Energy and Capital in a New-Keynesian Framework

Verónica Acurio Vásconez, Gaël Giraud,
Florent Mc Isaac, Ngoc Sang Pham

CES, PSE, University Paris I

March 27, 2014

Outline

Outline

Goals

- This paper constructs a New-Keynesian model with oil in the production function and in consumption.

- The model's parameters are estimated using Bayesian techniques.

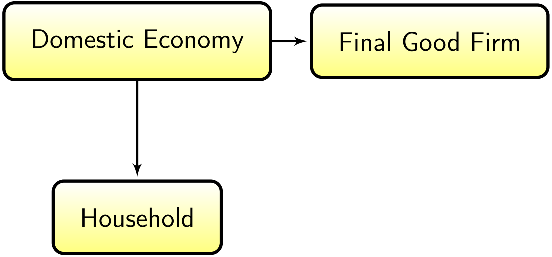
- We observe the impact of the oil shock in this economy.

Outline

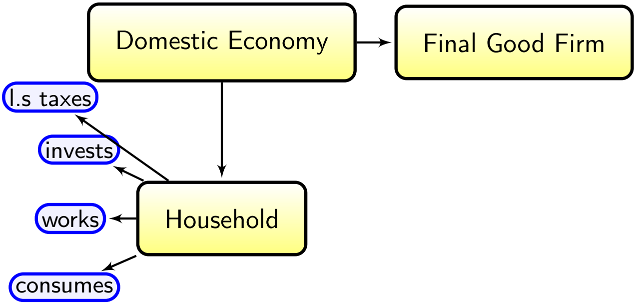
Model Structure

Domestic Economy

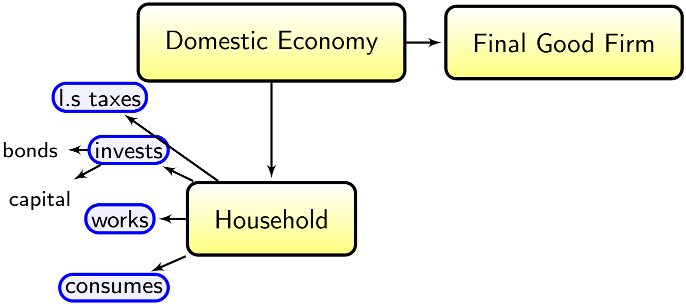
Model Structure



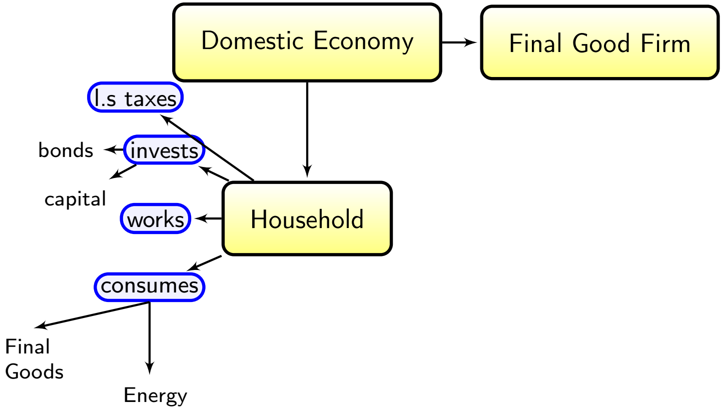
Model Structure



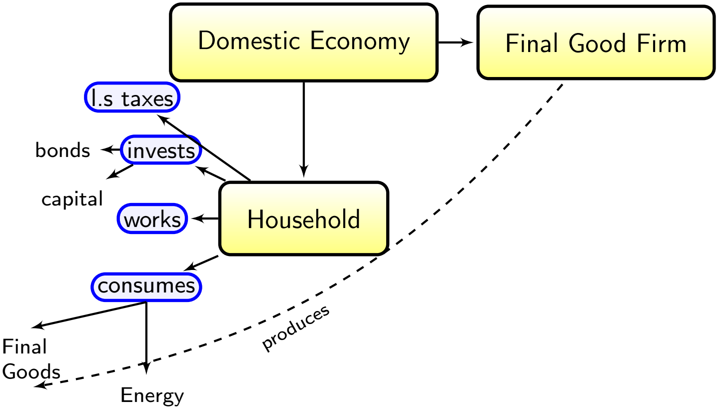
Model Structure



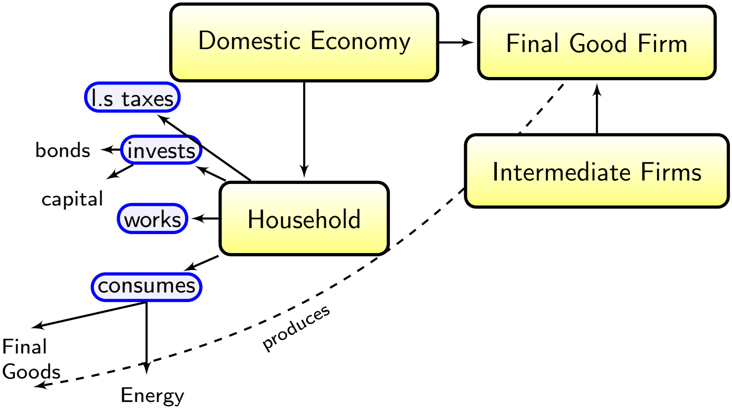
Model Structure



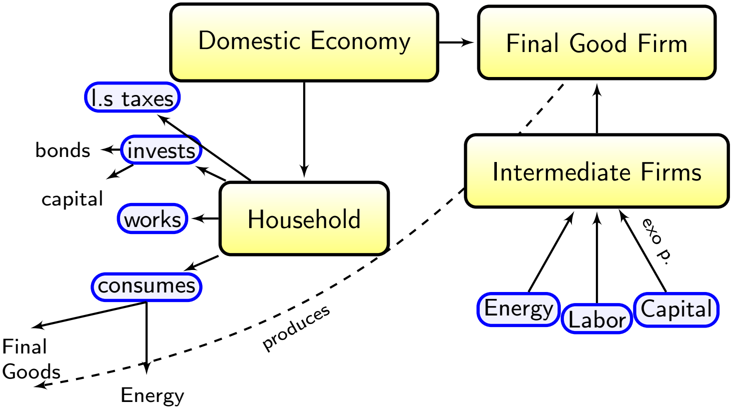
Model Structure



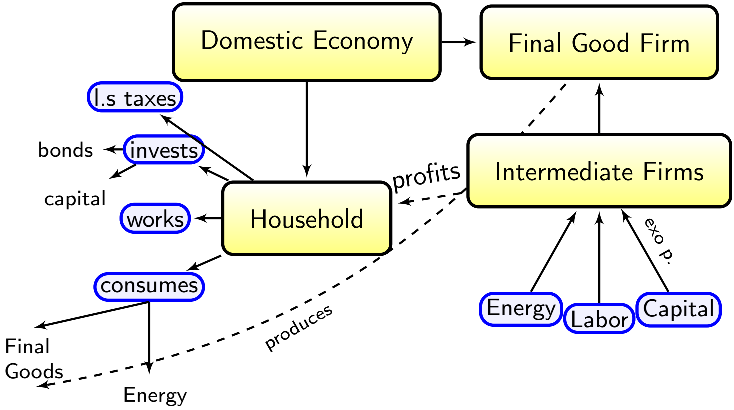
Model Structure



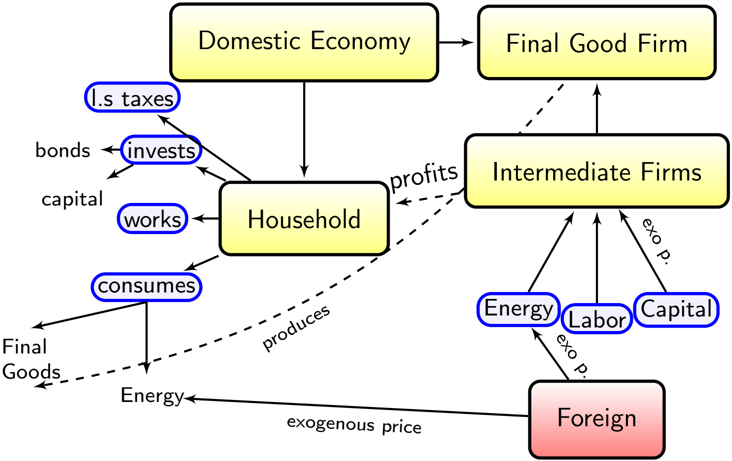
Model Structure



