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# Carbon dating: When is it beneficial to link ETSs?[*]

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### Abstract

This paper proposes a simple framework to evaluate the economic advantage of regulating carbon emissions by linking the emissions trading systems (ETSs) of two jurisdictions versus operating them under autarky. The ETSs are linked if the permits issued in one, and traded competitively across both, can be surrendered against emissions in the other. The paper's main innovation is in analyzing the sensitivity of aggregate and jurisdiction-specific economic advantage to the characteristics of the jurisdictions, in particular the uncertainty affecting the benefits of emissions. We decompose the economic advantage of linking into pair size, volatility and dependence effects. We show that when jurisdictions are ex ante identical and there are no tax distortions or sunk costs, the aggregate economic advantage is always non-negative and equally shared. It increases in pair size and in volatilities of jurisdiction-specific shocks but decreases in their correlation. In other words, there are only good and better links. With differences in ETS size the economic advantage is not equally shared, and the smaller jurisdiction receives a larger share. That is, linking partners may not value the link equally. When we additionally introduce sunk costs of linking, one jurisdiction may prefer an ETS under autarky to linking even when aggregate economic advantage is positive. A similar conclusion emerges with unilateral tax distortions affecting international permit trade. In an empirical application, we calibrate shock characteristics to the observed fluctuations in data from the world's 20 largest emitters and document substantial variation in economic advantage and its components.

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# 1 Introduction

Markets for emission permits have been an important climate policy tool in driving emission reduction efforts in a cost-effective and flexible way. World Bank (2014) identifies one region consisting of 31 nations, four individual nations and 13 sub-national jurisdictions which currently regulate carbon emissions using emissions trading systems (ETSs), and 13 additional ETSs are at various stages of development. Until recently the architecture supporting the emergence and expansion of these markets has largely been top-down, governed by international negotiations of emission reduction commitments by individual nations. However, the increasing number of planned and proposed trading systems at national, sub-national, and regional jurisdictions suggests that a bottom-up policy architecture in which these systems interact will be a significant element of the global climate change policy framework in the future. In fact, some of these trading programs have decided to link together, meaning one program recognizes the other program's permits for compliance in its system and vice versa.[1] For example, Quebec and California have chosen to link their programs. The Regional Greenhouse Gas Initiative (RGGI) in northeastern United States is effectively a system of linked ETSs, and so is the European Union Emission Trading System (EU ETS).

Connecting trading programs through linking strengthens their economic efficiency and has the potential to reinforce the international policy architecture politically. In this paper we study how the economic advantage of linking over autarky depends on the shocks affecting the linked jurisdictions, the jurisdictions' relative size, and tax distortions in international permit trade. Our theoretical model suggests that the value of linking two jurisdictions of equal size is nonnegative provided sunk costs of linking are not prohibitive.[2] Although true in aggregate, this may not hold at the country level when the sizes of the jurisdictions differ, or when there are distortions in international permit trade. Put differently, some linking partner matches, what we call carbon dates, generate greater value than others. It may even be the case that in some carbon dates one partner can be worse off than going it alone. When looking for a carbon date, our model can help answer the question: who is a good match?

To analyse this question we adapt the static model in Weitzman (1974) to two jurisdictions and to the climate change context. Our model is similar in spirit to the multi-firm case considered in Yohe (1976). However, in Yohe (1976) shocks are identically distributed and the comparative advantage of a uniform tax over a quantity standard is computed as function of industry size.[3] In this paper we are interested in the difference between the net benefits associated with two quantity instruments operated under linking and autarky. In our model, the trading programs are linked if the permits issued in one jurisdiction can be surrendered against compliance requirements in the other. Throughout we assume competitive permit trading and require that the decision to link must be made ex ante.

In this setting, we characterize the second-best policy and derive expressions for aggregate and individual advantages of linking over autarky. We decompose aggregate advantage into three readily interpretable quantities: the volatility effect, the dependence effect, and the pair size effect. Each of these quantities is related to shock properties and jurisdiction sizes, which we refer to as *pair characteristics*. In particular,

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[1] There are also examples of delinking. See Ranson and Stavins (2015) and Pizer and Yates (2015).

[2] The sunk costs may include but are not limited to negotiating the linking agreement, harmonizing the rules of the previously independent systems, and setting up a platform to facilitate the international transactions of permits.

[3] Our setup is also similar to De Meza and Van der Ploeg (1987) who study locational incentives and investment decisions of a multinational firm when production plants of equal size are subject to distinct shocks.

the volatility effect captures the total amount of volatility in the linked systems; the dependence effect is a simple function of the correlation and volatility of the shocks affecting the two jurisdictions; and the pair size effect scales the sum of the volatility and dependence effects.

Several papers consider the benefits and costs of linking (Flachsland et al. (2009), Jaffe et al. (2009), Ranson and Stavins (2015)). These papers describe three main mechanisms by which linking leads to cost savings: cost equalisation, reduced volatility and increased liquidity. When expected abatement costs in linking countries differ under autarky, the initially low-price jurisdiction exports permits at prices above its autarky cost and the initially high-price jurisdiction imports permits at prices below its autarky cost. A simple arbitrage argument guarantees that the expected price differential is instantly eliminated post linking. The analysis of cost savings can however be extended beyond this instantaneous arbitrage effect and this is where our paper makes a contribution.

As described by Flachsland et al. (2009) and Jaffe et al. (2009), linked jurisdictions will tend to experience reduced volatility because local shocks are spread over a larger market. Our analysis shows that this is not always the case. When the shocks are positively correlated, the initially low-volatility jurisdiction 'imports' some volatility from the initially high-volatility jurisdiction; yet, it is well compensated for doing so. The size of this compensation is determined by *pair characteristics* which is key to the value of the linking arrangement. The jurisdiction-specific shock variances contribute equally to the value of linking through the volatility effect. The dependence effect captures the correlation of shocks: it is decreasing in their covariance but in absolute value it can never be larger than the volatility effect. Finally, some papers consider the relevance of increased liquidity and argue that the largest economic benefit comes from linking large ETSs. Our pair size effect captures and qualifies this result. Pair size effect is increasing in each of its arguments, the sizes of the individual jurisdictions in a link. Crucially, the increase in the aggregate value consequent to an increase in the size of a given jurisdiction makes it and its partner better off, but not equally so because the latter captures a greater share of the increase in value.

To put these analytical results into context, we calibrate *pair characteristics* to historical emissions data and evaluate country specific and aggregate economic advantage of linking hypothetical ETSs in the world's 20 largest emitters. This exercise demonstrates that there is substantial, economically meaningful and policy relevant variation in the empirical counterparts of the effects described in the previous paragraph.

Some care must be taken with the interpretation of these results. In particular, we do not view them as a precise guide for jurisdictions currently contemplating a link, but rather as first pass analysis of the economic cost savings that feature in the policymakers' calculus. Surely the decision to create a link will be based on a variety of considerations beyond cost-effectiveness. While our results shed new light on how the bottom-up international architecture of tradable permit programs could evolve given the cost savings, the existing literature has explored other non-economic benefits as well as the considerable political and regulatory challenges that could arise in the context of linking.

For example, linking provides opportunities to improve the administration and governance of linked permit markets. Insofar as linking leads to the alignment of the administration and design of markets, it streamlines the compliance process and can lead to reduced administrative costs for business operating in those jurisdictions. Moreover, the benefits of linking can have ramifications that go beyond the geographical jurisdiction of the linking partners. Indeed, linking can lead to a leveling of the international playing field and to an improved support of global cooperation for tackling climate change. At the same time, the process of linking can require significant and costly efforts that may discourage it despite the potential benefits. These include the alignment of technical requirements (e.g. monitoring, reporting and verification (MRV) and tracking systems) and of design features (e.g. level of ambition, mode of allocation, inter-temporal flexibility, price management rules) all of which have to be negotiated. Papers focusing on various aspects of these issues include Flachsland et al. (2009), Ranson and Stavins (2015), Burtraw et al. (2013) and Bodansky et al. (2014).

The rest of the paper is organised as follows. Section 2 introduces the model, derives the second-best policies and defines jurisdiction-specific and aggregate advantage of linking over autarky. The analytical results are presented in Section 3, which contains the main proposition about the magnitude of the advantage, how it is shared and its components. Section 4 illustrates the analytical results calibrating the pair characteristics to historical emissions data. Section 5 extends the model to the case where there are tax distortions on international permit trades. A simulation exercise is used to show the impact of unilaterally imposed taxes. Section 6 concludes.

## 2 Theoretical model and equilibrium

When a regulator uses an emissions trading system (ETS) to constrain the aggregate polluting emissions of regulated sources, e.g. the polluting firms, a limited number of tradable emission permits are created which the firms must surrender in number equal to their emissions. The firms can obtain these permits from the regulator, who may auction or freely allocate them, or from other firms in the market at a mutually agreeable price. Competitive equilibrium ensures that regulated firms have exhausted trades at such price, and as a consequence all firms face the same permit price. When the two ETSs in different jurisdictions are linked, permits issued in one can be surrendered against emissions in the other. In what follows we assume that the prices of permits in the linked markets are also determined competitively.

### 2.1 Description of the environment

Our analysis relies on a simple static model that specializes Weitzman (1974) and Yohe (1976) to the case of quantity-based policies designed to regulate the climate change externality in different countries.[4] For simplicity, we consider the case of two ETSs being implemented by two countries indexed $\{i = 1, 2\}$. The total benefits from emissions are a function of the level of emissions $q_i \geq 0$ and are subject to country-specific shocks $\theta_i$

$$B_i(q_i, \theta_i) \quad = \quad b_0 + (b_1 + \theta_i)q_i - \frac{b_2}{2\psi_i} q_i^2 \quad \text{where} \quad i = 1, 2. \tag{1}$$

There are four parameters in the benefit function. Three of these, namely $b_0, b_1, b_2 \geq 0$, are identical across countries and capture constant, linear, and quadratic behavior of benefits with respect to emissions. The parameter $\psi_i > 0$ controls the level of emissions in country $i$. Specifically, $\psi_1 > \psi_2$ implies that emissions

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[4] The term jurisdiction is more appropriate since emissions trading systems can be set up, and linked, at sectoral, subnational, national or regional levels. We use country for brevity.

in country 1 are greater than in country 2 when both countries face the same shocks and permit prices.[5] Although it is reasonable to think of $\psi_i$ as the country level emissions, in principle directly comparable to the ETS size, $\psi_i > \psi_j$ does not necessarily imply that country $i$ is larger along other relevant dimensions.[6] Below we show that the sum $\psi_1 + \psi_2$ is a factor in determining the aggregate and country specific value of linking a particular pair of countries. Intuitively, the value of linking two small ETSs versus the alternative of operating them in autarky is smaller than the value of linking two large ETSs versus the alternative of two large separate systems.

