battle. At the opposite, if the innovation's private value is divided rather equally between the winner and the losers, then being defeated is not such a big deal so firms may not want to compete fiercely.

For simplicity, we address the case in which there are only two firms (i.e. $n = 2$) engaged in a single-stage innovation race and we define $\theta$ as the share of the innovation's private value that goes to the winner. We assume that it is larger than one half (i.e. $\theta \in \left[\frac{1}{2}, 1\right]$) so the winner collects a larger share than the loser. The rest of the model is unchanged. To simplify the expressions, let $A_i$ be the event *"Firm i wins the race"* and let $\overline{A_i}$ be the event *"Firm i loses the race"*. We thus have $\Pr(A_i = t) = \Pr(\tau_i = t)\Pr(\tau_{-i} > t)$ and $\Pr(\overline{A_i} = t) = \Pr(\tau_{-i} = t)\Pr(\tau_i > t)$. Firm $i$'s expected benefit is equal to

$$B(x_i, x_{-i}) = \int_0^{+\infty} \Pr(A_i = t) e^{-rt} \theta V(x_i) dt + \int_0^{+\infty} \Pr(\overline{A_i} = t) e^{-rt} (1-\theta) V(x_{-i}) dt$$

The left-hand side of that expression is her expected benefit when she wins the race and the right-hand side is firm $i$'s expected benefit when she is defeated by her rival. In the previous section, $\theta = 1$ so this latter part was equal to zero. Her expected cost is identical to that previously described. Again, firms are symmetric and they simultaneously choose whether or not to undertake preventive actions. The game has the same informational properties as the one introduced in the previous section. We search for symmetric pure-strategy NE. Both firms refraining from undertaking preventive actions is a NE whenever

$$b \leq b_0' = \frac{(hv(2\theta - 1) + c + \theta vr)(1-a)}{v\theta a(r + 2h)}$$

At the opposite, both firms undertaking such actions is a NE whenever

$$b \geq b_1' = \frac{(1-a)(ahv(2\theta - 1) + \theta vr + c)}{av(ah(2\theta - 1) + h + \theta r)}$$

Since $b_1' \leq b_0'$ and because the NE in which both firms undertake the preventive actions Pareto dominates that in which they do not, we conclude that the game admits a single symmetric NE such that both firms undertake preventive actions when $b \geq b_1'$ and they do not otherwise. We note that $b_1'$ is continuous and increasing with $\theta$. When the innovation's private value is equally divided among the winner and the loser (i.e. when $\theta = \frac{1}{2}$), then $b_1'$ is equal to $\underline{b}' = \frac{(vr+2c)(1-a)}{va(r+2h)}$. When the winner collects the whole prize (i.e. when $\theta = 1$) then $b_1'$ is equal to

$\overline{b}' = \frac{(1-a)(ahv+vr+c)}{av(r+ah+h)}$. Therefore, if $b \leq \underline{b}'$ or if $b \geq \overline{b}'$, the competitive threat has no impact on the equilibrium. When $b \in \left[\underline{b}', \overline{b}'\right]$, however, firms undertake preventive actions if

$$\theta \leq \widehat{\theta} = \frac{(1-a)(ahv(1+b)-c)}{(r+2ah)v(1-a(1+b))}$$

and they do not otherwise. The equilibrium is depicted in Figure 2.

**Lemma 2** *Proposition 1 holds when the degree of rivalry is measured through the importance of the competitive threat rather than the number of competitors.*

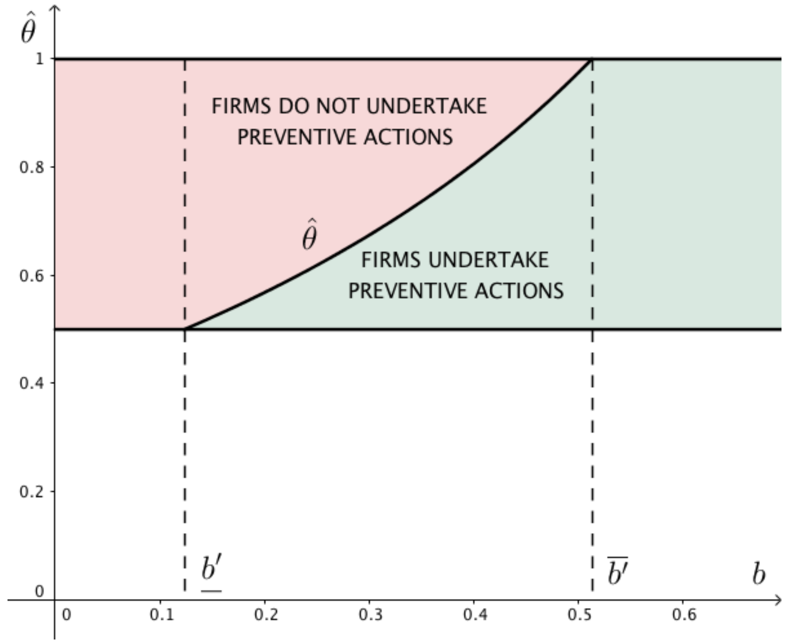


Figure 2: Equilibrium R&D strategies when rivalry is measured through the importance of the competitive threat

The comments that we made about Proposition 1 can be transposed into the current framework. In particular, if the degree of rivalry affects the equilibrium, then the larger the share of the innovation's private value that goes to the winner, the more difficult it is to sustain an equilibrium in which both firms undertake preventive actions. Indeed, if that value is shared rather equally between the winner and the loser (i.e. if the competitive threat is weak), then there is little incentive to achieve innovation first. In other words, the benefit associated with the implementation of an unsafe R&D strategy (i.e. achieving innovation sooner) is low and is likely to be offset by the cost of surrendering the larger prize. However, if the winner gets the lion's share (i.e. if the competitive threat is strong), then saving time becomes a crucial matter for both competitors and surrendering the larger prize in exchange of an increase in the speed of R&D proves to be optimal.

### 2.2.2 The size of the head start early entrants benefit from

So far, we assumed that firms entered the race at once. In practice, however, some firms act as precursors and open new lines of R&D while others jump on the bandwagon. In order to take into account this feature, we follow Fudenberg & *al.* (1983) and we study an alternative framework in which two firms (i.e. $n = 2$) enter a single-stage *winner-take-all* innovation race (i.e. $\theta = 1$) one after the other. In particular, a precursor firm acts as an *incumbent* ($I$) and starts R&D at time 0. At that moment, he decides whether or not to undertake preventive actions. At first, he does not face any rivalry. Yet, if he has not achieved innovation by time $T > 0$, an *entrant* firm ($E$) joins the race. At that moment, the *entrant* observes the R&D strategy implemented by the *incumbent* and she chooses her own (once and for all). The *incumbent* is unable to adjust his R&D strategy when the *entrant* joins the race (i.e. the *incumbent*'s R&D strategy is determined once and for all at time 0). From time $T$ on, both firms compete to be the first to introduce the innovation. In this framework, we follow Kamien & Schwartz (1972) and we capture the degree of rivalry through $T$, the *incumbent*'s *head start*, that is, the length of time during which he does not face any competition. The smaller the head start, the more intense rivalry. To simplify, we no longer assume that firms incur a per-period cost. Instead, we follow Loury (1979) and we suppose that R&D induces a fixed cost that is paid up front and which is small enough so engaging in R&D is always optimal.[7] The rest of the model is unchanged.

At time 0, the *incumbent*'s payoff $\Pi_I(x_I, x_E)$ is such that

$$\Pi_I(x_I, x_E) = \int_0^T \Pr(\tau_I = t)e^{-rt}V(x_I)dt + \int_T^{+\infty} \Pr(\tau_I = t)\Pr(\tau_E > t)e^{-rt}V(x_I)dt$$

with $\Pr(\tau_E > t) = e^{-h(x_E)(t-T)}$ since the *entrant* joins the race at time $T$. The left-hand side of this expression is the *incumbent*'s payoff when he achieves innovation before the *entrant* joins the race and the right-hand side is his payoff when he is the first to achieve innovation at the outcome of a race with the *entrant*.