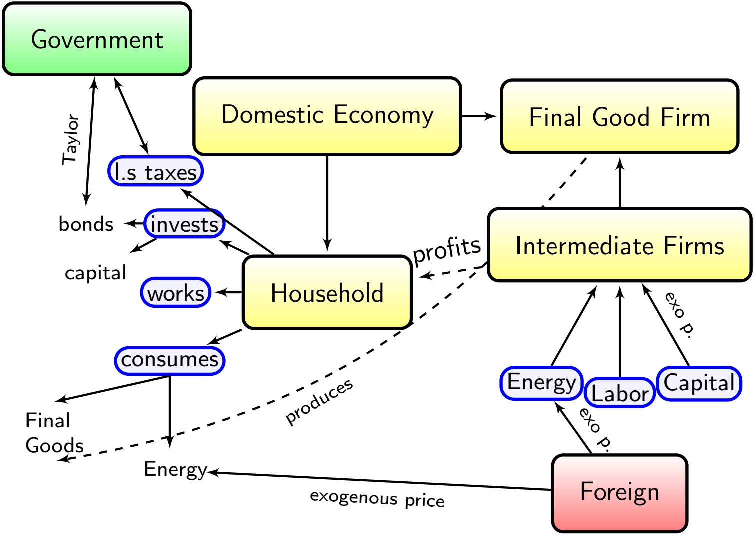
Model Structure



Model Structure



Model Structure



Outline

Household

Problem

$$\max \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t U(C_t, L_t) \right], \quad 0 < \beta < 1$$

s. t

$$P_{e,t} C_{e,t} + P_{q,t} C_{q,t} + P_{k,t} I_t + B_t + T_t$$
$$\leq (1 + i_{t-1}) B_{t-1} + W_t L_t + D_t + r_t^k P_{k,t} K_t$$

Household

Problem

$$\max \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t U(C_t, L_t) \right], \quad 0 < \beta < 1$$

s. t

$$P_{e,t} C_{e,t} + P_{q,t} C_{q,t} + P_{k,t} I_t + B_t + T_t$$
$$\leq (1 + i_{t-1}) B_{t-1} + W_t L_t + D_t + r_t^k P_{k,t} K_t$$

$$C_t := \Theta_x C_{e,t}^x C_{q,t}^{1-x}$$

$$\Theta_x := x^{-x}(1-x)^{-(1-x)}$$

Household

Problem

$$\max \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t U(C_t, L_t) \right], \quad 0 < \beta < 1$$

s. t

$$P_{e,t} C_{e,t} + P_{q,t} C_{q,t} + P_{k,t} I_t + B_t + T_t$$
$$\leq (1 + i_{t-1}) B_{t-1} + W_t L_t + D_t + r_t^k P_{k,t} K_t$$