We assume that country-specific shocks are limited to the intercepts of the marginal benefit schedules. These shocks capture the net effect of all factors that may influence emissions and their associated benefits such as business cycle and/or technology shocks, country-specific events, changes in the prices of factors of production, weather fluctuations etc. For example, a favorable aggregate total factor productivity shock would increase the benefits of emissions and in our model would correspond to $\theta_i > 0$.[7]

The shocks' distributions and their relation, their variance-covariance matrix, are at the heart of our analysis. Minimal restrictions are imposed on them. In particular, shocks are mean-zero, constant variance and possibly correlated random variables. That is, for $i = 1, 2$ we define

$$\begin{aligned} \mathbb{E}(\theta_i) &= 0; \\ V(\theta_i) &= \sigma_i^2; \\ \mathrm{Corr}(\theta_1, \theta_2) &= \rho \in [-1, 1]. \end{aligned} \tag{2}$$

Also, we assume that $b_1 + \theta_i > 0$ for every possible realization of the shock. This assumption ensures that, without regulation the marginal benefit of emissions is always positive and the emission control problem under investigation is non trivial. Further, we define $q_i^{BAU} = \psi_i b_1 / b_2$ as the business-as-usual emissions in the absence of regulation and for the average shock realization, $\mathbb{E}(\theta_i) = 0$.

Carbon dioxide is a uniformly mixed stock pollutant and total climate change damages in each country are a function of aggregate quantity emitted, $q_1 + q_2$. Accordingly, we have

$$D_i(q_1 + q_2) = d_0 + d_1(q_1 + q_2) + \frac{d_2}{2}(q_1 + q_2)^2, \tag{3}$$

where $d_0, d_1, d_2 \geq 0$. As with benefits, the coefficients $d_0$, $d_1$, and $d_2$ capture constant, linear and quadratic behavior. Note that the level of aggregate damages corresponds to the sum $D_1(q_1 + q_2) + D_2(q_1 + q_2)$.

The combination of shock characteristics and country sizes in a given pair is central to our analysis of the economic advantage of linking over autarky. We refer to it as *pair characteristics* which is given by the set $\{(\psi_1, \sigma_1), (\psi_2, \sigma_2), \rho\}$. Finally, we need to introduce the cost of linking which we denote by $\epsilon \geq 0$. We assume that aggregate sunk costs are exogenous, proportional to the size of the linked systems, and

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[5] There is an alternative interpretation: $b_2/\psi_i$ captures differences in country specific marginal abatement costs. $b_2/\psi_1 < b_2/\psi_2$ is observationally equivalent to marginal costs being higher in country 1 than in country 2 when both countries face the same shocks and target abatement.

[6] For example, on average Canadian emissions are greater than Brazilian emissions, e.g. $\psi_{CAN} > \psi_{BRA}$. This is true despite the fact that Brazil's real GDP and population are, respectively, twice and five times larger than Canada's.

[7] In the quantitative analysis below, we interpret these shocks as the cyclical components of emissions obtained using the Hodrick-Prescott filter on annual country emissions data.

shared according to country size. That is, given pair characteristics, the total linking cost is $(\psi_1 + \psi_2)\epsilon$ where country 1 incurs $\psi_1\epsilon$ and the rest accrue to country 2.

In passing, we note that the climate change context imposes weak conditions on some of the parameters of the model. These restrictions have been extensively discussed in the literature, e.g. Newell and Pizer (2003) and references therein, and are relatively uncontroversial. Specifically, abatement costs associated to greenhouse gases are convex while abatement benefits are approximately linear. In our setting these conditions imply $0 \approx d_2 \ll b_2$. Following Weitzman (1974) and Yohe (1976), we assume $d_1 + 2d_2 q^{BAU} > 0$ so that it is socially optimal to restrict emissions for the average shock realization.

## 2.2 Second best emissions

Given this set up, we solve the control problem in a second best world where emission caps must be fixed ex ante.[8] To this end, we maximize the expected aggregate net benefits. Formally, the program is

$$\max_{\{q_1 \geq 0, q_2 \geq 0\}} \quad \mathbb{E}\left[B_1(q_1, \theta_1) - D_1(q_1 + q_2) + B_2(q_2, \theta_2) - D_2(q_1 + q_2)\right]. \tag{4}$$

The solution to the problem in (4) is denoted by a pair of emissions quotas $\{\bar{q}_1, \bar{q}_2\}$ which is obtained by setting expected marginal benefits equal to *aggregate* marginal damages:

$$\begin{aligned} \bar{q}_i &= \frac{\psi_i (b_1 - 2d_1)}{2d_2(\psi_1 + \psi_2) + b_2}, \\ \bar{Q} &= \frac{(\psi_1 + \psi_2)(b_1 - 2d_1)}{2d_2(\psi_1 + \psi_2) + b_2}. \end{aligned} \tag{5}$$

We assume that regulators in each country set their emissions target equal to these quotas *under both autarky and linking* so that aggregate outcomes under the two regimes are comparable. As we will see, with second best quotas, expected autarky prices in each country and the expected linking price are equal.

## 2.3 Autarky and linking equilibria

Given these second-best quotas, a regulator in each country faces a choice between operating an ETS under autarky, where the equilibrium is denoted by the two pairs $\{(p_{A1}, q_{A1}), (p_{A2}, q_{A2})\}$, versus linking the system with the other country's ETS, in which case the equilibrium is given by the triple $\{p_L, q_{L1}, q_{L2}\}$. The regulators will take a decision comparing the level of expected net benefits under autarky and under linking. The next section is devoted to examining this comparison in detail. In what follows we characterize the autarky and linking equilibria.

We assume that permit trading is competitive, and that the systems are linked only when both regulators

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[8] The solution for the first best level of emissions is provided in the Appendix for reference.

decide to link. Under these conditions, the autarky equilibrium in country $i$ is given by

$$(p_{Ai}, q_{Ai}) = \begin{cases} (b_1 - \frac{b_2}{\psi_i}\bar{q}_i + \theta_i, \bar{q}_i) & \text{if}\, \theta_i > \frac{b_2}{\psi_i}\bar{q}_i - b_1, \\ \\ (0, \frac{\psi_i(b_1+\theta_i)}{b_2}) & \text{if}\, \theta_i > -b_1. \end{cases} \tag{6}$$

When the cap is binding, $q_{Ai} = \bar{q}_i$, the equilibrium price is positive and it is determined by the country-specific shock, $\theta_i$; whereas when the cap is not binding, $q_{Ai} < \bar{q}_i$, the equilibrium price is zero.[9] We refer to the former case as an *interior autarky equilibrium* (IAE). In the IAE, $p_{Ai}$ is increasing in $\theta_i$ and, unsurprisingly, does not depend on the other country's shock.

Under autarky the two regimes are isolated; there is no transmission channel for shocks. In fact, even if the countries are ex ante identical, *ex-post* prices typically differ, $p_{A1} \neq p_{A2}$. Such a price difference indicates that emission reductions are not efficiently allocated across countries. Conversely, under linking the ex post price difference that we would have observed under autarky is eliminated: permits flow from one country to the other until prices are equalized.[10] In particular, the country with the higher shock will import permits because regulated entities place a greater value on permits. Linking, therefore, increases the effective cap in the high-shock country and reduces it by the same amount in the low-shock country leaving the aggregate cap unchanged.

In order to characterize the equilibrium under linking we define $n \in [-\bar{q}_2, \bar{q}_1]$ as the number of permits exported from country 1 to country 2 with the understanding that when $n < 0$, country 1 imports permits. We define an *interior linking equilibrium* (ILE) as the region where $p_L > 0$ and $q_{L1} > 0$ and $q_{L2} > 0$.[11] Then, in an ILE, equilibrium price and quantities are given by

$$(p_L, q_{L1}, q_{L2}) = \left(K + \frac{\psi_1\theta_1 + \psi_2\theta_2}{\psi_1 + \psi_2}, \bar{q}_1 - n, \bar{q}_2 + n\right), \tag{7}$$

where

$$n = \frac{1}{b_2}\frac{\psi_1\psi_2}{(\psi_1 + \psi_2)}(\theta_2 - \theta_1),$$

and the constant $K$ is defined by

$$K = b_1 - \frac{b_2(b_1 - 2d_1)}{b_2 + 2d_2(\psi_1 + \psi_2)}.$$

It helps intuition to discuss the case of two countries of equal size in more detail. Without loss of generality, we set size to 1, i.e. $\psi_1 = \psi_2 = 1$. Note that in this case, $\bar{q}_1 = \bar{q}_2 = \bar{q}$. The top panel of Figure 1 illustrates the permit market equilibria under autarky and linking for a given pair of shocks realization where $0 = \theta_2 < \theta_1$. Country 1 faces a positive cost shock; the red solid line represents the marginal benefit curve consistent with $\theta_1 > 0$. Country 2 faces a zero shock and its marginal benefit curve is described by the green solid line. Thus, the marginal benefit of emissions is greater in country 1; consequently

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[9] The condition in Section 2.1 that $b_1 + \theta_i > 0$ for all realizations of $\theta_i$ rules out the possibility that both emissions and permit prices are zero.

[10] This is the market-based analogue of the outcome approximated using regulator imposed trading ratios in Holland and Yates (2015).

[11] The conditions on the pair $(\theta_i, \theta_j)$ for an interior and corner solutions are provided in the appendix. In this context, a corner solution corresponds to $n = -\bar{q}_2$, $n = \bar{q}_1$, or when $n = 0$ and $p_L = 0$.

$p_{A1} > p_{A2}$. When the two systems are linked, country 1 imports permits from country 2 until the price difference is arbitraged away, which occurs when $|n|$ permits are traded across systems. In this case the linking equilibrium is interior because $|n| < \bar{q}_2$ and $p_L > 0$. Similarly, the autarky equilibria are both interior because $0 < p_{A2} < p_{A1}$.

The bottom panel of Figure 1 illustrates a $(\theta_1, \theta_2)$ pair consistent with the equilibrium solutions just discussed. In addition, the shaded area in the figure indicates the shock pairs for which autarky and linking equilibria are simultaneously interior, i.e. where IAE and ILE intersect. The autarky equilibrium is interior, i.e. the cap is binding in each country, for all $(\theta_1, \theta_2)$ pairs in the region to the northeast of the intersection of the orange lines. Similarly, linking equilibrium is interior for all $(\theta_1, \theta_2)$ pairs between the positively sloped blue lines *and* to the northeast of the negatively sloped blue line. The positively-sloped lines constrain $n$ to the interval $(-\bar{q}, \bar{q})$. However, in a subset of this region where both shocks are large and negative, $p_L = 0$. In this subset, the aggregate permit demand is less than $2\bar{q}$ even when the price of permits is zero. In other words, the aggregate cap in the linked markets is not binding. This is ruled out by the definition of ILE, and identified in the graph as the region below the negatively sloped line.