At time $T$, the *entrant*'s payoff $\Pi_E(x_I, x_E)$ is such that

$$\Pi_E(x_I, x_E) = \int_0^{+\infty} \Pr(\tau_E = t)\Pr(\tau_I > t)e^{-rt}V(x_E)dt$$

Since R&D is memoryless, the *incumbent*'s head start is worthless once the *entrant* joins the race.[8]

In this dynamic game, the information is complete and perfect. We thus search for SPNE. Hence, we start by solving the subgames that start at time $T$. Next, we discuss the *incumbent*'s behaviour at time 0.

If the *incumbent* has not achieved innovation by time $T$, then the *entrant* joins the race. She observes the *incumbent*'s R&D strategy $x_I$ and she decides whether or not to undertake preventive actions in order to maximize $\Pi_E(x_E, x_I)$. Two subgames must be analyzed depending on the R&D strategy that the *incumbent*

[7] Since R&D costs are neutral in the analysis, we do not make them appear. Therefore, the amounts we indicate are net payoffs.

[8] If the *hazard rate* was determined by the stock of knowledge accumulated by the firm as time goes by (see e.g. Fundenberg & *al.*, 1983), then, for a given size of the head start, the *incumbent* would benefit from a stronger competitive advantage compared to the case in which R&D is memoryless.

has implemented at time 0. If he has not undertaken preventive actions, it is straightforward to show that the *entrant* undertakes preventive actions whenever

$$b \geq b_1'' = \frac{r+h}{r+2h}\frac{1-a}{a}$$

Likewise, if the *incumbent* has undertaken preventive actions, then so the *entrant* adopts the same R&D strategy whenever

$$b \geq b_0'' = \frac{r+ah}{r+h+ah}\frac{1-a}{a}$$

Since $b_1'' \geq b_0''$, the *entrant* always undertakes preventive actions if $b \geq b_1''$ and he never does not if $b \leq b_0''$. When $b \in [b_0'', b_1'']$, the *entrant* undertakes preventive actions if and only if the *incumbent* has undertaken such actions himself.

Now that we have described the *entrant*'s behaviour at time $T$, we turn to computing the equilibrium R&D strategy adopted by the *incumbent* at time 0. We show that if $b \leq \widehat{b} = \frac{r}{r+h}\frac{1-a}{a}$, then the *incumbent* never undertakes preventive actions. At the opposite, if $b \geq b_0''$, then the *incumbent* undertakes such actions whatever the degree of rivalry. Finally, if $b \in \left[\widehat{b}, b_0''\right]$, then we find that the *incumbent* undertakes preventive actions whenever the head start he benefits from is large enough.

**Lemma 3** *Proposition 1 holds when the degree of rivalry is measured through the size of the head start early entrants benefit from rather than the number of competitors.*

The proof is in the Appendix. Again, our comments about Proposition 1 can be transposed into the current framework. In particular, when the degree of rivalry affects the *incumbent*'s optimal R&D strategy, then the smaller the size of the head start, the more likely it is that he finds it optimal not to undertake prevention actions. Indeed, if the *incumbent* knows that he will face rivalry in a near future, then he is given strong incentives to achieve innovation before the entry occurs even if it leads to the surrendering of the larger prize. However, if the *incumbent* benefits from a significant head start, then he is given little incentive to rush R&D apart from the fact that the sooner innovation is achieved, the larger its discounted value.

### 2.2.3 Firms' relative progress within a multi-stage innovation race

In most of the literature regarding innovation races, authors assume that there is one single stage to complete for R&D to succeed (e.g. Loury, 1979; Lee & Wilde, 1981; Reinganum, 1981). Such an assumption has the advantage of keeping models tractable but it makes it impossible to capture the notion of *progress* within the race. Grossman & Shapiro (1985) are among the first to study innovation races with multi-stage R&D. In particular, they argue that *"in races other than sprints, strategy plays a critical role. The participants adjust their tactics as the race develops [...]".* In this section, we assume that R&D is made of several steps and capture the degree of rivalry through firms' relative progress within the race: the closer firms get to success and the more neck-to-neck they are, the more intense rivalry. Note that the degree of rivalry is not exogenous as was the case before. Indeed, firms' relative progress is the outcome of the strategies that they have implemented in the past. Therefore, we will not be able to proceed to direct comparisons with the results established earlier

To simplify, we consider the case for which there are only two firms (i.e. $n = 2$) who simultaneously enter a *winner-take-all* innovation race (i.e. $\theta = 1$). Two steps $s \in \{R, D\}$ must be completed one after the other for R&D to succeed. We shall refer to the first step as *Research* ($R$) and to the second as *Development ($D$).* *Research* must be completed before *Development* can be initiated. We use the term *state of the race* to denote firms' relative progress within the race. Therefore, a state of the race is a combination $(s_i, s_{-i})$ describing in which step firm $i$ and her rival are currently engaged. For each possible state of the race, firms simultaneously choose whether or not to undertake preventive actions.[9] In particular, let $x_i(s_i, s_{-i})$ indicate whether or not firm $i$ undertakes preventive actions in state $(s_i, s_{-i})$. As was the case earlier, $x_i(s_i, s_{-i}) = 0$ stands for no preventive actions and $x_i(s_i, s_{-i}) = 1$ means that firm $i$ undertakes such actions. Firm $i$'s R&D strategy $x_i$ is thus a mapping between each possible state of the race and a decision of whether or not to undertake preventive actions.

The first firm who completes *Development* achieves innovation and collects a prize whose value depends on whether or not she has completed each step while undertaking preventive actions. Let $m_i \in \{0, 1, 2\}$ indicate the number of steps that firm $i$ has completed while undertaking preventive actions. For reasons outlined earlier, undertaking preventive actions is rewarding in the sense that it reduces the size of the expected damage generated by the innovation and makes its introduction more profitable. With this in mind, we define $V(m_i)$ as the innovation's private value when firm $i$ wins the race and we assume that $V(0) = v > 0$, $V(1) = v(1+b)$ with $b > 0$[10] and $V(2) = v(1+b)^2$.

As was the case in the previous section, we assume that R&D induces a fixed cost that is paid up front and which is small enough so engaging in R&D is optimal for both firms. Also, we normalize the discount rate to zero.[11] The rest of the model

[9] In practice, switching from undertaking preventive actions to not taking them (or *vice versa*) from one state of the race to another is likely to induce a cost in the sense that it might require a restructuring of labor and capital or necessitate organizational changes. To simplify, we ignore those costs.

[10] Hence, the prize is the same regardless of which stage has been completed with preventive actions.

[11] Introducing a positive discount rate is appealing but it would lead to multiple equilibria and no evident coordination criterion.

is unchanged. In particular, the firm's *hazard* rate is equal to $h > 0$ when she does not undertake preventive actions and it is equal to $ah$ with $a \in ]0,1[$ when she does. To simplify the expressions let $A_i$ be the event *"Firm i takes the race to the next state"* and $\overline{A_i}$ be the event *"Firm i's rival takes the race to the next state"*. Hence, $\Pr(A_i = t) = \Pr(\tau_i = t)\Pr(\tau_{-i} > t)$ and $\Pr(\overline{A_i} = t) = \Pr(\tau_{-i} = t)\Pr(\tau_i > t)$. Also, let $\Pi_{s_i,s_{-i}}(x_i, x_{-i})$ be firm $i$'s expected payoff in state $(s_i, s_{-i})$.

In state $(D, D)$, both firms are engaged in *Development* so firm $i$ gets an expected payoff equal to

$$\Pi_{D,D}(x_i, x_{-i}) = \int_0^{+\infty} \Pr(A_i = t)e^{-rt}V(m_i)dt$$

In that case, the first firm who completes *Development* achieves innovation and collects its whole private value while the defeated firm gets nothing.