$$C_t := \Theta_x C_{e,t}^x C_{q,t}^{1-x}$$

$$\Theta_x := x^{-x}(1-x)^{-(1-x)}$$

$$U(C_t, L_t) = log(C_t) - \frac{L_t^{1+\phi}}{1+\phi}$$

Household

Problem

$$\max \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t U(C_t, L_t) \right], \quad 0 < \beta < 1$$

$$C_t := \Theta_x C_{e,t}^{x} C_{q,t}^{1-x}$$

$$\Theta_x := x^{-x}(1-x)^{-(1-x)}$$

$$U(C_t, L_t) = log(C_t) - \frac{L_t^{1+\phi}}{1+\phi}$$

s. t

$$P_{e,t}C_{e,t} + P_{q,t}C_{q,t} + P_{k,t}I_t + B_t + T_t$$
$$\leq (1 + i_{t-1})B_{t-1} + W_t L_t + D_t + r_t^k P_{k,t} K_t$$

$$C_{q,t} := \left(\int_0^1 C_{q,t}(i)^{1-\frac{1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon - 1}}$$

Household

Problem

$$\max \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t U(C_t, L_t) \right], \quad 0 < \beta < 1$$

s. t

$$P_{e,t} C_{e,t} + P_{q,t} C_{q,t} + P_{k,t} I_t + B_t + T_t$$
$$\leq (1 + i_{t-1}) B_{t-1} + W_t L_t + D_t + r_t^k P_{k,t} K_t$$

$$C_{q,t} := \left(\int_0^1 C_{q,t}(i)^{1-\frac{1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon - 1}}$$

$$\Theta_x := x^{-x}(1-x)^{-(1-x)}$$

$$C_t := \Theta_x C_{e,t}^x C_{q,t}^{1-x}$$

$$U(C_t, L_t) = log(C_t) - \frac{L_t^{1+\phi}}{1+\phi}$$

$$I_t := K_{t+1} - (1-\delta) K_t$$

Optimization

Household's Optimal Expenditure Allocation

Optimization

Household's Optimal Expenditure Allocation

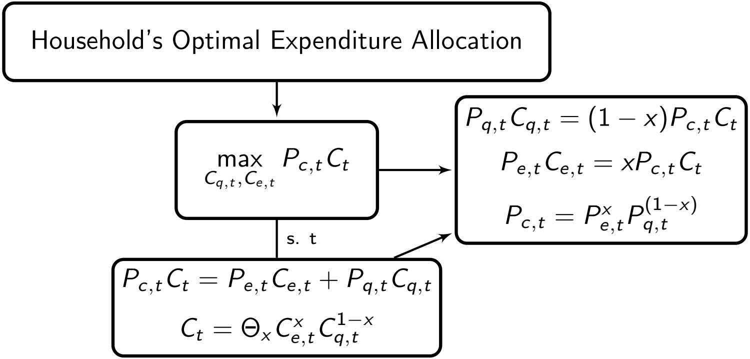
$$\max_{C_{q,t},C_{e,t}} \quad P_{c,t}C_t$$

s. t

$$P_{c,t}C_t = P_{e,t}C_{e,t} + P_{q,t}C_{q,t}$$

$$C_t = \Theta_x C_{e,t}^{x} C_{q,t}^{1-x}$$

Optimization

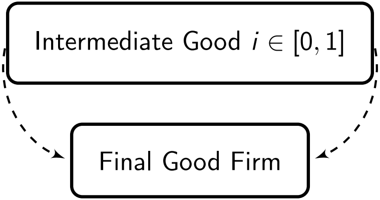


Outline

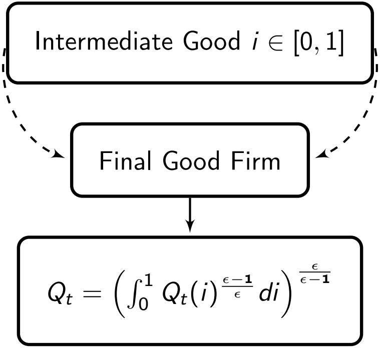
Final Good Producers

Final Good Firm

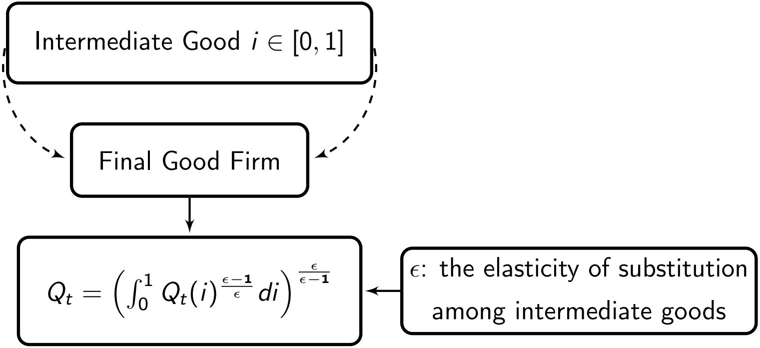
Final Good Producers



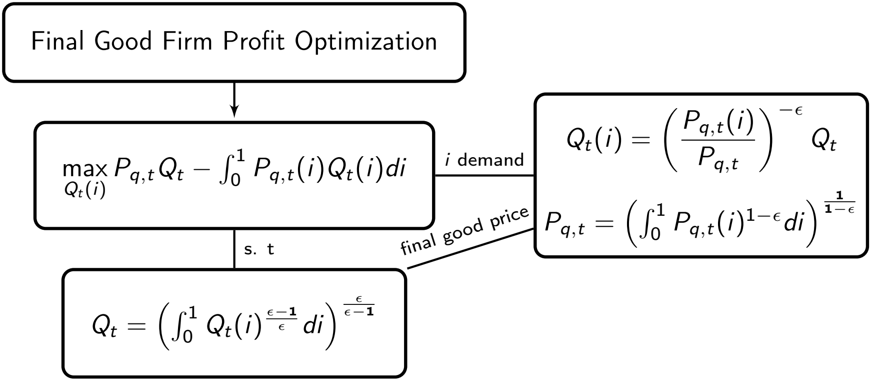
Final Good Producers



Final Good Producers



Final Good Producer Problem



Intermediate Good Firms

Intermediate Firms

Intermediate Good Firms

Intermediate Firms

↓

$$Q_t(i) = A_t E_t(i)^{\alpha_e} L_t(i)^{\alpha_\ell} K_t(i)^{\alpha_k}$$

$$\alpha_e, \alpha_\ell, \alpha_k \geq 0, \quad \alpha_e + \alpha_\ell + \alpha_k \leq 1$$