## 2.4 Welfare under linking versus autarky

We are finally in a position to address the question raised in the title of the paper. To that end we define the country-specific economic advantage of linking over autarky as the difference between the net benefits under linking minus the net benefits under autarky given exogenous sunk costs of linking, $\epsilon \geq 0$. We define aggregate economic advantage of linking over autarky as the sum of country-specific advantages. Formally, country-specific advantage can be written as the sum of private benefits net of permit costs, minus emission damages, plus initial permit holders' rents, under linking and under autarky, where the former must also account for the sunk costs of linking:

$$\begin{aligned}
\tilde{\delta}_1 &= [B_1(q_{L1}, \theta_1) - p_L q_{L1} - D_1(q_{L1} + q_{L2}) + p_L \bar{q}_1 - \psi_1 \epsilon] - [B_1(q_{A1}, \theta_1) - p_{A1} q_{A1} - D_1(q_{A1} + q_{A2}) + p_{A1} \bar{q}_1], \\
\tilde{\delta}_2 &= [B_2(q_{L2}, \theta_2) - p_L q_{L2} - D_2(q_{L1} + q_{L2}) + p_L \bar{q}_2 - \psi_2 \epsilon] - [B_2(q_{A2}, \theta_2) - p_{A2} q_{A2} - D_2(q_{A1} + q_{A2}) + p_{A2} \bar{q}_2], \\
\tilde{\Delta} &= \tilde{\delta}_1 + \tilde{\delta}_2.
\end{aligned}$$

We note that under autarky, permit costs and rents cancel out, since they represent a transfer between firms, or between the firms and the country's regulator. However, permit costs and rents differ under linking. In fact, when country 1 exports its permits, it reduces its emissions below $\bar{q}_1$ and sells unused permits at $p_L > p_{A1}$. Country 2, instead, imports permits and increases its emissions beyond its cap. Linking allows private benefits of emissions to increase, yet at a lower overall permit cost because $p_L < p_{A2}$.

One can simplify these expressions further by restricting attention to interior equilibria. To do so we drop the tilde to denote an interior equilibrium. Then,

$$\begin{aligned}
\delta_1 &= B_1(\bar{q}_1 - n, \theta_1) - B_1(\bar{q}_1, \theta_1) + p_L n - \psi_1 \epsilon, \\
\delta_2 &= B_2(\bar{q}_2 + n, \theta_2) - B_2(\bar{q}_2, \theta_2) - p_L n - \psi_2 \epsilon, \qquad (8) \\
\Delta &= [B_1(\bar{q}_1 - n, \theta_1) + B_2(\bar{q}_2 + n, \theta_2)] - [B_1(\bar{q}_1, \theta_1) + B_2(\bar{q}_2, \theta_2)] - (\psi_1 + \psi_2)\,\epsilon.
\end{aligned}$$

$\delta_i$ and $\Delta$ are random variables evaluated at equilibrium prices and allocations. In the next section, we show how $\mathbb{E}[\delta_i]$ and $\mathbb{E}[\Delta]$ depend on the variance-covariance matrix of the shocks. We will refer to the expected country-specific and expected aggregate advantage of linking over autarky by simply using the terms individual (or country-specific) and aggregate advantages, respectively. Below we will also restrict our attention to interior equilibria. In essence this is a restriction that the cap is sufficiently stringent given the volatility of the shocks. Such a restriction allows substantial simplifications, in particular the damages under autarky and linking are equal. Moreover, under this restriction there is a uniquely determined linking price $p_L$.[12]

## 3 Analytical results

This section derives an expression for the country-specific and aggregate advantage of linking and discusses how its distinct components relate to the shock characteristics and the sizes of the ETSs being linked. Throughout, we assume that all parameters other than the *pair characteristics* are the same across countries and that the second best quotas in (5) are imposed so autarky and linking equilibria are described by the expressions in (6) and (7). Proposition 1 is the fundamental result of this paper.

**Proposition 1.** *Fix pair characteristics* $\{(\psi_1, \sigma_1), (\psi_2, \sigma_2), \rho\}$ *and let* $\epsilon \geq 0$. *Define pair size effect (PSE), volatility effect (VE) and dependence effect (DE) as*

$$\begin{aligned}
PSE(\psi_1, \psi_2) &= \frac{\psi_1 \psi_2}{2b_2\,(\psi_1 + \psi_2)}, \\
VE(\sigma_1, \sigma_2) &= \sigma_1^2 + \sigma_2^2, \\
DE(\sigma_1, \sigma_2, \rho) &= -2\sigma_1 \sigma_2 \rho.
\end{aligned}$$

*Then*

$$\begin{aligned}
E\,[\Delta] &= PSE\,(VE + DE) - (\psi_1 + \psi_2)\,\epsilon, \\
E\,[\delta_1] &= \frac{\psi_2}{\psi_1 + \psi_2} PSE\,(VE + DE) - \psi_1 \epsilon, \\
E\,[\delta_2] &= \frac{\psi_1}{\psi_1 + \psi_2} PSE\,(VE + DE) - \psi_2 \epsilon.
\end{aligned}$$

*Proof: see Appendix.*

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[12] See the Appendix for more detail.

We start by making a few observations about the general properties of the pair size, volatility and dependence effects defined in the proposition. First, $PSE$ is increasing in each of its arguments so that the larger linked systems generate greater economic value, all else equal. Crucially, the increase in $E\left[\Delta\right]$ due to a small increase in, say, $\psi_1$ is not equally shared and we come back to this point later. Second, $VE$ is always positive and increasing in each of its arguments. Unlike $\psi_i$, $\sigma_i^2$ equally contribute to the individual advantages, $E\left[\delta_i\right]$. Third, $DE$ is decreasing in $\rho$, may be positive or negative depending on the sign of $\rho$ but it can never be larger than VE in absolute value, i.e. $|DE| \leq VE$.

Next, we consider extreme examples of the pair characteristics and discuss aggregate advantage and individual countries' incentives to link. Suppose shocks to marginal benefit curves are equal and perfectly positively correlated, e.g. ($\sigma_1 = \sigma_2$, $\rho = 1$), when linking costs are negligible then $\mathbb{E}[\Delta] = VE + DE = 0$, regardless of $PSE$. Countries are indifferent between running an ETS in autarky or linking their ETSs. In effect, as far as sectoral coverage and economic conditions are concerned, there is only one large jurisdiction implementing the same ETS.

The opposite extreme occurs when the shocks to country 1 have the opposite sign as those in country 2, e.g. ($\sigma_1 = \sigma_2$, $\rho = -1$). In this case the advantage of linking is maximum, and as long as there is some uncertainty and linking costs are negligible, linking is the preferred option. More generally, provided the benefit fluctuations in the two countries are of equal volatility but not perfectly positively correlated ($\sigma_1 = \sigma_2$, $-1 < \rho < 1$) it will always be preferable to link ETSs. Further, the incentive to connect the systems is always inversely related to the correlation coefficient.

We now turn to the influence of the differences in the volatilities of the country-specific shocks and maintain the assumption that $\epsilon \approx 0$. When volatilities are not equal ($\sigma_1 \neq \sigma_2$), linking is advantageous even if the shocks are perfectly correlated in the two countries ($\rho = 1$). This can be seen directly from the fact that the $VE$ depends on *the sum of the two shock variances* whereas $DE$ is *the product of two standard deviations.*

A subtle implication of this is that when only one country is subject to shocks ($0 = \sigma_1 < \sigma_2$), it is beneficial for both countries to link since they are individually better off under linking than under autarky. In this case, there are two offsetting effects on the linking decision. In autarky, the marginal benefit in country 1 is constant whereas marginal benefit in country 2 depends on the shock realization. Consequently, ex post autarky permit price levels will differ almost surely. So running the ETS in country 1 in autarky forgoes the gains arising from mutually beneficial permit transactions. When linking, country 1 accepts to 'import' some volatility from country 2; yet, it is well compensated for doing so.

The discussion so far abstracts from sunk costs and the cases where $\psi_1 \neq \psi_2$ matter. We first illustrate the implications of positive linking costs, $\epsilon > 0$, with the aid of the following corollary but assume the ETSs are of equal size, an assumption we will relax shortly.

**Corollary 1.** *Maintain the conditions in Proposition 1 but assume* $\psi_1 = \psi_2$. *For a given* $\epsilon > 0$, *it is possible to find pair characteristics such that* $\mathbb{E}[\delta_i] \lesseqgtr 0$.

Without loss of generality, when the pair characteristics is $\{(1, \sigma_1), (1, \sigma_2), \rho\}$, $PSE = 1/4b_2$ and

$$E\left[\delta_1\right] = E\left[\delta_2\right] = \frac{1}{8b_2}\left(\sigma_1^2 + \sigma_1^2 - 2\sigma_1\sigma_2\rho\right) - \epsilon.$$

A weaker claim than this corollary is that, for any given $\epsilon > 0$, there exist a pair characteristics $\{(1,\sigma_1),(1,\sigma_2),\rho\}$ such that the country specific economic advantages are negative. Note that when $\psi_1 = \psi_2$, countries equally share any economic advantage or disadvantage due to the linked systems. As we illustrate with the next corollary, when $\psi_1 \neq \psi_2$ economic advantage is no longer equally shared and may indeed be positive in aggregate while it is negative for one of the two countries.

**Corollary 2.** *Maintain the conditions in Proposition 1 but assume* $\psi_1 \in (0,1)$ *and* $\psi_2 = 1$ *. Then* $\mathbb{E}[\delta_2] < 0 < \mathbb{E}[\delta_1]$ and $\mathbb{E}[\Delta] > 0$ *when* $\hat{\epsilon}$ *satisfies*

$$\frac{\psi_1}{(1+\psi_1)} PSE(VE+DE) < \hat{\epsilon} < \frac{1}{(1+\psi_1)} PSE(VE+DE).$$

In words, when the ETS in country 1 is smaller, say because the country itself is smaller, or alternatively the scope of the coverage is smaller, we may obtain a situation where country 1 benefits from the linking arrangement but country 2 loses. This will be true despite the fact $E[\Delta] > 0$ but requires that $\epsilon$ is in the interval identified in the corollary. It is important to note that this interval is typically non-empty because $0 < \psi_1 < 1$ and completely identified given the pair characteristics.[13]

To summarize, in the absence of sunk costs aggregate net benefits of linking are always greater than or equal to the aggregate net benefits of autarky. When ETSs have the same size, the economic advantage of linking is equally shared, meaning linking two systems operating under autarky results in a Pareto improvement. Perhaps surprisingly, the Pareto improvement is available even when the volatility of shocks is very different across countries and for all $\rho < 1$. When sunk costs are positive, linking may not be Pareto improving. Put differently, not all carbon dates are created equal. Given sunk costs, some will lead to successful linking arrangements while others should not even reach the first date stage, at least from a cost effectiveness perspective. When sizes of the ETSs vary, the larger country may be driven towards autarky. Put more starkly, size does matter in carbon dates.