In state $(D, R)$, firm $i$ is ahead (i.e. engaged in *Development* while her rival is still working on *Research*) so firm $i$'s expected payoff is equal to

$$\Pi_{D,R}(x_i, x_{-i}) = \int_0^{+\infty} \Pr(A_i = t)e^{-rt}V(m_i)dt + \int_0^{+\infty} \Pr(\overline{A_i} = t)e^{-rt}\Pi_{D,D}(x_i, x_{-i})dt$$

The first term is firm $i$'s expected payoff when she defeats her rival before the latter catches up whereas the second term is her expected payoff when she loses her edge.

At the opposite, in state $(R, D)$, firm $i$ is behind (i.e. is engaged in *Research* while her rival is working on *Development* so firm $i$'s expected payoff is equal to

$$\Pi_{R,D}(x_i, x_{-i}) = \int_0^{+\infty} \Pr(A_i = t)e^{-rt}\Pi_{D,D}(x_i, x_{-i})dt$$

At this state of the game, firm $i$ either catches up or loses the race.

Finally, in state $(R, R)$, both firms are engaged in *Research* so firm $i$ gets an expected payoff equal to

$$\Pi_{R,R}(x_i, x_{-i}) = \int_0^{+\infty} \Pr(A_i = t)e^{-rt}\Pi_{D,R}(x_i, x_{-i})dt + \int_0^{+\infty} \Pr(\overline{A_i} = t)e^{-rt}\Pi_{R,D}(x_i, x_{-i})dt$$

The first term is firm $i$'s expected payoff when she takes the lead whereas the second term is her expected payoff when she loses ground.

The timing of the game goes as follows: at the beginning of the race (i.e. in state $(R, R)$), firms simultaneously choose whether or not to undertake preventive actions. As soon as the race develops (i.e. passes to the next state), firms learn the past of the game and they simultaneously decide whether or not to undertake preventive actions for the current state of the race. We thus face a dynamic game whose information is complete and perfect so we search for (pure-strategy) SPNE. We solve the game backwards so we first focus on the subgames that start once both firms are engaged in *Development*. Next, we solve the subgames in which one firm is ahead and conducts *Development* while her rival has not completed *Research* yet. Finally, we address the case in which both firms are engaged in *Research* and we solve the whole game.

When both firms are engaged in *Development*, it is immediate to show that they both undertake preventive actions if $b \geq \widetilde{b}(a) = \frac{1-a}{1+a}$ and that they both from taking those actions otherwise. Hence, both firms undertaking preventive actions is optimal whenever the prize gap (i.e. the extra benefit factor associated with preventive actions) is sufficiently large with respect to the *hazard* gap (i.e. the extent to which undertaking those actions is expected to slow R&D down). Since $\widetilde{b}$ is decreasing, the smaller the *hazard* gap, the smaller the prize gap threshold above which both firms undertaking preventive actions form an equilibrium. An interesting feature of those subgames is that their NE is unaffected by the race's past. That is, the question of knowing whether or not firms have undertaken preventive actions to complete *Research* is irrelevant. Finally, pay attention to the fact that those subgames might admit two symmetric NE. Yet, we show that the equilibrium in which both firms undertake preventive actions Pareto dominates that in which they do not and we assume that firms coordinate on the Pareto dominant equilibrium.[12]

We now turn to solving the subgames that start once one firm is ahead. In that case, there is one *leader* who is engaged in *Development* and one *follower* who conducts *Research*. Let us define $\widehat{b}(a) = \frac{1-a}{4a}$ and

$$\phi(a) = \begin{cases} \widetilde{b}(a) & \text{if } a \leq \frac{1}{3} \\ \widehat{b}(a) & \quad a \geq \frac{1}{3} \end{cases}$$

In that case, the *leader* undertakes preventive actions if $b \geq \phi(a)$ whereas the *follower* undertakes such actions whenever $b \geq \widetilde{b}(a)$. If $b \in \left[\phi(a), \widetilde{b}(a)\right]$, then an asymmetric equilibrium arises: the *leader* undertakes preventive actions whereas the *follower* does not. Again, we observe that the past of the race does not influence the equilibrium of those subgames. Note that $\widetilde{b}(a) \geq \phi(a)$ so the *leader* is more likely to undertake preventive actions than the *follower*. Indeed, the *follower* can hardly afford the luxury of undertaking preventive actions and thus reducing the speed of R&D when her rival is about to achieve innovation. Besides, we observe that if the *leader* loses his edge (i.e. if the *follower* catches up), then he either stays on course (i.e. does not change his behavior) or he stops undertaking preventive actions whereas the *follower* always stays on course.

Finally, we address the case in which both firms are engaged in *Research*. Our conclusions are depicted in Figure 3. Let us define $\underline{b}(a) = \frac{2}{3}\frac{1-a}{1+a}$, $\overline{b}(a) = \frac{1}{4}\frac{\sqrt{8a^3+25a^2+2a+1}-4a^2-a-1}{a(1+a)}$ and

$$\psi(a) = \begin{cases} \underline{b}(a) \text{ if } a \leq \frac{3}{5} \\ \widehat{b}(a) \text{ if } a \in \left[\frac{3}{5}, \frac{1+2\sqrt{7}}{9}\right] \\ \overline{b}(a) \text{ if } a \geq \frac{1+2\sqrt{7}}{9} \end{cases}$$

[12] Note that we shall use this Pareto dominance argument multiple times throughout the analysis.

Tedious but routine calculations allow us to show that both firms undertake preventive actions if $b \geq \psi(a)$ and that they do not otherwise. Since $\psi(a) \leq \widetilde{b}(a)$, firms are more inclined to undertake such actions when they are both engaged in *Research* than the case in which they are both engaged in *Development*. In the latter state of the race - and unlike the former - firms are engaged in a "sudden death game" so the time that can be saved by renouncing to undertake preventive actions is more valuable. The fact that $\psi(a) \leq \widetilde{b}(a)$ also implies that if a firm falls behind, then she either stays on course or she stops undertaking preventive actions. Finally, since $\psi(a) \lesseqgtr \phi(a)$, it is not clear whether taking the lead makes firms stay on course, start undertaking preventive actions or stop taking them.

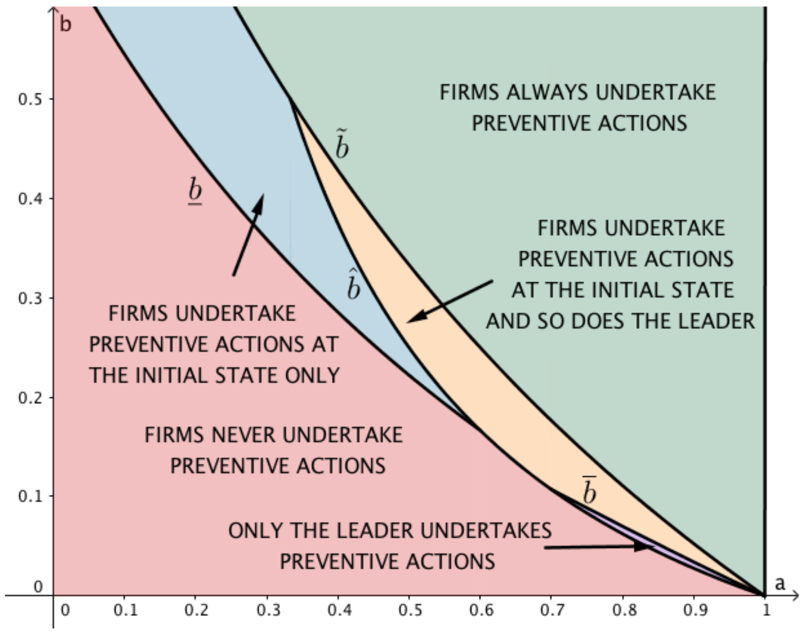


Figure 3: Equilibrium R&D strategies when rivalry is measured through frms' relative progress within a multi-stage innovation race

We observe that if the prize gap is sufficiently large or sufficiently small, then firms' decision of whether or not to undertake preventive actions is the same at all states of the race. In the former case (i.e. when $b \geq \widetilde{b}(a)$), they always undertake such actions whereas they never do in the latter (i.e. when $b \leq \min\{\psi(a), \phi(a)\}$). This echoes Proposition 1 since it shows that the degree of rivalry does not affect the equilibrium R&D strategies when undertaking preventive actions is either very rewarding or, at the opposite, not quite worth it. Only when $b \in \left[\min\{\psi(a), \phi(a)\}, \widetilde{b}\right]$ will the firms adjust their tactics as the race develops. We focus on this latter case and we distinguish three different *scenarios*.