Intermediate Good Firms

Intermediate Firms

$$Q_t(i) = A_t E_t(i)^{\alpha_e} L_t(i)^{\alpha_\ell} K_t(i)^{\alpha_k}$$

$$\alpha_e, \alpha_\ell, \alpha_k \geq 0, \quad \alpha_e + \alpha_\ell + \alpha_k \leq 1$$

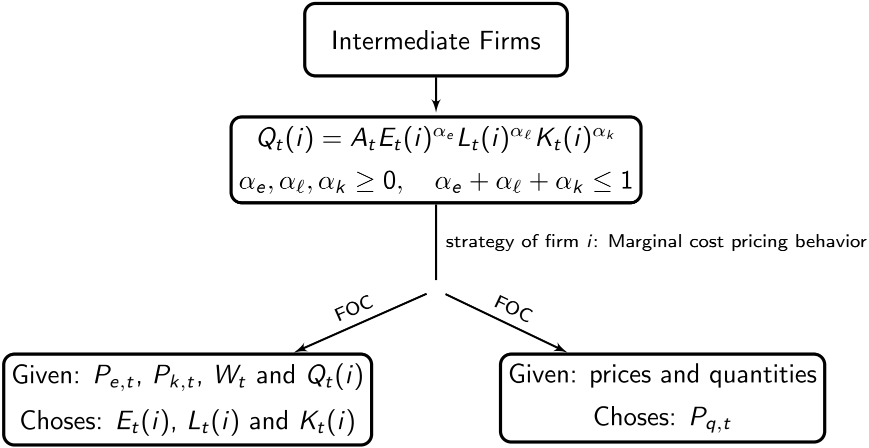
strategy of firm i: Marginal cost pricing behavior

FOC

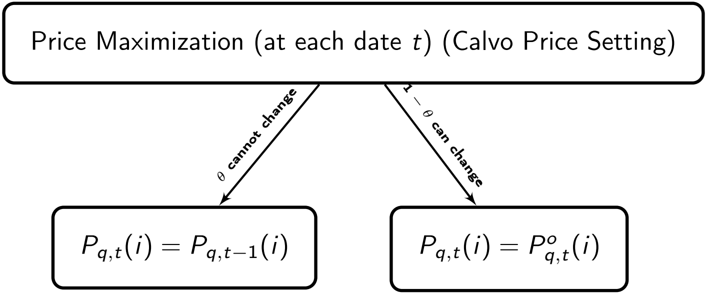
Given: $P_{e,t}$, $P_{k,t}$, W_t and $Q_t(i)$

Choses: $E_t(i)$, $L_t(i)$ and $K_t(i)$

Intermediate Good Firms



Price Optimization



Outline

GDP and GDP Deflator Definition

GDP
(in value added)

↓

$$P_{y,t}Y_t = P_{q,t}Q_t - P_{e,t}E_t$$

GDP and GDP Deflator Definition

GDP (in value added)