## 4 Empirical application

In this section we calibrate pair characteristics to historical emissions data and illustrate the analytical results presented in Section 3. Our aim is to make the case that there is substantial, economically meaningful and policy relevant variation in the empirical counterparts of country specific and aggregate economic advantage, $E[\delta_i]$ and $E[\Delta]$, as well as their components, $PSE$, $VE$, and $DE$.

To that end, we obtain annual carbon dioxide emissions data covering 1950-2012 from WRI (2015) and denote each country $i$ and year $t$ entry by $e_{it}$. We focus on a restricted country sample consisting of world's 20 largest emitters in 2012. These countries are responsible for about 80% of global carbon dioxide emissions in that year.[14] We assume the existence of a hypothetical second best ETS in every country

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[13] The interval will be trivially empty when $(VE + DE) = 0$, a case we discussed above.

[14] The sample includes Australia (AUS), Brazil (BRA), Canada (CAN), China (CHN), Germany (DEU), France (FRA), Great Britain (GBR), Indonesia (IDN), India (IND), Iran (IRN), Italy (ITA), Japan (JPN), Korea (KOR), Mexico (MEX), Poland (POL), Russia (RUS), Saudi Arabia (SAU), Turkey (TUR), United States (USA) and South Africa (ZAF). Data for Saudi Arabia starts in 1953 and Iranian from 1950-1960 is excluded because it contains extreme fluctuations due to political unrest.

covering all of the country's carbon dioxide emissions. We use 2012 emissions to proxy the size of these ETSs and normalize the largest ETS, that of China, to 1. In other words, we set $\psi_{CHN} = 1$.

With 20 countries in our sample there are 190 possible linking arrangements we evaluate using our model. Before doing that, we must first calculate the empirical counterparts of $V(\theta_i)$ and $\mathrm{Corr}(\theta_i, \theta_j)$. To do so, we adopt the methodology described in detail in Doda (2014). Briefly, we use the HP filter introduced by Hodrick and Prescott (1997) to decompose historical carbon dioxide emissions into trend and cyclical components.[15] For each country's time series, the HP filter produces two time series $\{e^t_{it}, e^c_{it}; t = 1950, \ldots, 2012\}$ such that $e_{it} = e^t_{it} + e^c_{it}$. We interpret the cyclical components $e^c_{it}$ as being governed by the underlying country-specific shocks $\theta_i$. Given our interpretation of the shocks as business cycle and/or technology shocks, country-specific events, changes in the prices of factors of production, weather fluctuations etc. we consider this interpretation legitimate. It is worth mentioning that we are implicitly assuming the properties of $e^c_{it}$ are not affected by the recent introduction of climate change policies in some of the advanced countries. We conjecture this is innocuous as most of these policies affect only a portion of the aggregate emissions and do so only in the last few years of our sample.[16]

We denote the standard deviation of $e^c_{it}$ in country $i$ by $\sigma(e^c_{it})$. Similarly, the correlation coefficient between $e^c_{it}$ and $e^c_{jt}$ is $\mathrm{Corr}(e^c_{it}, e^c_{jt})$. Summary statistics for $\psi_i$, $\sigma(e^c_{it})$ and $\mathrm{Corr}(e^c_{it}, e^c_{jt})$ are reported in Table 1. These quantities allow us to construct the variance-covariance matrix of the underlying shocks $\theta$s. Specifically, given our model we have

$$\sigma_i = \frac{b_2}{\psi_i}\sigma(e^c_{it}) \quad \text{and} \quad \rho_{ij} = \mathrm{Corr}(e^c_{it}, e^c_{jt}).$$

In what follows, we assume $\epsilon = 0$ and we normalize $b_2 = 0.5$ so that the pair size effect in Proposition 1 becomes $PSE = \psi_1\psi_2/(\psi_1 + \psi_2)$. This corresponds to changing the units in which $b_2$ is measured. The normalization is innocuous because our discussion and insights are about the relative magnitudes of $E[\Delta]$, $E[\delta_i]$, $PSE$, $VE$, and $DE$ across pairs.

Using these assumptions and calibrated pair characteristics, we compute the quantities in Proposition 1. Table 2 reports the summary statistics for $E[\Delta]$, $PSE$, $VE$, and $DE$ including and excluding Saudi Arabia from the sample. This is because $\sigma(e^c_{SAUt})$ is extremely high, twice as large as the second largest in our sample. Such a large standard deviation is reflected in an extreme volatility effect whenever Saudi Arabia is a linking partner and, consequently, makes Saudi Arabia the preferred partner for every other country in the sample. This is clearly observable in the differences between the left and right panels of Table 2

Our analytical results indicate that the country specific value of a pair depends on the sum of $VE$ and $DE$ scaled by $PSE$. Figures 2-4 report the country-pair results sorted in decreasing order of the individual advantage obtained by the countries we focus on, namely Saudi Arabia (SAU), China (CHN), Germany (DEU) and the UK (GBR). In particular, these figures contain $E[\Delta]$ and $E[\delta_i]$ (top diagram), $VE$ and $DE$ (mid diagram), and $PSE$ (lower diagram). The results reported in these figures are representative of

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[15] We use the optimal penalty parameter $\lambda = 6.25$ for annual data. We refer to Doda (2014) for a discussion about the calibration procedure.

[16] Preliminary analysis restricting the sample to pre-climate change policy era produces similar results.

the distribution of these effects over the entire sample, and demonstrate the significant empirical variation in individual and aggregate values due to linking.

Figure 2 illustrates the empirical results for the linking arrangements with the most volatile possible partner, Saudi Arabia. As noted, the volatility of Saudi Arabia's emissions is very high and largely determines the $VE$, and dominates the $DE$ across all possible links. Consequently, the $PSE$ becomes the main factor determining the ranking of the possible links. In other words, for Saudi Arabia the sum $VE + DE$ offers little help in the identification of the preferred linking partners because it varies little across pairs. Pair sizes, however, vary substantially and therefore determine the ranking.

Based on the results for Saudi Arabia one might expect that the larger the size of the combined ETSs, the larger the economic advantage of linking over autarky. However, Saudi Arabia is a very special example. Figure 3 illustrates this is not the case by reporting the results for China, the largest possible linking partner.[17] The largest $PSE$ is between China and the U.S. Yet, the $VE$ and $DE$ corresponding to this link are virtually zero. In other words, the calibrated pair characteristics of China and US are such that a link between this particular pair generates small benefits compared to other possible pairs. For example, China's ETS would be more cost-effective when running linked to Iran and Indonesia, the top two links, respectively. In these links $VE$ and $DE$ are the largest and drive the linking partner selection.

Thus, $VE$ and $DE$ matter in a carbon date, especially when the $PSE$ of possible links is similar. Figure 4 reports the results for Germany (left column) and illustrates this point. The $PSE$ of Germany with Australia and with Italy are virtually identical. Consequently, $VE$ and $DE$ makes the difference in these pairs. The $VE$ of the pair Germany-Italy is larger than that of the pair Germany and Australia. Yet, the link with Australia is preferred. This is because the correlation of emissions in Germany and Italy is positive and the corresponding $DE$ inverts the linking order based solely on $VE$. The same outcome can be observed for German links with Mexico and Brazil.

Finally, we observe that the individual incentives to link may differ significantly from the aggregate value generated by the link. Figure 4 reports the calibrated results for the UK (right column) and illustrates this point. Let us focus our attention on two groups: South Africa, Mexico and Korea on the one hand, and China, the U.S. and India on the other. From the perspective of a social planner, the links with South Africa, Mexico and Korea are all better than the links with China, the U.S. and India. Howe veer, the latter three links are all preferred from an individual perspective. This is because UK is significantly smaller than China, the U.S. and India and the corresponding $PSE$ are larger than the one with South Africa, Mexico and Korea. Moreover, the volatility and dependence effects of the UK pairing with China, U.S. and India are approximately identical. Therefore, it is the $PSE$ that drives the linking partner selection.

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[17] Note that we have excluded Saudi Arabia from Figures 3 and 4.

## 5 Extension with unilateral taxes on international permit transactions

In this section we extend our model to interventions on international permit trade and how they impact the advantage of linking over autarky. Specifically, we study unilateral taxes on non-domestic permit trade. For simplicity, we restrict our attention to the case where $\psi_1 = \psi_2 = 1$, which implies $\bar{q}_1 = \bar{q}_2 = \bar{q}$. We assume that country 1's regulator imposes a proportional tax $\tau^x$ on permit exports and a tax of $\tau^m$ on permit imports.

Why would country 2 choose to participate in a linking arrangement with country 1 under these circumstances? When the latter imposes taxes it in effect expropriates some of the benefits which the former would otherwise receive. To be clear, distortionary taxes will unambiguously reduce $\mathbb{E}[\Delta]$ and $\mathbb{E}[\delta_2]$ but the latter may remain positive so that it is in country 2's self interest to link regardless. If the taxes imposed by country 1 are not too high, $\mathbb{E}[\delta_1]$ with taxes can be greater than $\mathbb{E}[\delta_1]$ without taxes so the taxes pay off from country 1's individual perspective.

Distortionary taxes may be viewed as a way of implementing international transfers when lump-sum transfers between countries, which are more efficient because they are free from the deadweight losses, are not available. For example, small per unit taxes imposed by country 1 may be more attractive politically in country 2 than increasing country 1's share of total permits, i.e. its quota, post linking. It may be a necessary 'dowry' to country 1 which otherwise may not regulate emissions at all.[18] We do not model these political economy aspects of the problem explicitly but show that there are unilateral incentives to impose taxes.