First, when $b \in [\psi(a), \phi(a)]$, both firms undertake preventive actions at the initial state of the race. As soon as one of them takes the lead, they both stop undertaking preventive actions for the rest of the race. Second, if $b \in \left[\max\{\psi(a), \phi(a)\}, \widetilde{b}\right]$, then both firms undertake preventive actions at the initial state of the race. Once one of them takes the lead, the firm who is ahead keeps undertaking preventive actions whereas the one who is late stops undertaking such actions. If the *follower* catches up, then both firms are neck-to-neck in *Development* and none of them undertakes preventive actions. Last, if $b \in [\phi(a), \psi(a)]$, then firms do not undertake preventive actions when they are working on the same step. If one of them is ahead, then none of them undertakes preventive actions at the initial state of the race. Once one of them takes the lead, she starts undertaking those actions and the *follower* does not adjust her tactics. Finally, if the *follower* catches up, then firms are neck-to-neck in *Development* and none of them undertakes preventive actions.

In a nutshell, when firms are engaged in the same step, preventive actions are more likely to be taken in *Research* than in *Development*. If one firm is ahead, then the *leader* is more likely to undertake preventive actions than the *follower*. When those actions are either very rewarding or, at the opposite, not quite worth it, the degree of rivalry does not make the firms adjust their tactics as the race develops. In the former case, firms undertake preventive actions at all states of the race whereas they never take those actions in the latter. However, if there is a balance between the prize gap and the hazard gap, firms adapt their tactics as the race develops. If they lose their edge or if they fall behind, then firms either stay on course or stop undertaking preventive actions. If they catch up, then they stay on course. Finally, if they take the lead, then they either stay on course, start undertaking preventive actions or stop taking them.

In this section, we have established that if there is a balance between the prize gap and the hazard gap then rivalry makes the implementation of preventive actions more difficult to be sustained as an equilibrium. In the next section, we explore the second set of R&D strategies in which firms may undertake corrective actions aimed at addressing a mistake. This alternative approach will allow us determine whether or not our main findings are driven by a specific class of strategies.

## 3 Corrective actions

When firms are not fully aware of the consequences induced by their actions, the environment they face is uncertain. Our previous framework included two types of uncertainty: *technological uncertainty* (i.e. firms' ability to achieve innovation) and *competitive uncertainty* (i.e. firms' chances to be the first to achieve innovation). Yet, there is a third type of uncertainty that we left aside: *product uncertainty*.[13] By that, we refer to the presence of uncertainty in the relation between firms' R&D strategy and the characteristics of the innovation. So far, the implications of each R&D strategy, and, specifically, their impact on the value of the innovation, were indeed perfectly understood *ex ante*. Yet, R&D is, above all, a *learning-by-doing* process: attempts might fail or lead to unexpected outcomes so it might be

[13] If prizes are understood as expected values rather than certain amounts, then our previous framework included product uncertainty.

necessary to go back and correct previous mistakes.

In this section, we introduce product uncertainty in the sense that firms are unable to determine whether their innovation will turn out beneficial or harmful at the time they are engaged in R&D. This uncertainty dissipates once innovation is achieved and firms then decide whether to introduce their innovation as such or to go back to R&D in order to undertake corrective actions. As we shall see, undertaking such actions is appealing because it increases the profitability of the innovation. However, engaging in a *second round of R&D* requires time, resources and it exposes firms to the risk of being defeated by one of their rivals. This latter threat is particularly important when undertaking corrective actions transmits information to rival firms.

The main question we investigate is whether or not rivalry - as measured by the number of competitors - makes firms less inclined to undertake corrective actions once they realize that they have made a mistake. To that end, we consider that an even natural number of firms $n$ are engaged in a single-stage *winner-take-all* innovation race. At the time they start R&D, firms decide which of two equally costly paths $p \in \{R, W\}$ to take. A path includes all the relevant features of R&D such as the technology which is used, the agenda that is decided, etc. The *right path* $(R)$ leads to a beneficial innovation and the *wrong path* $(W)$ leads to a harmful innovation. For reasons outlined earlier, the former type of innovation is more profitable than the latter. Let $V(p)$ be the private value of the innovation when the winner has taken path $p$. We assume that the harmful innovation is worth $v > 0$ whereas the beneficial innovation is worth $v + b$ with $b > 0$.

Our framework features product uncertainty because the firms do not know which path leads where at the time they decide which one to take. In particular, we assume that they initially believe each path to be equally likely to lead to the harmful innovation. Hence, firms choose one path at random so one half (type-$R$ firms) takes the right path whereas the other half (type-$W$ firms) takes the wrong path Firms are assumed to observe the path their rivals are on. Once they have taken a path, firms choose what amount of resources to devote to R&D. This amount determines both the per-period cost of R&D and the *hazard rate* associated with the exponentially distributed random variable describing the time at which R&D succeeds. We assume that they all invest the same amount of resources so each of them is first to achieve innovation (i.e. to take the lead) with probability $\frac{1}{n}$.

The *leader* ($L$) discovers the path he was on so he learns whether his innovation has turned out beneficial or harmful.[14] Then, he can either introduce the innovation and collect its value or he can go back to R&D and undertake corrective actions.[15] In our model, such actions consist in switching from the wrong path to the right path. Once the *leader* goes back to R&D he instantly surrenders the former prize that he had access to. We assume that switching from one path to the other induced a fixed cost $S < b$. This change of course might indeed require a restructuring of labor and capital or necessitate organizational changes. Besides, in order to take into account the fact that the *leader* may benefit from his experience as he engages in a *second round of R&D*, we assume that his *hazard* rate is scaled up by factor $1 + \gamma$ with $\gamma \geq 0$. The greater $\gamma$ the more substantial the experience.

[14] Note that even though firms do not know which path leads where at the time they enter the race, they are aware that one path leads to a beneficial innovation whereas the other leads to a harmful innovation. It implies that if the *leader* learns that he was on the wrong path, he finds out how to set things right and correct his previous mistake. If this was not the case, then it could take the *leader* several attempts to eventually achieve a beneficial innovation.

[15] In order to rule out successive restarts, we assume that R&D can be restarted only once in the whole industry.

If the *leader* learns that his innovation is beneficial (i.e. if he was on the right path), then there is no need to undertake corrective actions since he already has access to the larger prize. In that case, he introduces the innovation. However, if the *leader* learns that his innovation is harmful (i.e. if he was on the wrong path), then he can either introduce it anyway and collect the smaller prize or he can go back to R&D and undertake corrective actions. Yet, there are three forces that might deter the *leader* from doing so.

First, undertaking corrective actions is costly because the *leader* has to switch from the wrong path to the right path and because he has to devote resources to a *second round* of R&D. If those costs are too important with respect to the prize gap (i.e. the difference between the private value of the beneficial and the harmful innovation), then the *leader* will be better off introducing the harmful innovation. Second, when the *leader* engages in a *second round* of R&D, he might be defeated by one of the *followers* and lose the smaller prize that he previously had access to. Last, when the *leader* undertakes corrective actions, he transmits information to the *followers* and the race might intensify. Indeed, since firms know which path their rivals are on, switching from one path to the other is an observable move. Therefore, when the *leader* undertakes corrective actions, product uncertainty vanishes and *followers* learn which path leads where.