↓

$$P_{y,t} Y_t = P_{q,t} Q_t - P_{e,t} E_t$$

GDP Deflator

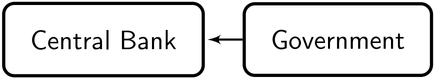
↓

$$P_{y,t} = P_{c,t}$$

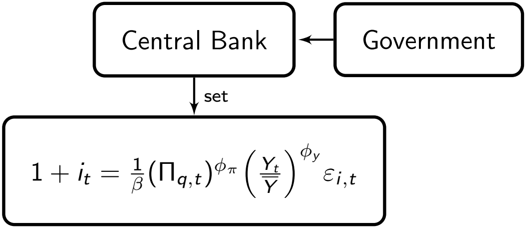
Government

Government

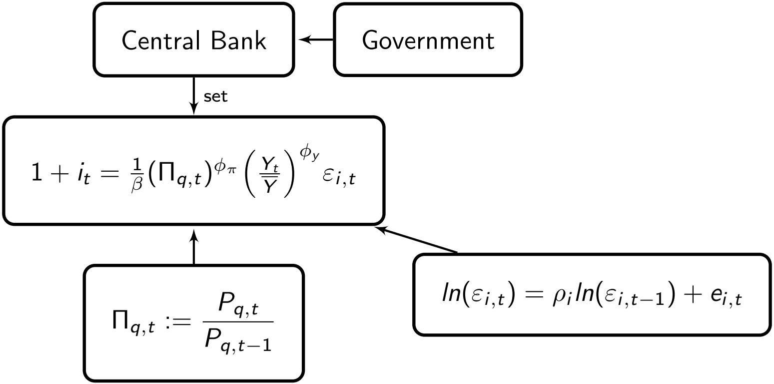
Government



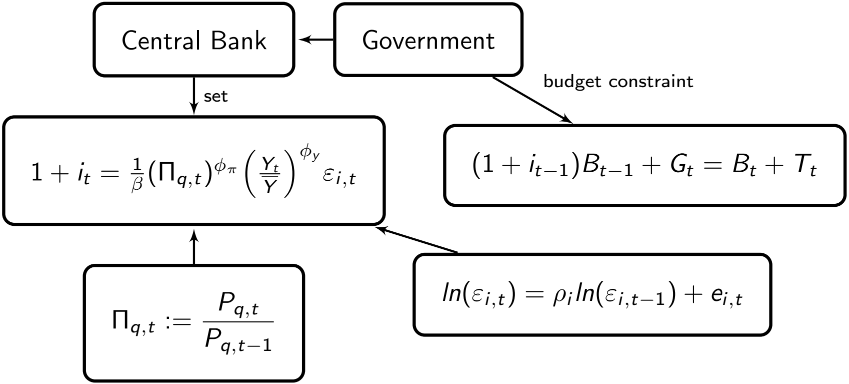
Government


Central Bank
Government
set
$1 + i_t = \frac{1}{\beta}(\Pi_{q,t})^{\phi_\pi}\left(\frac{Y_t}{\overline{Y}}\right)^{\phi_y}\varepsilon_{i,t}$

Government

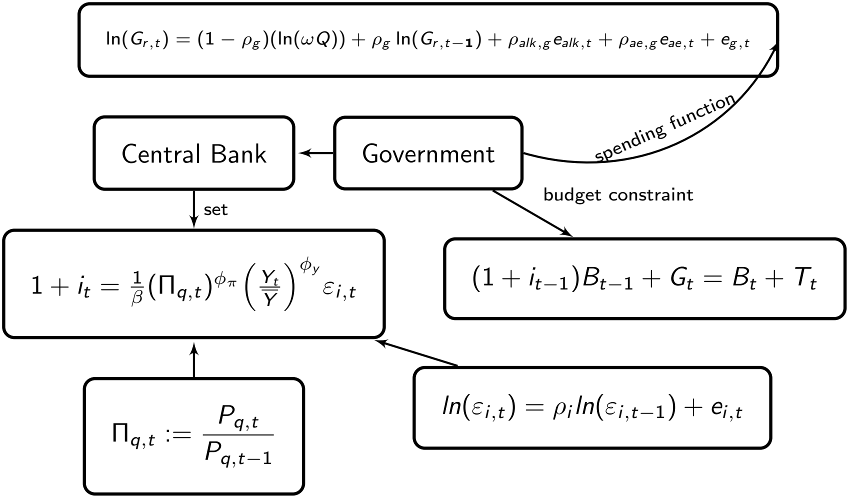

Central Bank
Government
set
$1 + i_t = \frac{1}{\beta}(\Pi_{q,t})^{\phi_\pi} \left(\frac{Y_t}{\overline{Y}}\right)^{\phi_y} \varepsilon_{i,t}$
$\Pi_{q,t} := \frac{P_{q,t}}{P_{q,t-1}}$
$ln(\varepsilon_{i,t}) = \rho_i ln(\varepsilon_{i,t-1}) + e_{i,t}$

Government

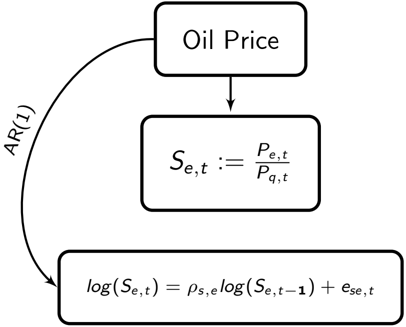


Central Bank
Government
set
budget constraint
$1 + i_t = \frac{1}{\beta}(\Pi_{q,t})^{\phi_\pi}\left(\frac{Y_t}{\overline{Y}}\right)^{\phi_y}\varepsilon_{i,t}$
$(1 + i_{t-1})B_{t-1} + G_t = B_t + T_t$
$\Pi_{q,t} := \frac{P_{q,t}}{P_{q,t-1}}$
$ln(\varepsilon_{i,t}) = \rho_i ln(\varepsilon_{i,t-1}) + e_{i,t}$

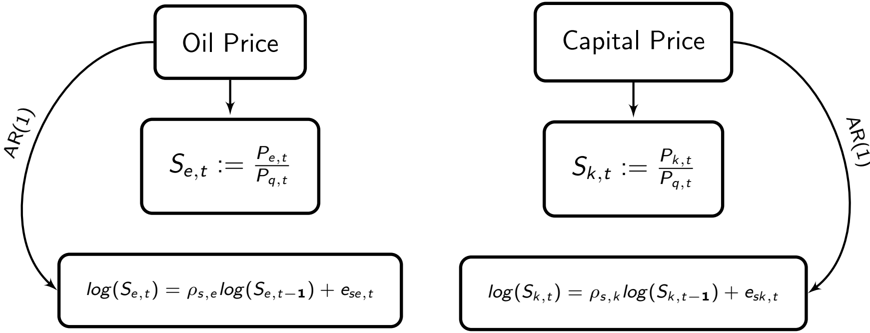
Government


$\ln(G_{r,t}) = (1-\rho_g)(\ln(\omega Q)) + \rho_g \ln(G_{r,t-1}) + \rho_{alk,g} e_{alk,t} + \rho_{ae,g} e_{ae,t} + e_{g,t}$
spending function
Central Bank
Government
set
budget constraint
$1 + i_t = \frac{1}{\beta}(\Pi_{q,t})^{\phi_\pi} \left(\frac{Y_t}{\bar{Y}}\right)^{\phi_y} \varepsilon_{i,t}$
$(1 + i_{t-1})B_{t-1} + G_t = B_t + T_t$
$\Pi_{q,t} := \frac{P_{q,t}}{P_{q,t-1}}$
$ln(\varepsilon_{i,t}) = \rho_i ln(\varepsilon_{i,t-1}) + e_{i,t}$