The autarky equilibria are the same as in (6) because by construction taxes only apply to the international transactions. The linking equilibrium now depends on country 1's tax choice, $(\tau^x, \tau^m)$. Such tax levels generate a wedge between the permit price paid by the buyer and that received by the seller. Consequently, under linking there are two distinct prices, one for each country. In an ILE, the resulting equilibrium prices are given by

$$(p_{L1}, p_{L2}) = \begin{cases} ((1-\tau^x)p_{L2},\ b_1 + \theta_2 - b_2\left[\bar{q} + n(\tau^x,\tau^m)\right]) & \text{if } (1-\tau^x)p_{A2} > p_{A1} \\ \\ (b_1 + \theta_1 - b_2\bar{q},\ b_1 + \theta_2 - b_2\bar{q}) & \text{if } (1-\tau^x)p_{A2} \le p_{A1} \le \frac{p_{A2}}{(1-\tau^m)} \\ \\ (b_1 + \theta_1 - b_2\left[\bar{q} - n(\tau^x,\tau^m)\right],\ (1-\tau^m)p_{L1}) & \text{if } p_{A2} < (1-\tau^m)p_{A1} \end{cases} \tag{9}$$

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[18] See for example Victor (2015).

where

$$n(\tau^x, \tau^m) = \begin{cases} \frac{-\tau^x(b_1 - b_2\bar{q}) + (1-\tau^x)\theta_2 - \theta_1}{b_2(2-\tau^x)} & \text{if } (1-\tau^x)p_{A2} > p_{A1} \\ 0 & \text{if } (1-\tau^x)p_{A2} \leq p_{A1} \leq \frac{p_{A2}}{(1-\tau^m)} \\ \frac{\tau^m(b_1 - b_2\bar{q}) + \theta_2 - (1-\tau^m)\theta_1}{b_2(2-\tau^m)} & \text{if } p_{A2} < (1-\tau^m)p_{A1} \end{cases}$$

We emphasize the dependence of the internationally exchanged quantity $n$ on $(\tau^x, \tau^m)$ to distinguish it from $n$ in the previous sections.

It is important to highlight that above we have omitted the conditions for an ILE for brevity. Yet, we do not neglect them. These conditions are identical to those associated with (7) and are satisfied in an ILE by assumption. The conditions given in (9) impose additional restrictions because some international permit trades which are otherwise mutually beneficial no longer take place in the presence of positive taxes. Intuitively, given $(\tau^x, \tau^m)$, the difference between the shocks must be large enough for both the permit importer and exporter to gain from the permit exchange.

Going back to $(\theta_1, \theta_2)$ plane in Figure 1, the pair $(\tau^x, \tau^m)$ creates a no-trade band around the $45^{\circ}$ degree line. The width of this band is determined by the level of the taxes. Also, the band will not be symmetric around the $45^{\circ}$degree line when $\tau^x \neq \tau^m$. Moreover, a high level of permit export and import taxes may completely eliminate the ILE. Finally, when both taxes are zero, the expressions in (9) are identical to the one in (7) with $\psi_1 = \psi_2 = 1$. We summarize our result with distortionary taxes in the following proposition.

**Proposition 2.** *Maintain the conditions in Proposition 1 but assume $\psi_1 = \psi_2 = 1$. Let $\delta_i(\tau^x, \tau^m)$ denote the economic advantage of linking in country $i = 1, 2$ when country 1 unilaterally imposes $(\tau^x, \tau^m)$. Then in interior equilibria $\exists(\bar{\tau}^x, \bar{\tau}^m) \in \mathbb{R}^2_{++}$* such that

$$\begin{aligned} \mathbb{E}[\delta_1(\bar{\tau}^x, \bar{\tau}^m)] &> \mathbb{E}[\delta_1(0,0)] \\ \mathbb{E}[\delta_2(\bar{\tau}^x, \bar{\tau}^m)] &\lesseqgtr 0 \\ \mathbb{E}[\delta_1(\bar{\tau}^x, \bar{\tau}^m)] + \mathbb{E}[\delta_2(\bar{\tau}^x, \bar{\tau}^m)] &< \mathbb{E}[\delta_1(0,0)] + \mathbb{E}[\delta_2(0,0)]. \end{aligned}$$

*provided $\epsilon$ is sufficiently small so $\mathbb{E}[\delta_1(0,0) + \delta_2(0,0)] > 0$.*

*Proof: see Appendix.*

This proposition states that it is possible to find a pair of unilateral ad valorem taxes on international permit transactions such that when country 1 imposes these taxes it is better off with taxes than without. On the other hand, country 2 is always worse off relative to the case when taxes are zero. It may be better or worse off relative to autarky depending on the level of sunk costs. In other words, the economic advantage of linking is dissipated, and re-distributed away from country 2 with distortionary taxes.

In this sense, the distortionary taxes are a double-edged sword. On the one hand, the potential for capturing some of the benefits from your linking partner through taxes can persuade countries which would otherwise not regulate their emissions to take climate action if their putative partners are willing and able to turn a blind eye to this expropriation for political economy reasons. On the other hand, taxes create a time consistency problem by giving an incentive to countries to impose taxes after the linking arrangement is in place. Needless to say, this works to limit the number of potentially successful linking partner matches, or carbon dates.

We illustrate Proposition 2 using numerical simulations because a closed form solution to $\mathbb{E}[\delta_i(\bar{\tau}^x, \bar{\tau}^m)]$ is not available without making an assumption about the joint distribution of the shocks. Rather than imposing a simple distribution to maintain analytical tractability, we proceed by evaluating the expectations under the assumption that the shocks $\theta_i$ are jointly normally distributed with $\sigma_1 = \sigma_2 = 0.456$ and $\rho = 0.127$, which are the average values for these statistics when Saudi Arabia is excluded from the sample discussed in the previous section. We maintain $b_2 = 0.5$. Moreover, we choose the other required parameters so that $0 \approx d_2 \ll b_2$ and $\bar{q} = 0.5q^{BAU}$.

For simplicity and without loss of generality, we set the two taxes equal to a common value $\bar{\tau}$ which we vary in the interval $[0, 0.5]$. For each $\bar{\tau}$ we draw 50,000 shock pairs $(\theta_1, \theta_2)$ from the joint distribution and evaluate $\mathbb{E}[\delta_1(\bar{\tau}, \bar{\tau})]$, $\mathbb{E}[\delta_2(\bar{\tau}, \bar{\tau})]$ and $\mathbb{E}[\Delta(\bar{\tau}, \bar{\tau})]$, which are illustrated in Figure 5 with blue, green and red lines, respectively. Before discussing the results of the simulations, we emphasize that these calculations are merely to illustrate proposition 2 and should not be taken as an attempt at full calibration.

When $\bar{\tau} > 0$, we observe that $\mathbb{E}[\delta_1(\bar{\tau}, \bar{\tau})]$ and $\mathbb{E}[\delta_2(\bar{\tau}, \bar{\tau})]$ differ despite the fact that $\psi_1 = \psi_2$. The graph shows that when $\bar{\tau} > 0$, country 1 always receives a larger fraction of the aggregate value generated under the linking arrangement. It is only when $\bar{\tau}$ is less than about 0.18 that country 1 prefers linking with taxes than without. Moreover, there is an advantage-maximizing tax rate of approximately 0.09 for country 1. Beyond this level increasing the tax rate reduces the tax base, i.e. the number of international permit transactions, by more. For all positive taxes in this figure the gain of country 1 is smaller than the loss of country 2. To see this note that in the figure the aggregate advantage of linking is decreasing in $\bar{\tau}$.

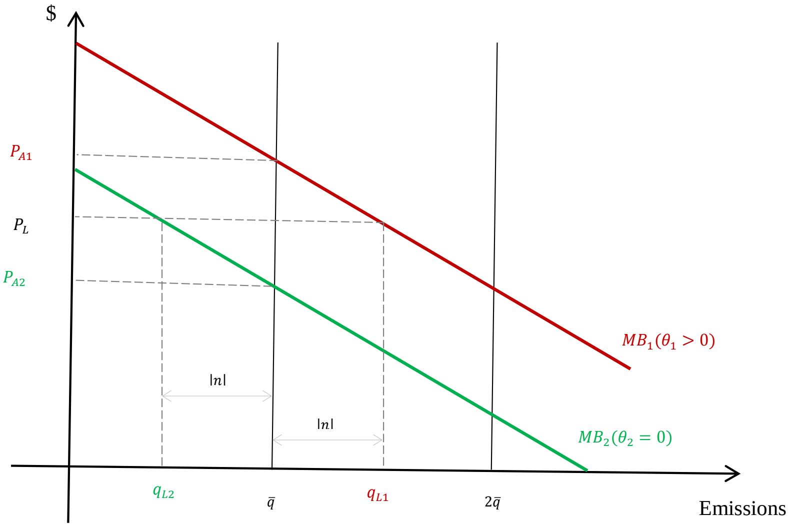
Finally, consider the effect of sunk costs in this extension with distortionary taxes. Suppose these costs are given by a lower horizontal line in the figure. When they are present and equally shared, country 2 may find that it is better off under autarky than in a linking arrangement with country 1. Specifically, when $\bar{\tau}$ is greater than about 0.06, which we note is less than the advantage maximizing tax rate from the perspective of country 1, country 2 is better of under autarky, i.e. $\mathbb{E}[\delta_2(\bar{\tau}, \bar{\tau})] < 0$. This suggests that if country 2 expects country 1 to impose such taxes, it may not carbon date with country 1. If the taxes are a surprise to country 2, then the odds of a future carbon date may be diminished.