If the *leader* has taken the wrong path and goes back to R&D as for undertaking corrective actions, then we assume that *followers* can adjust their own trajectory (i.e. decide whether or not to switch from one path to the other). Note that there are two types of *followers*: those who are on the right path (type-$R$ *follower*) and those who are on the wrong path (type-$W$ *follower*). There are $\frac{n}{2}$ type-$R$ *followers* and $\frac{n}{2}-1$ type-$W$ *followers*. Clearly, type-$R$ *followers* remain on the right path. However, type-$W$ *followers* have an interest in adjusting their trajectory and switching from the wrong path to the right path in order to get a larger prize in the event in which they win the race. However, this move would induce a switching cost so the question of whether type-$W$ *follower*s are better off adjusting their trajectory will require some attention. Finally, once the *followers* have decided whether or not to switch from one path to the other, all firms (including the *leader*) simultaneously determine the amount of resources that they devote to this *second round* of R&D.

We focus on the game that starts once a *leader* emerges.[16] Indeed, our primary interest is to determine the conditions under which a *leader* who has initially taken the wrong path is better off undertaking corrective actions rather than introducing the harmful innovation as such. Let us summarize the timing of that (sub)game: first, the *leader* decides whether to introduce the innovation or to undertake corrective actions. In the former case, the game ends and payoffs are realized. In the latter case, *followers* observe the *leader*'s move and they simultaneously decide whether or not to switch from one path to the other. Finally, all firms simultaneously decide what amount of resources to devote to this *second round* of R&D.

We thus face a dynamic game whose information is perfect and complete. We do not allow firms to randomize their strategies and we search for SPNE. Therefore, we solve the game backwards and we start by studying the last stage at which all firms simultaneously choose what amount of resources to devote to the *second round* of R&D. We assume the equilibrium to be interior.[17] There are actually two

[16]Recall that the past of the game can be summarized as follows: firms choose one path at random, they devote the same amount of resources to the *first round* of R&D and each of them takes the lead with equal probability.

[17]That is, all types of firms devote a strictly positive amount of resources to the *second round* of R&D so they get a positive expected payoff.

subgames to explore depending on whether or not type-$W$ *followers* have switched from the wrong path to the right path.

Let $x_i \geq 0$ be the amount of resources that firm $i$ devotes to the *second round* of R&D. Recall that this amount determines the flow cost of R&D. Besides, let $\tau_i$ be the random variable describing the time at which firm $i$ achieves innovation. It is assumed to follow an exponential distribution of parameter (*hazard* rate) $h(x_i)$ with $h' > 0$ and $h'' \leq 0$ if firm $i$ is a *follower* and of parameter $(1+\gamma)h(x_i)$ if firm $i$ is the *leader* since his experience makes him more efficient. To simplify notations, we define

$$C = \int_0^{+\infty} \Pr(\tau_i = t \text{ or } \tau_{-i} = t) \left\{ \int_0^t e^{-rs} x_i ds \right\} dt$$

as the expected cost associated with the resources devoted by firm $i$ to the *second round* of R&D. The index $-i$ refers to firm $i$'s rivals and $r$ is the discount rate. Also, let $A_i$ be the event: "*Firm $i$ wins the second round of R&D*" so $\Pr(A_i = t) = \Pr(\tau_i = t)\Pr(\tau_{-i} > t)$. Finally, let $\Pi_i(x_i, x_{-i})$ be firm $i$'s expected payoff when she invests $x_i$ in the *second round* of R&D while her competitor's R&D investments are given by the vector $x_{-i}$.

If firm $i$ is the *leader*, then his expected payoff is equal to

$$\Pi_L(x_i, x_{-i}) = \int_0^{+\infty} \Pr(A_i = t) e^{-rt} V(R) dt - C - S$$

If firm $i$ is a type-$R$ *follower*, then her expected profit is equal to

$$\Pi_R(x_i, x_{-i}) = \int_0^{+\infty} \Pr(A_i = t) e^{-rt} V(R) dt - C$$

If firm $i$ is a type-$W$ *follower* who has adjusted her trajectory, then her expected payoff is equal to

$$\Pi_W(x_i, x_{-i}) = \int_0^{+\infty} \Pr(A_i = t) e^{-rt} V(R) dt - C - S$$

Finally, if firm $i$ is a type-$W$ *follower* who has decided to remain on the wrong path, then her expected payoff is equal to

$$\Pi_W(x_i, x_{-i}) = \int_0^{+\infty} \Pr(A_i = t)e^{-rt}v(W)dt - C$$

In each subgame, there exists a single NE in which firms that have the same type devote the same amount of resources to this *second round* of R&D. Let $x^* = (x_L^*, x_R^*, x_W^*)$ (resp. $x^{**} = (x_L^{**}, x_R^{**}, x_W^{**})$) be those amounts when type-$W$ *followers* adjust their trajectory (resp. when type-$W$ *followers* remain on the wrong path). In both cases, we expect the *leader* and type-$R$ *followers* to devote a larger amount of resources to the *second round* of R&D than to the first because uncertainty has vanished so they know that they are on the path that leads to the larger prize. The same applies for type-$W$ *followers* if they adjust their trajectory. If they do not, we presume that they reduce their initial effort to achieve innovation since their profit incentive is weaker.

When type-$W$ *followers* have adjusted their trajectory, we show that the NE induces the following expected payoffs:

$$\Pi_L(x^*) = V(R) - \frac{1}{(1+\gamma)h'(x_L^*)} - S$$

$$\Pi_R(x^*) = V(R) - \frac{1}{h'(x_R^*)}$$

$$\Pi_W(x^*) = V(R) - \frac{1}{h'(x_W^*)} - S$$

Likewise, when type-$W$ firms remain on the wrong path, we show that the NE induces the following expected payoffs

$$\Pi_L(x^{**}) = V(R) - \frac{1}{(1+\gamma)h'(x_L^{**})} - S$$

$$\Pi_R(x^{**}) = V(R) - \frac{1}{h'(x_R^{**})}$$

$$\Pi_W(x^{**}) = V(W) - \frac{1}{h'(x^{**}_W)}$$

We now turn to the previous stage of the game in which the *follower*s simultaneously decide whether or not to adjust their trajectory. As mentioned earlier, type-$R$ *followers* stay on the right path. As for type-$W$ *followers*, they switch from the wrong path to the right path whenever $\Pi_W(x^*) \geq \Pi_W(x^{**})$ which is equivalent to

$$b - S \geq \frac{1}{h'(x^*_W)} - \frac{1}{h'(x^{**}_W)} \tag{1}$$

Therefore, the extra profitability (net of the switching cost) associated with the introduction of a beneficial innovation must be sufficiently large to induce type-$W$ *followers* to adjust their trajectory and take the right path. To simplify notations, let us define $\widetilde{x}_i$ as the equilibrium amount of resources that is devoted by firm $i$ to the *second round* of R&D. Hence, $\widetilde{x}_i = x^*_i$ if type-$W$ *followers* adjust their strategy (i.e. if inequality (1) is true) and $\widetilde{x}_i = x^{**}_i$ if type-$W$ *followers* remain on the wrong path (i.e. if inequality (1) is false).

Finally, we solve the stage of the game at which the *leader* decides whether or not to go back to R&D and undertake corrective actions. As explained earlier, if he discovers that his innovation is beneficial, then he introduces it and he gets the larger prize. However, if he learns that his innovation is harmful, then he can either introduce it and gets the smaller prize of he can undertake corrective actions and get $\Pi_L(\widetilde{x})$. Hence, the *leader* undertakes corrective actions whenever

$$\Pi_L(\widetilde{x}) \geq V(W) \Leftrightarrow h'(\widetilde{x}_L) \geq \frac{1}{(b-S)(1+\gamma)} \tag{2}$$

In can be checked that the assumption according to which the NE of the investment subgame is interior implies that the amount of resources devoted by the *leader* to the *second round* of R&D increases with the number of competitors so both

$h'(\widetilde{x}_L)$ and $\Pi_L(\widetilde{x})$ decrease with the number of competitors.[18] In other words, the *leader* responds to an increase in rivalry by stepping up his own effort to achieve innovation. Note that the idea that firms react agressively to an increase in rivalry has already been put forward by Lee & Wilde (1981).