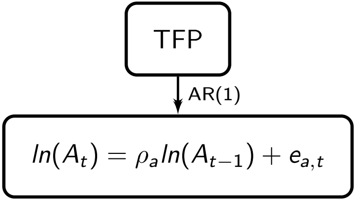
Other Shocks



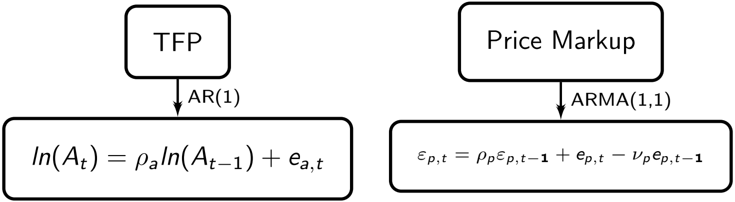
Other Shocks



Other Shocks



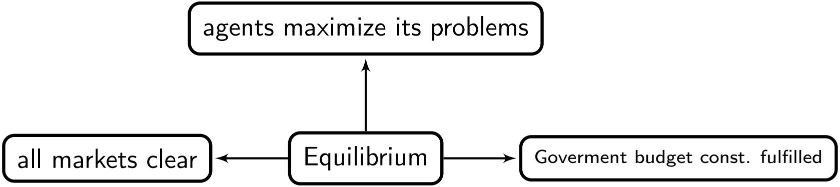
Other Shocks



Definition of Equilibrium

Equilibrium

Definition of Equilibrium



Outline

Data

Observed Variable	Transformation
invobs	$detrend\left(ln\left(\frac{\frac{PFI}{GDPDEF}}{LNSIndex}\right)*100\right)$
yobs	$detrend\left(ln\left(\frac{GDPC09}{LNSIndex}\right)*100\right)$
labobs	$ln\left(\frac{Averagehours*CE16OVIndex}{LNSIndex}\right)*100-mean\left(ln\left(\frac{Averagehours*CE16OVIndex}{LNSIndex}\right)*100\right)$
infobs	$ln\left(\frac{GDPDEF}{GDPDEF(-1)}\right)*100-mean\left(ln\left(\frac{GDPDEF}{GDPDEF(-1)}\right)*100\right)$
iobs	$\left(ln\left(1+\frac{FEDFUND}{400}\right)-mean\left(ln\left(1+\frac{FEDFUND}{400}\right)\right)\right)*100$
eobs	$ln\left(\frac{TotalSAOil}{LNSIndex}\right)*100-mean\left(ln\left(\frac{TotalSAOil}{LNSIndex}\right)*100\right)$

Calibrated Parameters

β	δ	ω	x	ϵ
0.99	0.025	0.18	0.023	8

Table : Calibrated Parameters

Estimation Results - θ estimated

Parameter		Prior distribution	Posterior distribution			
			Mode	Mean	10%	90%
θ estimated						
Capital elasticity	α_k	IGamma(0.1,2)	0.3728	0.3599	0.3380	0.3822
Labor elasticity	α_ℓ	IGamma(0.4,2)	0.6424	0.6411	0.6111	0.6745
Oil elasticity	α_e	IGamma(0.6,2)	0.1234	0.1254	0.1051	0.1460
Inverse Frisch elasticity	ϕ	IGamma(1.17,0.5)	0.6209	0.6308	0.4736	0.8019
Taylor rule response to inflation	ϕ_π	Normal(1.2,0.1)	1.2235	1.2253	1.0686	1.3558
Taylor rule response to output	ϕ_y	Normal(0.5,0.1)	0.8020	0.7882	0.6884	0.8876
Calvo price parameter	θ	Beta(0.5,0.1)	0.9812	0.9812	0.9380	0.9883

Table : Prior and Posterior Distribution of Structural Parameters

Estimation Results - θ estimated

Table : Prior and Posterior Distribution of Shock Parameters

Parameter		Prior distribution	Posterior distribution			
			Mode	Mean	10%	90%
Autoregressive parameters						
Technology	ρ_a	Beta(0.5,0.2)	0.8619	0.8481	0.7960	0.8999
Real oil price	ρ_{se}	Beta(0.5,0.2)	0.5761	0.5611	0.4629	0.6669
Real capital price	ρ_{sk}	Beta(0.5,0.2)	0.7210	0.7080	0.6647	0.7524
Price markup1	ρ_p	Beta(0.5,0.2)	0.9418	0.9283	0.8955	0.9640
Price markup2	ν_p	Beta(0.5,0.2)	0.9796	0.9760	0.9610	0.9913
Government	ρ_g	Beta(0.5,0.2)	0.9058	0.8995	0.8712	0.9258
Tech. in Gov.	ρ_{ag}	Beta(0.5,0.2)	0.6904	0.6127	0.3549	0.9472
Monetary	ρ_i	Beta(0.5,0.2)	0.9399	0.9308	0.9035	0.9581
Standard deviations						
Technology	σ_a	IGamma(1,2)	0.4361	0.4435	0.3901	0.4942
Real oil price	σ_{se}	IGamma(1,2)	2.0000	1.9373	1.8652	2.000
Real capital price	σ_{sk}	IGamma(1,2)	0.7740	0.7675	0.6379	0.8781
Price markup	σ_p	IGamma(1,2)	0.1814	0.1854	0.1615	0.2094
Government	σ_g	IGamma(1,2)	2.0000	1.7921	1.5508	1.9998
Monetary	σ_i	IGamma(1,2)	0.5410	0.4566	0.3859	0.5205