# 6 Conclusions

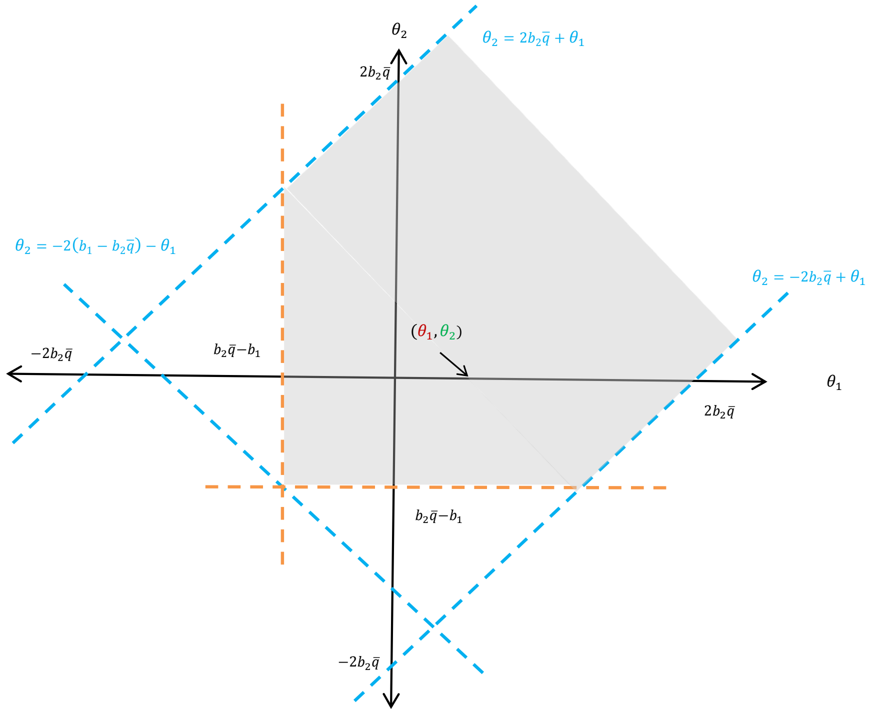
In this paper we use a simple and standard framework to evaluate the economic advantage of linking ETSs in two jurisdictions versus operating them under autarky. The paper demonstrates that the jurisdiction-specific characteristics are crucial in determining the value of linking arrangements, both in aggregate and from each jurisdiction's perspective. We characterize analytically the economic advantage of linking versus autarky and decompose it into three readily interpretable components: the pair size effect, the volatility effect and the dependence effect. We also identify the conditions under which one, but not the other jurisdiction, is worse off relative to autarky even when the aggregate value of the linking arrangement is positive. Our empirical application demonstrates that there is significant variation in individual and aggregate economic advantage of linking. Furthermore, our numerical simulations show that unilateral distortions can destroy potentially valuable linking arrangements. In other words, when carbon dating, it is important to chose your date carefully.