We further show that $h'(\widetilde{x}_L)$ is continuous with respect to the number of competitors and that it tends to $\frac{1}{(1+\gamma)(v+b)} \leq \frac{1}{(b-S)(1+\gamma)}$ as $n$tends to $+\infty$. Assume that inequality (2) is true when $n = 2$,[19] then we are able to establish that $\Pi_L(\widetilde{x}) \geq V(W) \Leftrightarrow n \leq \widehat{n}'$ with

$$\widehat{n}' = \frac{2(h(\widetilde{x_L})(1+\gamma)(b-S) + (v+S)(h(\widetilde{x_W}) - r) - \widetilde{x_L})}{(v+S)(h(\widetilde{x_R}) + h(\widetilde{x_W}))}$$

We are now able to answer the question of whether rivalry makes the *leader* less inclined to undertake corrective actions when he discovers that his innovation has turned out harmful.

**Proposition 4** *The leader undertakes corrective actions if rivalry is sufficiently weak and he introduces the harmful innovation otherwise.*

Therefore, unlike preventive actions, rivalry unambiguously makes the implementation of corrective actions more difficult to be sustained as an equilibrium. Clearly, the more intense rivalry, the more keenly contested the *second round* of R&D so the more appealing it is for the *leader* to be satisfied with the smaller prize rather than attempting to get a larger one at the risk of losing everything.

Comparative statics on $\widehat{n}'$ are somewhat complicated to study because of its implicit form. However, we expect $\widehat{n}'$ to be increasing with $\gamma$ since the more experienced the *leader* the stronger his competitive advantage in the *second round*

[18] By assumption, the NE is interior. Therefore, it must be the case that $(1+\gamma)h(\widetilde{x_L})V(R) - \widetilde{x_L} \geq 0$. We show that this condition implies $h'(\widetilde{x_L}) \leq 0$.

[19] Otherwise, the *leader* does not undertake corrective actions whatever the degree of rivalry.

of R&D. Moreover, we presume that $\widehat{n}'$ decreases with $S$ since the greater the switching cost, the less incentivized the *leader* to undertake corrective actions. For opposite reasons, we suspect $\widehat{n}'$ to be increasing with $b$. Finally, $\widehat{n}'$ should be decreasing with $r$ since the greater the discount rate, the more appealing it is for the *leader* to collect the smaller prize today rather than a potential larger prize in the future.

# 4 Overlapping steps

In this last section, we explore the third set of R&D strategies in which firms decide whether or not to overlap (i.e. conduct simultaneously) several of the steps R&D is made up of. As noted by Scherer (1967): *"R&D is in many ways a heuristic process. Each sequential step provides knowledge useful in the next step. Time can be saved by overlapping steps, but then one takes actions (e.g. conducts tests) without all the knowledge prior steps have furnished. As more and more actions are based on a given amount of prior knowledge, more and more costly mistakes are made"*. In this section, we assume that the innovation induces a greater damage when firms overlap steps and we investigate whether or not rivalry - as measured by the number of competitors - makes firms more inclined to break the sequentiality of R&D.

To that end, we consider a model in which $n \in \{1, 2\}$ symmetric firms[20] are engaged in an innovation race. Remaining in the race induces a per-period cost $c > 0$ which is assumed to be small enough so firms never find it optimal to abandon R&D. Two equally difficult steps must be completed for innovation to be achieved. One is called *Research* ($R$) and the other *Development* ($D$). In the previous section,

[20] Studying the $n$-firm case would be interesting because it would allow a more accurate measurement of the degree of rivalry. Yet, this generalization makes our model too tedious to provide any meaningful insights.

firms had to complete *Research* before they could initiate *Development*. We relax this assumption and we explore the case in which firms can also overlap both steps (i.e. conduct *Research* and *Development* simultaneously). At the time she enters the race, firm $i$ chooses what R&D strategy $x_i \in \{O, S\}$ to implement: strategy $O$ (*Overlapping*) consists in initiating *Research* and *Development* at once whereas strategy $S$ (*Sequential*) consists in initiating *Development* as soon as *Research* is finished. The firms simultaneously choose their R&D strategy. By assumption, harmful innovations are more likely to arise when steps are overlapped, the expected value of the innovation $V(x_i)$ is larger when steps have been conducted one at the time. In particular, we set $V(O) = v > 0$ and $V(S) = v(1+b)$ with $b > 0$.[21] We refer to $b$ as the *prize gap* in the sense that it measures the extent to which overlapping steps curtails the private value of the innovation.

The time at which a firm completes step $s \in \{R, D\}$ is a random variable that is assumed to follow an exponential distribution of parameter $h(m)$ where $m$ is the fraction of the firm's - predetermined - resources which is devoted to that step. We assume $h' > 0$ and $h'' < 0$ so R&D features decreasing returns to scale. When firms do not overlap steps, their whole resources are devoted to *Research* first and to *Development* afterwards so if $x_i = S$, then $\forall s : m = 1$. However, when firms opt for conducting *Research* and *Development* at once, they first have to split their resources into two parts: one fraction is allocated to conducting *Research* while the other is used to undertake *Development*. The concavity of the *hazard rate* function implies that the firm should divide her resources into two halves.[22] As

[21] When firms choose to conduct *Research* and *Development* simultaneously, the order in which those two steps are completed is irrelevant. That is, our model includes some form of inertia within each step. Namely, once *Research* or *Development* is initiated, firms can hardly take advantage of the arrival of new information and adjust their trajectory at the time they succeed in completing one step.

[22] With increasing returns to scale, a firm overlapping *Research* and *Development* that has not

soon as one step has been completed (either *Research* or *Development*), then firms allocate their whole resources to the remaining step. Hence, if $x_i = O$, then $m = \frac{1}{2}$ when no step has been completed yet and $m = 1$ when one step has already been completed. We further assume that $h(1) = h > 0$ and $h(\frac{1}{2}) = ah$ with $a \in [\frac{1}{2}, 1]$ since R&D features decreasing returns to scale. Obviously, the more concave $h$, (i.e. the larger $a$), the more appealing it is to overlap R&D. We refer to $a$ as the *hazard gap* in the sense that it measures the extent to which overlapping steps hastens R&D.

We define $\tau_i^k$ with $k \in \{1, 2\}$ as the random variables describing the time at which firm $i$ completes the single step she is working on (if $k = 1$) and the first of the two steps she conducts (if $k = 2$). Hence, $\Pr(\tau_i^1 = t) = he^{-ht}$ and $\Pr(\tau_i^2 = t) = 2ahe^{-2aht}$. Finally, future amounts are discounted at rate $r > 0$.

## 4.1 One-firm case

We first focus on the case in which a single firm is engaged in R&D.[23] Rivalry shall be introduced later on. Let $\Pi_q(x)$ denote her expected profit when she has adopted the R&D strategy $x$ and when she has already completed $q \in \{0, 1\}$ step. First, suppose that this firm has chosen to conduct steps one at the time (i.e. $x = S$). When she initiates *Development*, her expected payoff is equal to

$$\Pi_1(S) = \int_0^{+\infty} \Pr(\tau^1 = t) \left\{ e^{-rt} V(S) - \int_0^t ce^{-rs} ds \right\} dt$$

At the time she initiates *Research*, her expected profit is equal to

$$\Pi_0(S) = \int_0^{+\infty} \Pr(\tau^1 = t) \left\{ e^{-rt} \Pi_1(S) - \int_0^t ce^{-rs} ds \right\} dt$$

completed any step yet would allocate all her resources to *Research* or to *Development*. In that case, overlapping steps can never be optimal since it does not allow the firm to save time and it yields a lower payoff.

[23]Since there is only one firm, we drop the index $i$.