Estimation Results - θ calibrated

Parameter		Prior distribution	Posterior distribution			
			Mode	Mean	10%	90%
θ calibrated						
Capital elasticity	α_k	IGamma(0.2,2)	0.3918	0.3809	0.3624	0.3989
Labor elasticity	α_ℓ	IGamma(0.4,2)	0.5947	0.5966	0.5622	0.6305
Oil elasticity	α_e	IGamma(0.5,2)	0.1132	0.1177	0.0915	0.1434
Inverse Frisch elasticity	ϕ	IGamma(1.17,0.5)	1.2562	1.2625	0.9073	1.6069
Taylor rule response to inflation	ϕ_π	Normal(1.2,0.1)	1.5236	1.5307	1.3883	1.6722
Taylor rule response to output	ϕ_y	Normal(0.5,0.1)	0.0265	0.0214	0.0001	0.0402

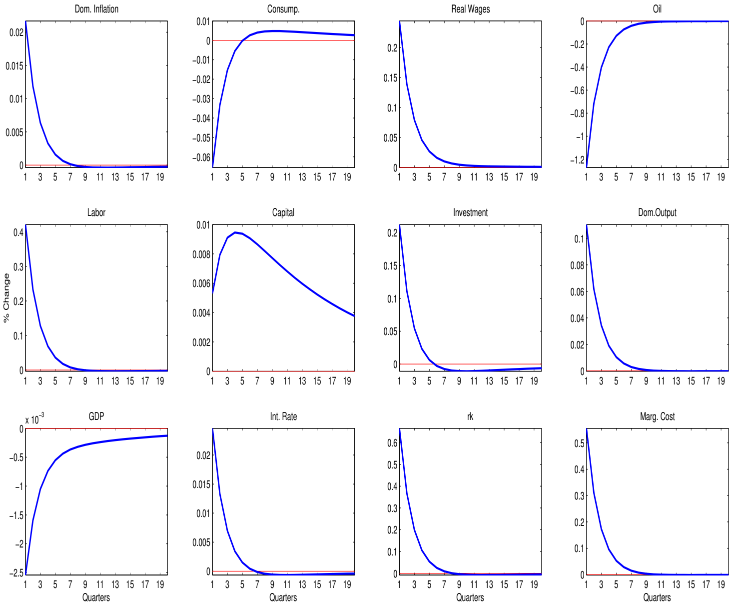
Table : Prior and Posterior Distribution of Structural Parameters

Estimation Results - θ calibrated

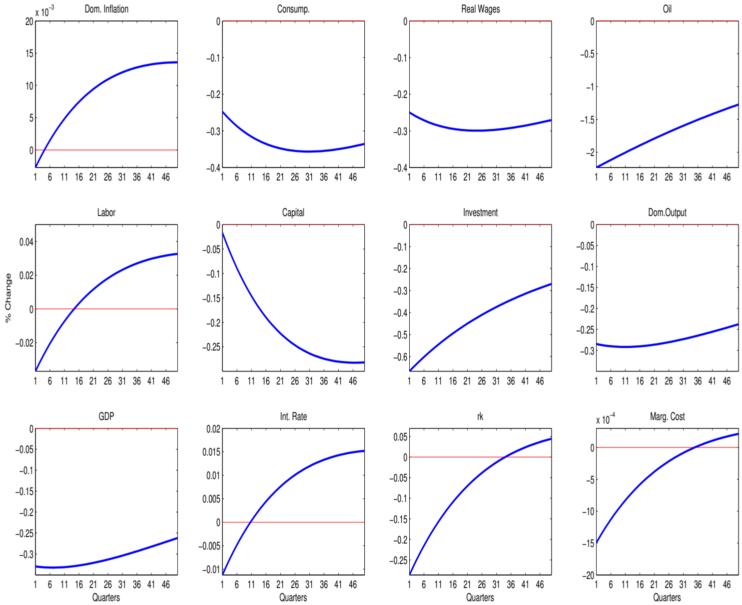
Table : Prior and Posterior Distribution of Shock Parameters

Parameter		Prior distribution	Posterior distribution			
			Mode	Mean	10%	90%
Autoregressive parameters						
Technology	ρ_a	Beta(0.5,0.2)	0.9605	0.9401	0.9033	0.9774
Real oil price	ρ_{se}	Beta(0.5,0.2)	0.9934	0.9872	0.9754	0.9977
Real capital price	ρ_{sk}	Beta(0.5,0.2)	0.8940	0.8924	0.8483	0.9314
Price markup1	ρ_p	Beta(0.5,0.2)	0.9839	0.9621	0.9299	0.9971
Price markup2	ν_p	Beta(0.5,0.2)	0.1652	0.1711	0.0593	0.2758
Government	ρ_g	Beta(0.5,0.2)	0.9373	0.9312	0.9061	0.9560
Tech. in Gov.	ρ_{ag}	Beta(0.5,0.2)	0.7129	0.6589	0.3808	0.9541
Monetary	ρ_i	Beta(0.5,0.2)	0.1914	0.2104	0.1249	0.2856
Standard deviations						
Technology	σ_a	IGamma(1,2)	0.4538	0.4542	0.3981	0.5078
Real oil price	σ_{se}	IGamma(1,2)	2.0000	1.9475	1.8842	2.000
Real capital price	σ_{sk}	IGamma(1,2)	0.5459	0.5750	0.4722	0.6714
Price markup	σ_p	IGamma(1,2)	0.4235	0.4645	0.2868	0.6602
Government	σ_g	IGamma(1,2)	2.0000	1.8359	1.6425	2.000
Monetary	σ_i	IGamma(1,2)	0.4778	0.4769	0.4062	0.54555