# Figures

Figure 1: Autarky and Linking

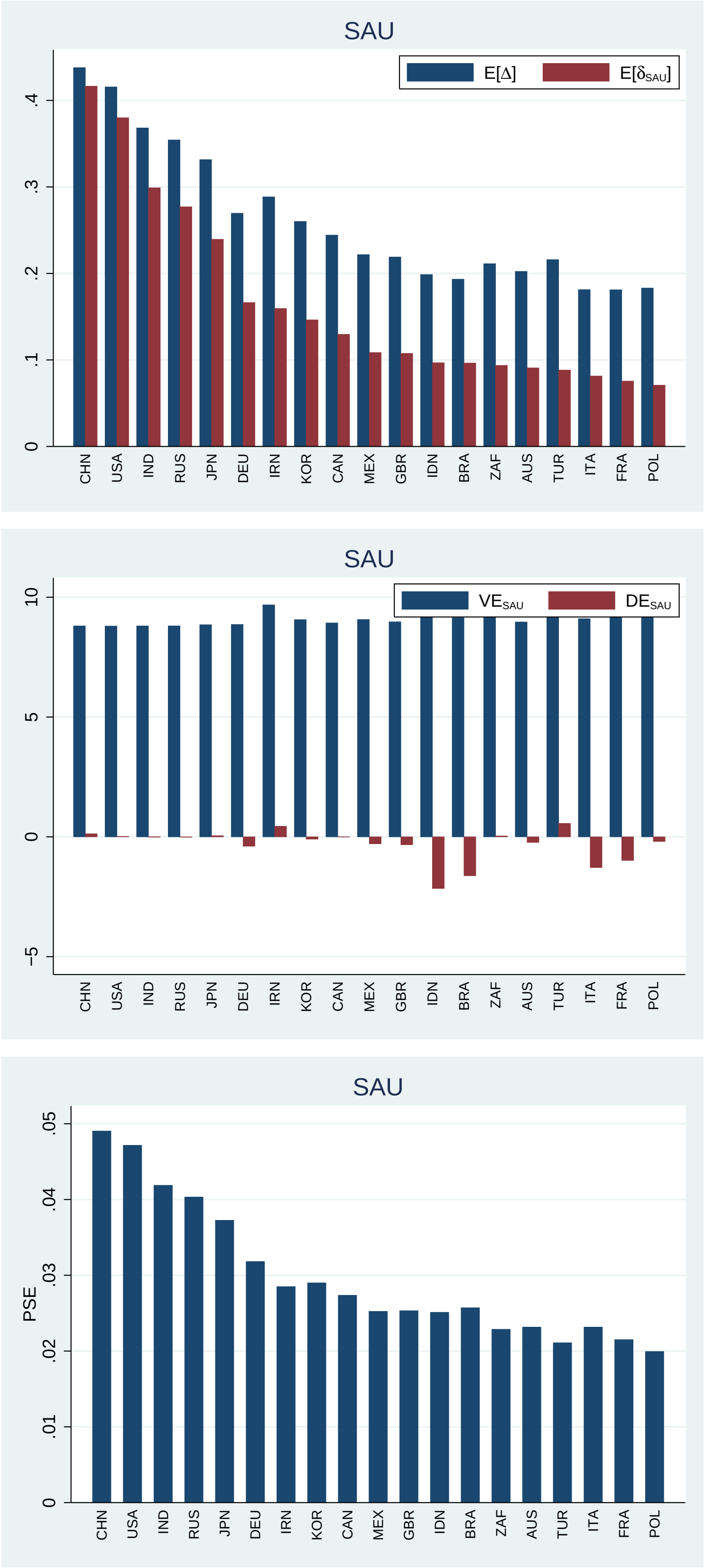


PANEL A: Permit Market Equilibrium



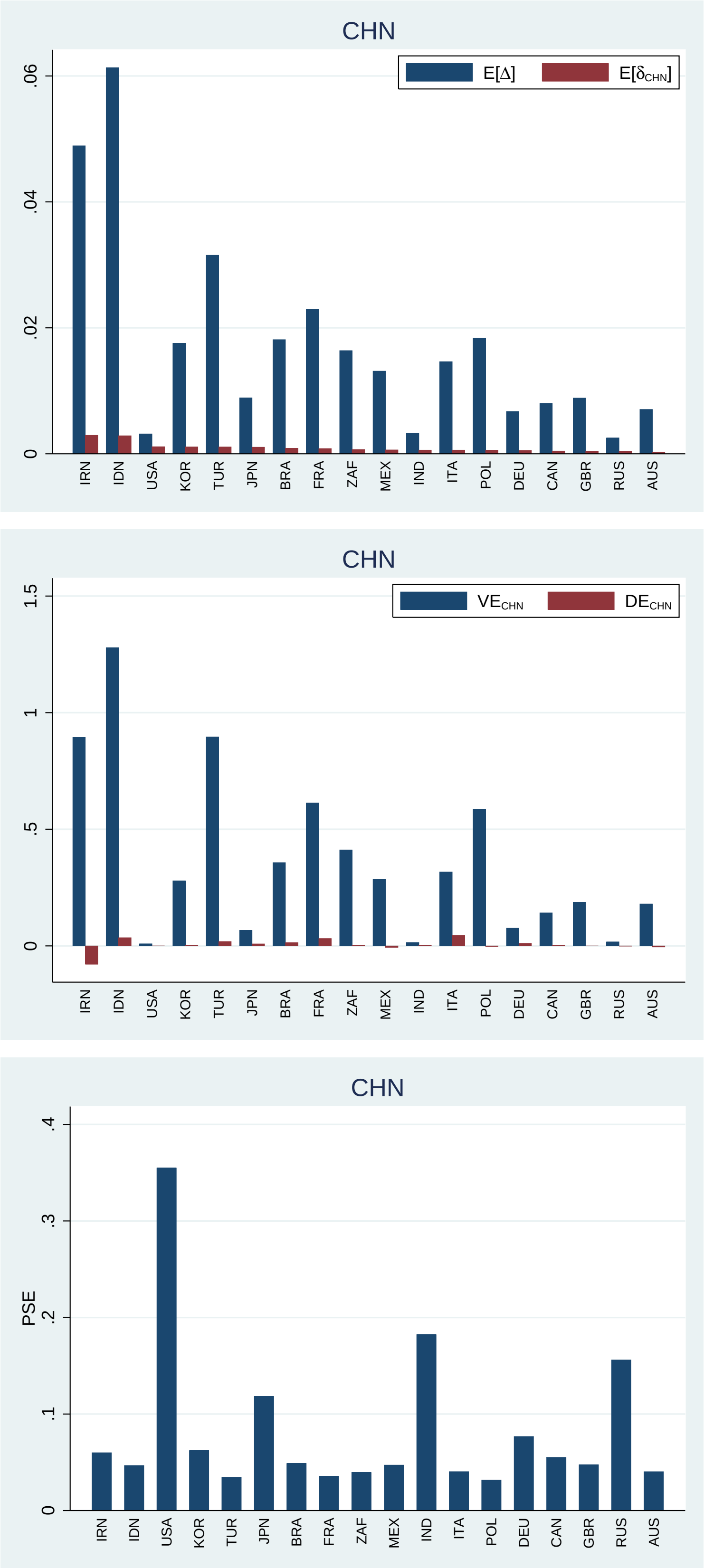
PANEL B: Intersection of IAE and ILE

Figure 2: Linking partners for the country with the greatest $VE$



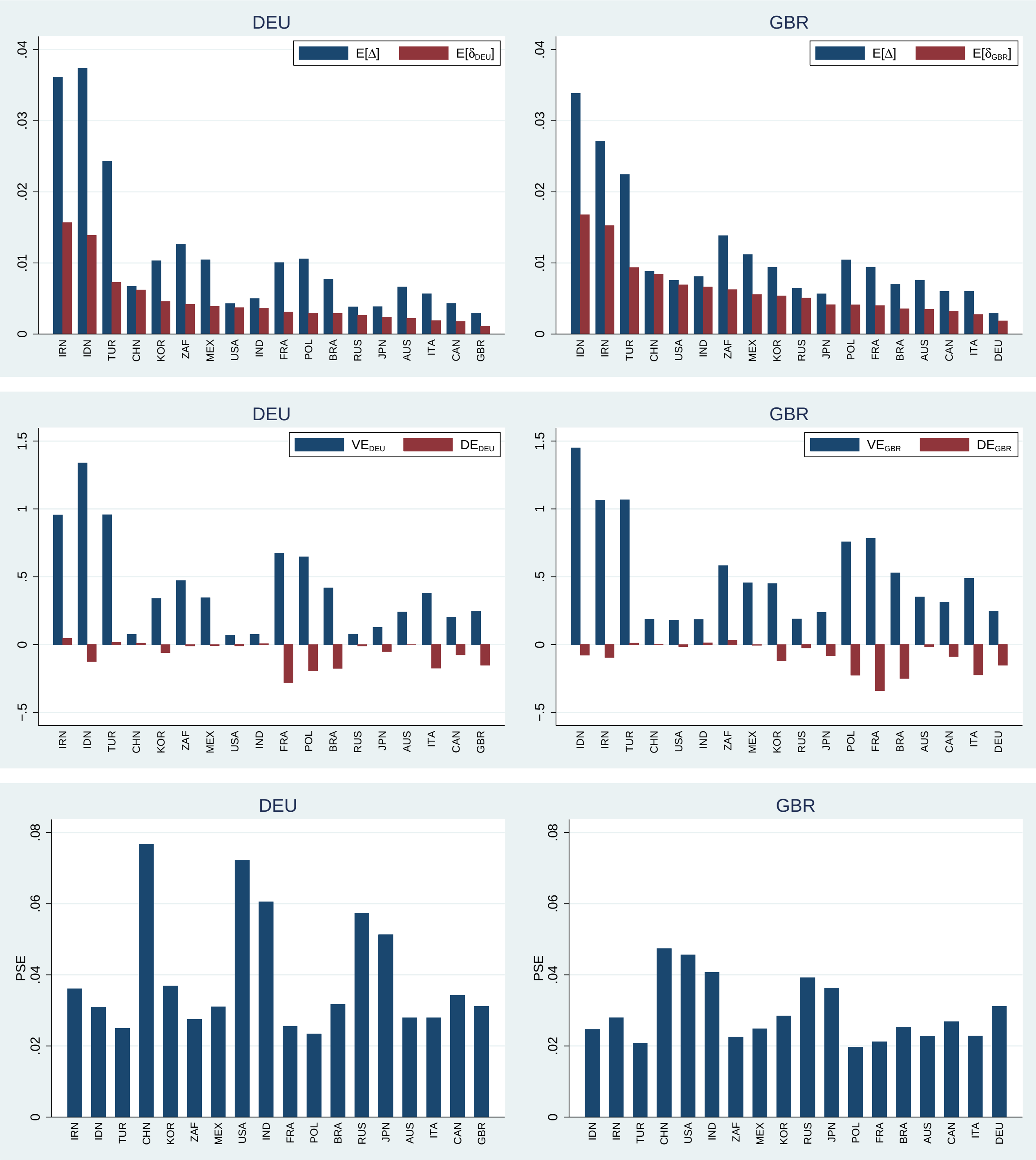
**Note:** Linking partners sorted in decreasing order of $E[\delta_i]$, making the left most pair the most desirable linking arrangement for country $i$.

Figure 3: Linking partners for the largest emitter



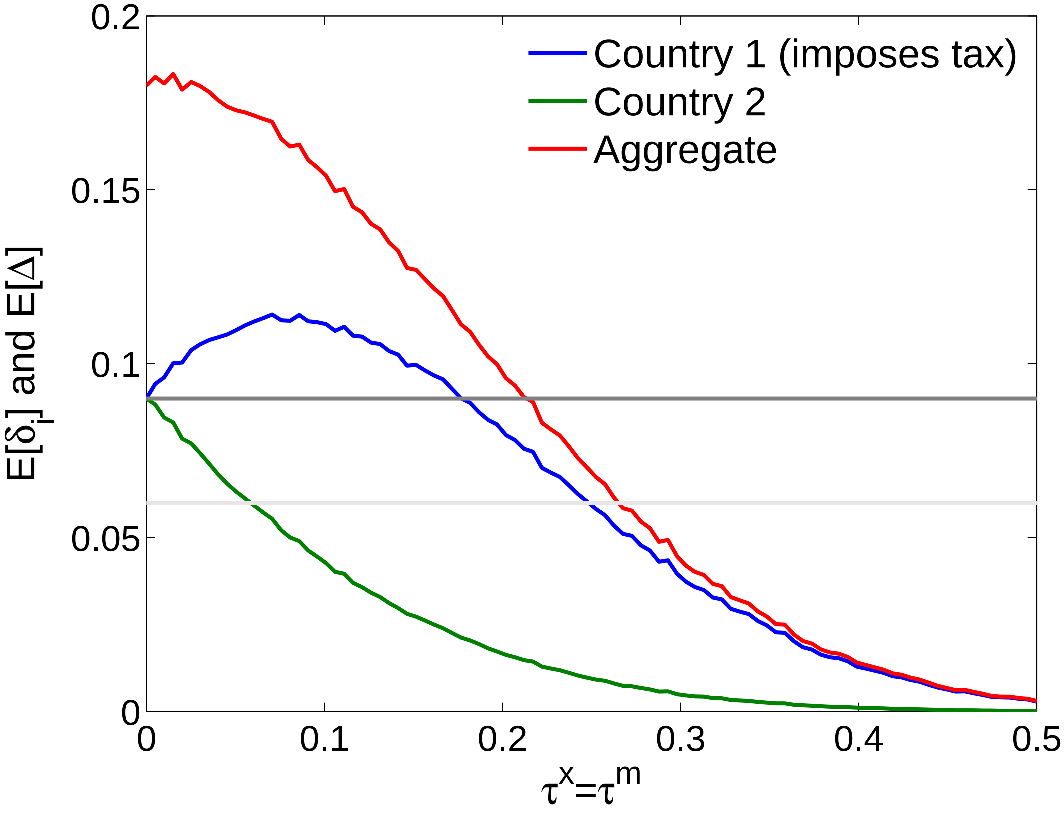
**Note:** Linking partners sorted in decreasing order of $E[\delta_i]$, making the left most pair the most desirable linking arrangement for country $i$. Saudi Arabia excluded.

Figure 4: Linking partners for Germany and United Kingdom



**Note:** Linking partners sorted in decreasing order of $E[\delta_i]$, making the left most pair the most desirable linking arrangement for country $i$. Saudi Arabia excluded.

Figure 5: Illustration of proposition 2



# Tables

Table 1: Summary statistics: $\sigma(e^c_{it})$ and $Corr(e^c_{it}, e^c_{jt})$

| | **mean** | **sdev** | **min** | **max** | **N** |
|---|---|---|---|---|---|
| $\psi_i$ | 0.142 | 0.234 | 0.033<br>POL | 1<br>CHN | 20 |
| $\sigma(e^c_{it})$ | 0.038 | 0.032 | 0.017<br>AUS | 0.153<br>SAU | 20 |
| $Corr(e^c_{it}, e^c_{jt})$ | 0.122 | 0.218 | -0.459<br>CHN- ITA | 0.688<br>DEU-FRA | 190 |

Table 2: Summary statistics: $E[\Delta]$, $PSE$, $VE$, and $DE$

| | **190 pairs including SAU** | | | | **171 pairs excluding SAU** | | | |
|---|---|---|---|---|---|---|---|---|
| | **mean** | **sdev** | **min** | **max** | **mean** | **sdev** | **min** | **max** |
| $E[\Delta]$ | 0.042 | 0.079 | 0.001<br>RUS-USA | 0.438<br>CHN-SAU | 0.018 | 0.013 | 0.001<br>RUS-USA | 0.061<br>CHN-IDN |
| $PSE$ | 0.038 | 0.033 | 0.017 | 0.355 | 0.039 | 0.035 | 0.017 | 0.355 |
| $VE$ | 1.527 | 2.585 | 0.008 | 10.064 | 0.682 | 0.480 | 0.008 | 2.159 |
| $DE$ | -0.084 | 0.254 | -2.153 | 0.560 | -0.056 | 0.112 | -0.520 | 0.439 |

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# Appendix

## First-best emissions levels

We characterize the first-best emissions levels by postulating a social planner who maximizes the aggregate net benefits by conditioning country-specific emissions levels to the realization of the shocks. Formally the planner solves

$$\max_{\{q_1 \geq 0, q_2 \geq 0\}} \quad [B_1(q_1, \theta_1) - D_1(q_1 + q_2)] + [B_2(q_2, \theta_2) - D_2(q_1 + q_2)] \quad \text{given } \theta_1 \text{ and } \theta_2. \tag{10}$$

Denoting the optimal solution as $(q_1^*, q_1^*)$

$$(q_1^*, q_2^*) \quad = \quad \begin{cases} \left( \frac{(2d_2\psi_2 + b_2)\theta_1 - 2d_2\psi_2\theta_2 + b_2(b_1 - 2d_1)}{b_2\left[\frac{b_2}{\psi_1} + 2d_2\left(1 + \frac{\psi_2}{\psi_1}\right)\right]}, \frac{(2d_2\psi_1 + b_2)\theta_2 - 2d_2\psi_1\theta_1 + b_2(b_1 - 2d_1)}{b_2\left[\frac{b_2}{\psi_2} + 2d_2\left(1 + \frac{\psi_1}{\psi_2}\right)\right]} \right) & \text{if} \begin{pmatrix} \theta_2 < \frac{2d_2\psi_2 + b_2}{2d_2\psi_2}\theta_1 + \frac{b_2(b_1 - 2d_1)}{2d_2\psi_2} \\ \& \\ \theta_2 > \frac{2d_2\psi_1}{2d_2\psi_1 + b_2}\theta_1 - \frac{b_2(b_1 - 2d_1)}{2d_2\psi_1 + b_2} \end{pmatrix} \\ \\ \left( \frac{b_1 - 2d_1 + \theta_1}{\frac{b_2}{\psi_1} + 2d_2}, 0 \right) & \text{if} \begin{pmatrix} \theta_1 > 2d_1 - b_1 \\ \& \\ \theta_2 \leq 2d_1 - b_1 + 2d_2 q_1^* \end{pmatrix} \\ \\ \left( 0, \frac{b_1 - 2d_1 + \theta_2}{\frac{b_2}{\psi_2} + 2d_2} \right) & \text{if} \begin{pmatrix} \theta_1 \leq 2d_1 - b_1 + 2d_2 q_2^* \\ \& \\ \theta_2 > 2d_1 - b_1 \end{pmatrix} \\ \\ (0, 0) & \text{if} \begin{pmatrix} \theta_1 \leq 2d_1 - b_1 \\ \& \\ \theta_2 \leq 2d_1 - b_1 \end{pmatrix} \end{cases}$$

In the first case where optimal emissions are positive in both countries, we observe that country $i$'s optimal emissions are continuous, increasing in its own shocks and decreasing in the other country's shocks. The solution is symmetric in the sense that the coefficients of own and other country's shocks, $(2d_2\psi_i + b_2)$ and $-2d_2\psi_i$ respectively, are identical. When countries are of equal size, i.e. $\psi_1 = \psi_2$, a small increase in $\theta_i$ results in an increase in total emissions because the implied increase in emissions in country $i$ is greater than the decline in $j$. All else constant, emissions in country $i$ is increasing in $\psi_i$ but decreasing in $\psi_j$.