Now, suppose that this firm has decided to conduct *Research* & *Development* at once. As soon as she completes one step (either *Research* or *Development*), she no longer splits her resources into two halves. In that case, her expected profit is equal to

$$\Pi_1(O) = \int_0^{+\infty} \Pr(\tau^1 = t) \left\{ e^{-rt} V(O) - \int_0^t ce^{-rs} ds \right\} dt$$

When this firm has neither completed *Research* nor *Development*, then she allocates half of her resources to each step. In that case, her expected profit is equal to

$$\Pi_0(O) = \int_0^{+\infty} \Pr(\tau^2 = t) \ \left\{ e^{-rt} \Pi_1(O) - \int_0^t ce^{-rs} ds \right\} dt$$

Hence, the firm is better off adopting the R&D strategy that consists in conducting one step after the other whenever $\Pi_0(S) \geq \Pi_0(O)$. We show that this inequality is true so long as

$$b \geq \widehat{b} = \frac{(vr + c)(2a - 1)}{v(r + 2ah)}$$

We observe that the prize gap has to be large enough for the firm to find it optimal to conduct one step at the time. Note that $\widehat{b}$ increases with $a$ because the greater the *hazard* gap (i.e. the smaller $a$) the larger the cost associated with dividing resources so the less appealing it is to overlap steps. Also, note that $\widehat{b}$ increases with $r$ because the more discounted future amounts, the more tempting it is for the firm to overlap steps as for achieving innovation sooner. Likewise, $\widehat{b}$ increases with $c$ because overlapping steps allows the first to save time and to incur R&D costs from a shorter period of time. Finally, note that $\widehat{b}$ decreases with both $v$ and $h$ because the larger $v$, the wider the prize gap and the larger $h$, the more costly the division of resources.

## 4.2 Two-firm case

Now that we have described the optimal R&D strategy of a firm facing no rivalry, we turn to the case in which two firms are engaged in a *winner-take-all* innovation race. The main question is to determine whether or not the introduction of rivalry makes firms more inclined to overlap steps. Recall that firms simultaneously choose their R&D strategy at the time they enter the race so we are interested in a static game with complete but imperfect information. We do not allow firms to randomize their strategies and we search for NE in which both firms adopt the same strategy. Let $\Pi_{q_i,q_{-i}}(x_i, x_{-i})$ be firm $i$'s expected profit when she adopts the R&D strategy $x_i$, has already completed $q_i$ step and faces a rival who has chosen the R&D strategy $x_{-i}$ and has already completed $q_{-i}$ step. To simplify the expressions, we use the following notations: first, let

$$C = \int_0^{+\infty} \Pr(\tau_i^k = t \text{ or } \tau_{-i}^k = t) \left\{ \int_0^t ce^{-rs} ds \right\} dt$$

be firm $i$'s expected cost from one state of the race to the next. Also, let $A_i$ be the event, *"Firm i takes the race to the next state"* so

$$\Pr(A_i = t) = \Pr(\tau_i^k = t) \Pr(\tau_{-i}^k > t)$$

Finally, let $\overline{A_i}$ be the event *"Firm i's rival takes the race to the next state"* so

$$\Pr(\overline{A_i} = t) = \Pr(\tau_{-i}^k = t) \Pr(\tau_i^k > t)$$

Once both firms have completed one step, firm $i$'s expected payoff is equal to

$$\Pi_{1,1}(x_i, x_{-i}) = \int_0^{+\infty} \Pr(A_i = t) e^{-rt} V(x_i) dt - C$$

In that case, the first firm who completes the remaining step wins the race, collects the whole prize while her rival gets nothing.

When firm $i$ takes the lead and completes one step before her rival does, her expected payoff is equal to

$$\Pi_{1,0}(x_i, x_{-i}) = \int_0^{+\infty} \Pr(A_i = t) e^{-rt} V(x_i) dt + \int_0^{+\infty} \Pr(\overline{A_i} = t) e^{-rt} \Pi_{1,1}(x_i, x_{-i}) dt - C$$

The first term is firm $i$'s expected benefit when she achieves innovation before her rival catches up and the second term is her expected payoff when she loses her edge.

When firm $i$ falls behind (i.e. when her rival completes one step before she does), her expected payoff is equal to

$$\Pi_{0,1}(x_i, x_{-i}) = \int_0^{+\infty} \Pr(A_i = t) e^{-rt} \Pi_{1,1}(x_i, x_{-i}) dt - C$$

Finally, when none of the firms have completed any step yet, each of them gets an expected payoff equal to

$$\Pi_{0,0}(x_i, x_{-i}) = \int_0^{+\infty} \Pr(A_i = t) e^{-rt} \Pi_{1,0}(x_i, x_{-i}) dt + \int_0^{+\infty} \Pr(\overline{A_i} = t) e^{-rt} \Pi_{0,1}(x_i, x_{-i}) dt - C$$

The first term is the firm's expected payoff when she takes the lead and the second term is her expected payoff when she falls behind.

## 4.3 Rivalry and optimal R&D strategy

We now turn to describing the conditions under which both firms overlapping steps and both firms conducting one step at the time form a NE. Let us define

$$\widehat{b}' = \frac{(2a-1)(4v(1+a)h^3 + (2c(3+4a) + vr(9+8a))h^2 + r(c+vr)(2h(3+a)+r))}{(r+h(1+2a))^2(r+4h)v}$$

and

$$\overline{b} = \frac{(2a-1)(8av(1+a)h^3 + 2(2(c+v)(2a^2r+1) + a(4c+7vr))h^2 + r(c+vr)(2h(2+3a)+r))}{(r+4ah)(2(2+a(3+2a))h^2 + 4r(1+a)h + r^2)v}$$

Both firms choosing not to overlap *Research* and *Development* is a NE whenever $\Pi_{0,0}(S,S) \geq \Pi_{0,0}(O,S)$. We show that this true for all $b \geq \widehat{b}'$. At the opposite, both firms choosing to overlap *Research* and *Development* is a NE whenever $\Pi_{0,0}(O,O) \geq \Pi_{0,0}(S,O)$. We show that this true for all $b \leq \overline{b}$. Note that $\overline{b} \geq \widehat{b}'$[24]

[24]The inequality is true so long as $c \leq 2ahv$. However, the assumption that firms never abandon R&D implies that the inequality is true. To see this, consider (for instance) that both

so if $b \geq \bar{b}$, there is a single NE in which both firms conduct one step at the time. Likewise, if $b \leq \widehat{b}'$, then there exists a single NE in which both firms overlap *Research* and *Development.* However, if $b \in \left[\widehat{b}', \bar{b}\right]$ those two outcomes form a NE.

**Claim 5** *The Nash Equilibrium in which both firms choose not to overlap steps Pareto dominates that in which they do.*

The Proof is in the Appendix. We assume that firms coordinate on the Pareto dominant equilibrium. Therefore, the equilibrium is such that firms conduct one step at the time when $b \geq \widehat{b}'$, and they overlap *Research* and *Development* otherwise. As was the case in the one-firm case, the prize gap must be sufficiently large for both firms choosing not to overlap steps to constitute an equilibrium. Note that the comparative statics results that we previously established remain valid in the two-firm case.

We are finally able to answer our main question, namely: has the introduction of rivalry made firms more inclined to overlap steps? The assumption that firms never abandon R&D makes it straightforward to establish that $\widehat{b} \leq \widehat{b}'$. Therefore, the profit gap threshold above which firms do not overlap steps is larger when rivalry is introduced. If $b \geq \widehat{b}'$ or if $b \leq \widehat{b}$, then introduction of one rival firm has not affected the equilibrium R&D strategies. In the former case, firms conduct one step at the time whatever the degree of rivalry (i.e. whether or not they face a competitor). In the latter case, firms always overlap *Research* and *Development.* If $b \in \left[\widehat{b}, \widehat{b}'\right]$, however, then the introduction of a rival firm has changed the equilibrium R&D strategies. In particular, as depicted in Figure 4, when a firm faces no rivalry, then she conducts one step at the time whereas she - and her rival - overlaps *Research* and *Development* in the presence of a competitor.

firms overlap *Research* and *Development.* In that case, firms do not drop out the race (i.e. whatever its state) so long as $c \leq \frac{2ah^2v}{r+2h+2ah}$. Since $\frac{2ah^2v}{r+2h+2ah} \leq 2ahv$, we conclude that $\bar{b} \geq \widehat{b}'$ is always true.