Outline



IRF to a Real Oil Price Shock. Case: θ Estimated



IRF to a Real Oil Price Shock. Case: θ Calibrated

Optimization

First Order Conditions

Euler:

$$1 = \beta \mathbb{E}_t \left[(1+i_t) \frac{C_t}{C_{t+1}} \frac{P_{c,t}}{P_{c,t+1}} \right]$$

competive labor supply sch.:

$$\frac{W_t}{P_{c,t}} = C_t L_t^{\phi}$$

Fisher:

$$1 = \beta \mathbb{E}_t \left[\frac{C_t}{C_{t+1}} \frac{P_{c,t}}{P_{c,t+1}} \frac{P_{k,t+1}}{P_{k,t}} (r_{t+1}^k + 1 - \delta) \right]$$

No Ponzi Scheme

Transversality condition (no Ponzi Scheme)

$$\lim_{k\to\infty} \mathbb{E}_t \left(\frac{B_{t+k}}{\prod_{s=0}^{t+k-1} (1+i_{s-1})} \right) \geq 0, \quad \forall t.$$

Stochastic Discount Factor

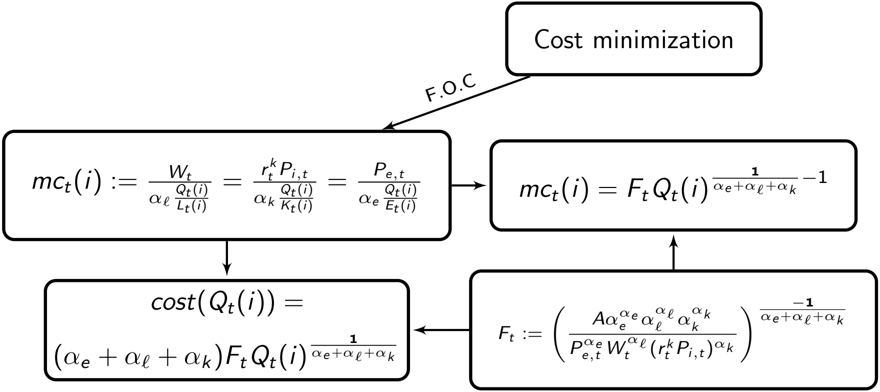
1. from date t to date $t+1$

$$d_{t,t+1} := \frac{\beta U_C(C_{t+1}, L_{t+1})}{U_C(C_t, L_t)} \frac{P_{c,t}}{P_{c,t+1}}, i.e, \quad \frac{1}{1+i_t} = \mathbb{E}_t(d_{t,t+1}).$$

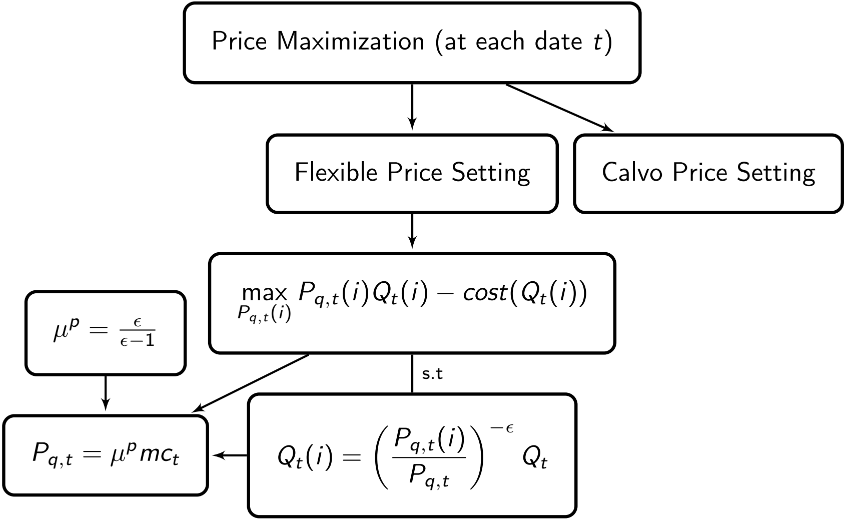
2. from date t to date $t+k$

$$d_{t,t+k} := \prod_{s=t}^{t+k-1} \Delta_s^{s+1}, \text{ then}, \quad d_{t,t+k} := \frac{\beta^k U_C(C_{t+k}, L_{t+k})}{U_C(C_t, L_t)} \frac{P_{c,t}}{P_{c,t+k}}.$$

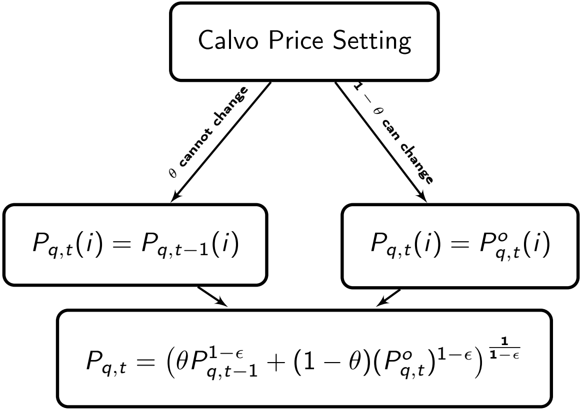
Cost Minimization



Price Optimization



Calvo Price Setting



Calvo Price Setting

Calvo Price Setting Problem

$$\max_{P_{q,t}(i)} \mathbb{E}_t \left[\sum_{k=0}^{\infty} \theta^k d_{t,t+k} \left[P_{q,t}(i) Q_{t,t+k}(i) - cost(Q_{t,t+k}(i)) \right] \right]$$

s.t

$$Q_{t,t+k}(i) = \left(\frac{P_{q,t}(i)}{P_{q,t+k}} \right)^{-\epsilon} Q_{t+k}, \quad \forall k \geq 0$$

Calvo Price Setting