**Proposition 6** *If there is a balance between the prize gap and the hazard gap, then firms are better off conducting one step at the time when they face no rivalry whereas they find it optimal to overlap both steps when they do.*

However, if overlapping *Research* and *Development* is either very rewarding or, at the opposite, not quite worth it, then firms implement the same R&D strategy regardless of whether or not they face a rival. In the former case, overlapping both steps is always optimal whereas it never is in the latter.

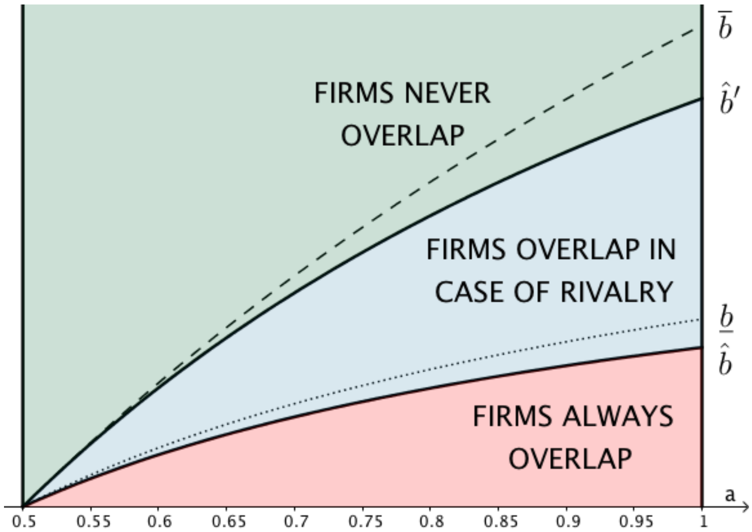


Figure 4: Equilibrium R&D strategies when firms decide whether or not to overlap several of the steps R&D is made of

Hence, we draw very similar conclusions to those of Section 2, when firms had to decide whether or not to undertake preventive actions. In both cases, the tradeoff is between an increased private value of the innovation and a reduction in the speed of R&D. When firms do not face any rivalry, they are not exposed to the threat of being defeated so there is little incentive for them to rush R&D apart from the fact that the sooner innovation is achieved, the smaller the cost induced by that process and the larger the innovation's discounted value. Introducing rivalry makes the time that is saved by overlapping steps more valuable. Also, breaking the sequentiality of R&D - and surrendering the larger prize - is more costly in the two-firm case.

## 5 Concluding remarks

In this paper, we analyzed the R&D strategy implemented by firms when they are engaged in an innovation race. Competitors have an interest in adopting safe R&D strategies because they enhance the private value of the innovation. However, safety has the disadvantage of slowing R&D down so the cost of that process is greater and firms are exposed to a higher risk of being defeated by one or their rivals. We investigated whether or not rivalry makes firms more inclined to implement unsafe R&D strategies that foster the emergence of harmful innovations. In particular, in order to determine the extent to which our conclusions rely on a specific approach, we explored successively three sets of R&D strategies: the decision of whether or not to undertake preventive actions, corrective actions or to overlap several of the steps R&D is made of. Likewise, we provided several measurement methods for the degree of rivalry including the number of competitors, the importance of the competitive threat, the size of the head start early entrants benefit from and firms' relative progress within a multi-stage innovation race.

When firms decide whether or not to undertake preventive actions, we showed that if such actions are either very rewarding or, at the opposite, not quite worth it, then the degree of rivalry does not affect the equilibrium R&D strategies. In the former case, firms always undertake preventive actions whereas they never do in the latter. However, if there is a balance between the prize gap and the *hazard* gap, then firms undertake preventive actions so long as the degree of rivalry is weak and they do not otherwise. We drew similar conclusions in the case for which firms decide whether or not to overlap several of the steps R&D is made of. Finally, when firms are given the opportunity to undertake corrective actions once they realize that they have made a mistake, we found that the more intense rivalry, the more difficult it is to sustain an equilibrium in which such actions are taken.

Our findings puts forward an additional argument in favour of narrow and infinitely-lived patents in the sense that the narrower the patent, the fewer the firms competing for the same industrial property title. Besides, our conclusions indicate that the public agency should not subsidize competing firms in the sense that feeding rivalry might foster the emergence of harmful innovations. Also, our work mitigates the appeal of contests that deliberately create rivalry. Finally, it stresses the importance of rewarding safe R&D strategies in order to make firms immune to rivalry.

# 6 Appendix

## 6.1 Proof of Lemma 3

We define $x_E(x_I)$ as the *entrant*'s optimal R&D strategy when the *incumbent* has adopted strategy $x_I$ and $G(T) = \Pi_I(1, x_E(1)) - \Pi_I(0, x_E(0))$ as the gap between the *incumbent*'s payoff when he undertakes preventive actions and when he does not, for a given degree of rivalry. $G$ is continuous with respect to $T$ on $\mathbb{R}^+$. Besides, note that $G(+\infty) > 0 \Leftrightarrow b \geq \widehat{b} = \frac{r}{r+h}\frac{1-a}{a}$ with $\widehat{b} \leq b_0''$. The *incumbent* undertakes preventive actions whenever $G(T) \geq 0$. If $b \leq b_0''$, then $x_E(0) = x_E(1) = 0$. In that case, $G$ has the following properties: its intercept is negative, it is decreasing on $[0, T_0]$ with $T_0 = \frac{1}{h(1-a)} \ln(\frac{(r+ah+h)}{a(r+2h)(1+b)})$ and increasing on $[T_0, +\infty[$. Hence, if $b \leq \widehat{b}$, then $G(T)$ is always negative. If $b \in \left[\widehat{b}, b_0''\right]$, however, the intermediate value theorem allows us conclude that there exists a single threshold $\widehat{T} > 0$ for the *incumbent*'s head start that must be exceeded for undertaking preventive actions to be optimal. Formally, $\forall b \in \left[\widehat{b}, b_0''\right]$, $\exists! \widehat{T} > 0 : \forall T \leq \widehat{T}, G(T) < 0$ and $\forall T \geq \widehat{T}$,

$G(T) \geq 0$. Unfortunately, this threshold $\widehat{T}$ cannot be computed analytically. If $b \geq b_1''$, then $x_E(1) = x_E(0) = 1$. In that case, $G$ has the following properties: its intercept is positive and it is increasing on $\mathbb{R}^+$. Therefore, if $b \geq b_1''$, then $G(T) \geq 0$. Finally, if $b \in [b_0'', b_1'']$, then $x_E(1) = 1$ and $x_E(0) = 0$. In that case, it is straightforward to show that $G(T) \geq M(T) = \Pi_I(1, x_E(1)) - \Pi_I(0, x_E(1))$. For all $b \in [b_0'', b_1'']$, $M$ has the following properties: its intercept is positive and it is increasing on $\mathbb{R}^+$. Therefore, since $M(T) \geq 0$ for all $b \in [b_0'', b_1'']$, so is $G(T)$.

## 6.2 Proof of Claim 1

First, note that $\Pi_{0,0}(S, S) \geq \Pi_{0,0}(O, O)$ is equivalent to

$$b \geq \underline{b} = \frac{(2a-1)(12ah^2 + 4rah + 4h^2 + 5hr + r^2)(2c + vr)}{(r + h + 2ah)(r + 4ah)(r + 4h)v}$$

Second, we show that $\underline{b} \leq \widehat{b}'$. To that end, we show that the inequality is true for all

$$c \leq \frac{2ah^2v(3r + 4h + 4ah)}{r^2 + 8a^2h^2 + 4h^2 + 8ah^2 + 6rah + 4hr}$$

Recall that the assumption that firms never drop out the race requires (at least) that $c \leq \frac{2ah^2v}{r+2h+2ah}$. Since $\frac{2ah^2v}{r+2h+2ah} \leq \frac{2ah^2v(3r+4h+4ah)}{r^2+8a^2h^2+4h^2+8ah^2+6rah+4hr}$ we are able to conclude that the inequality $\Pi_{0,0}(S, S) \geq \Pi_{0,0}(O, O)$ is always true.

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