In the remaining cases, the non-negativity constraints on emissions require that when the shocks satisfy the conditions specified, there will be a corner solution. For example, when $\theta_i$ is small relative to $\theta_j$, it may be optimal for $i$ not to emit at all. These correspond to second and third cases. In the final case the shocks in both countries are sufficiently low so that $q_i^* = q_j^* = Q^* = 0$.

### Linking equilibrium for all shocks

The following is the complete characterization of the linking equilibrium.

$$
(p_L, q_{L1}, q_{L2}) = \begin{cases}
\left(K + \frac{\psi_1\theta_1+\psi_2\theta_2}{\psi_1+\psi_2}, \bar{q}_1 - n, \bar{q}_2 + n\right) & \text{if} \quad \begin{array}{c} \theta_2 < b_2\frac{\psi_1+\psi_2}{\psi_1\psi_2}\bar{q}_1 + \theta_1 \\ \& \\ \theta_2 > -b_2\frac{\psi_1+\psi_2}{\psi_1\psi_2}\bar{q}_2 + \theta_1 \\ \& \\ \theta_2 > -\frac{2}{\psi_2}K - \frac{\psi_1}{\psi_2}\theta_1 \end{array} \qquad (L1) \\[2em]
\left(b_1 + \theta_1 - \frac{b_2}{\psi_1}\left(\bar{q}_1 + \bar{q}_2\right), \bar{q}_1 + \bar{q}_2, 0\right) & \text{if} \quad \begin{array}{c} \theta_2 \leq -b_2\frac{\psi_1+\psi_2}{\psi_1\psi_2}\bar{q}_2 + \theta_1 \\ \& \\ \theta_1 > -\left[b_1 + \frac{b_2}{\psi_1}\left(\bar{q}_1 + \bar{q}_2\right)\right] \end{array} \qquad (L2) \\[2em]
\left(b_1 + \theta_2 - \frac{b_2}{\psi_2}\left(\bar{q}_1 + \bar{q}_2\right), 0, \bar{q}_1 + \bar{q}_2\right) & \text{if} \quad \begin{array}{c} \theta_2 \geq b_2\frac{\psi_1+\psi_2}{\psi_1\psi_2}\bar{q}_1 + \theta_1 \\ \& \\ \theta_1 > -\left[b_1 + \frac{b_2}{\psi_2}\left(\bar{q}_1 + \bar{q}_2\right)\right] \end{array} \qquad (L3) \\[2em]
\left(0, \frac{\psi_1(b_1+\theta_1)}{b_2}, \frac{\psi_2(b_1+\theta_2)}{b_2}\right) & \text{otherwise} \qquad (L4)
\end{cases}
$$

where the constant $K$ is defined in the text. In cases $L2$ and $L3$, $p_L$ is not uniquely determined because there is a difference between the valuation of the last permit between the buyer and the seller. In particular, $MB^{buyer}(\bar{q}_1 + \bar{q}_2) > MB^{seller}(0)$. In principle, any $p_L$between these extremes will be mutually beneficial. Above we have assumed that seller has all the bargaining power and can keep all the surplus from the trade, i.e. $p_L = MB^{buyer}(\bar{q}_1 + \bar{q}_2)$.

We emphasize that an assumption on bargaining powers is not required in the ILE, which is one of the main reasons why we exclusively focus on ILE in the paper.

## Proof of Proposition 1

We first evaluate the country-specific economic advantage of linking over autarky restricting our attention to interior equilibria. Substituting $n = \frac{1}{b_2} \frac{\psi_1 \psi_2}{(\psi_1+\psi_2)} (\theta_2 - \theta_1)$ in the first line of Equation (8), we obtain

$$\begin{aligned}
\delta_1 &= -n(b_1 + \theta_1) - \frac{b_2}{2\psi_1}(-2\bar{q}_1 n + n^2) + p_L n - \psi_1 \epsilon \\
&= n\Big[\frac{\psi_1\theta_1 + \psi_2\theta_2}{\psi_1 + \psi_2} - \theta_1 - \frac{b_2}{2\psi_1} n\Big] - \psi_1 \epsilon \\
&= n\frac{\psi_2}{2(\psi_1 + \psi_2)}(\theta_2 - \theta_1) - \psi_1 \epsilon \\
&= \frac{\psi_1 \psi_2^2}{(\psi_1 + \psi_2)^2} \frac{(\theta_2 - \theta_1)^2}{2b_2} - \psi_1 \epsilon.
\end{aligned}$$

Evaluating the second line of Equation (8), we obtain

$$\delta_2 = \frac{\psi_1^2 \psi_2}{(\psi_1 + \psi_2)^2} \frac{(\theta_2 - \theta_1)^2}{2b_2} - \psi_2 \epsilon.$$

The aggregate economic advantage of linking over autarky corresponds to the sum of country-specific advantages:

$$\Delta = \delta_1 + \delta_2 = \frac{1}{2b_2} \frac{\psi_1 \psi_2}{\psi_1 + \psi_2} (\theta_2 - \theta_1)^2 - (\psi_1 + \psi_2)\epsilon.$$

Using (2), we derive the expression for the expected aggregate economic advantage:

$$\begin{aligned}
E[\Delta] &= \frac{1}{2b_2} \frac{\psi_1 \psi_2}{\psi_1 + \psi_2} E(\theta_2 - \theta_1)^2 - (\psi_1 + \psi_2)\epsilon \\
&= \frac{1}{2b_2} \frac{\psi_1 \psi_2}{\psi_1 + \psi_2} (\sigma_1^2 + \sigma_2^2 - 2\sigma_1\sigma_2\rho) - (\psi_1 + \psi_2)\epsilon.
\end{aligned}$$

## Proof of Proposition 2

In proving this proposition the expressions for the country-specific economic advantage of linking over autarky in the presence of taxes are useful:

$$\begin{aligned}
\delta_1(\tau^x,\tau^m) &= B_1(\bar{q}-n(\tau^x,\tau^m),\theta_1) - B_1(\bar{q},\theta_1) + p_{L1}n(\tau^x,\tau^m) + (p_{L2}-p_{L1})n(\tau^x,\tau^m) - \epsilon,\\
\delta_2(\tau^x,\tau^m) &= B_2(\bar{q}+n(\tau^x,\tau^m),\theta_2) - B_2(\bar{q},\theta_2) - p_{L2}n(\tau^x,\tau^m) - \epsilon.
\end{aligned}$$

where taxes are imposed by country 1. The tax pair $(\tau^x,\tau^m)$ generates a wedge between the permit price paid by the buyer and that received by the seller. Consequently, under linking there are two distinct prices, $p_{L1}$ and $p_{L2}$. Then $(p_{L2}-p_{L1})n(\tau^x,\tau^m)$ corresponds to the tax revenues collected by country 1. Given $\delta_1(\tau^x,\tau^m)$ and $\delta_2(\tau^x,\tau^m)$, the aggregate economic advantage of linking over autarky in the presence of taxes is

$$\Delta(\tau^x,\tau^m) = [B_1(\bar{q}-n(\tau^x,\tau^m),\theta_1) + B_2(\bar{q}+n(\tau^x,\tau^m),\theta_2)] - [B_1(\bar{q},\theta_1) + B_2(\bar{q},\theta_2)] - 2\epsilon.$$

The first two parts of proposition 2 state that it is possible to find a pair of unilateral ad valorem taxes on international permit transactions such that when country 1 imposes these taxes it is better off, $\mathbb{E}[\delta_1(\bar{\tau}^x,\bar{\tau}^m)] > \mathbb{E}[\delta_1(0,0)]$, and country 2 is worse off, $\mathbb{E}[\delta_2(\bar{\tau}^x,\bar{\tau}^m)] < \mathbb{E}[\delta_2(0,0)]$.

We start by describing our strategy. First, note that given shocks $(\theta_1,\theta_2)$, international permit transactions, $n(0,0)$, are almost surely non zero in the interior equilibrium. Second, given a tax pair $(\tau^x,\tau^m)$, we have, $n(\tau^x,\tau^m) \gtreqless 0$. Finally, when $\tau^x \to 0$ and $\tau^m \to 0$ the probability that $n(\tau^x,\tau^m) = 0$ becomes negligible. Using these observations, below we show that for small taxes, country 1 is always – i.e. for all possible shock realisations – better off relative to the case of no taxes, regardless of whether it is a permit importer or exporter. On the contrary, country 2 is always worse off.

We begin by considering the case where $n(0,0) > 0$ so that country 1's export tax is relevant

$$\begin{aligned}
\frac{\partial\delta_1}{\partial\tau^x} &= B_1'(\bar{q}-n(\tau^x,\tau^m),\theta_1)\Big[\frac{-\partial n(\tau^x,\tau^m)}{\partial\tau^x}\Big] + \Big[p_{L2}\frac{\partial n(\tau^x,\tau^m)}{\partial\tau^x} + \frac{\partial p_{L2}}{\partial\tau^x}n(\tau^x,\tau^m)\Big]\\
&= B_1'(\bar{q}-n(\tau^x,\tau^m),\theta_1)\Big[\frac{-\partial n(\tau^x,\tau^m)}{\partial\tau^x}\Big] + \frac{\partial n(\tau^x,\tau^m)}{\partial\tau^x}\Big[b_1+\theta_2-b_2(\bar{q}+n(\tau^x,\tau^m)) - b_2 n(\tau^x,\tau^m)\Big]\\
&= -\frac{\partial n(\tau^x,\tau^m)}{\partial\tau^x}\Big[B_1'(\bar{q}-n(\tau^x,\tau^m),\theta_1) - \Big[b_1+\theta_2-b_2\bar{q}-2b_2\Big[\frac{-\tau^x(b_1-b_2\bar{q})+(1-\tau^x)\theta_2-\theta_1}{b_2(2-\tau^x)}\Big]\Big]\Big]
\end{aligned}$$

where $B_1'$ represents the marginal benefits of emissions in country 1. The second equality uses $p_{L2} = b_1+\theta_2-b_2\left[\bar{q}+n(\tau^x,\tau^m)\right]$ and its derivative with respect to $\tau^x$. Taking the limit, recalling that $n(\tau^x,\tau^m)$ decreases in $\tau^x$ and that the linking price is higher than the autarky price when country 1 exports permits, we have

$$\lim_{\tau^x\to 0}\frac{\partial\delta_1}{\partial\tau^x} = -\frac{\partial n(\tau^x,\tau^m)}{\partial\tau^x}\Big[p_L - p_{A1}\Big] > 0. \tag{11}$$

Next we consider the case where $n(0,0) < 0$ so that country 1's import tax is relevant

$$\frac{\partial \delta_1}{\partial \tau^m} \;=\; B_1'(\bar{q} - n(\tau^x, \tau^m), \theta_1)\Big[\frac{-\partial n(\tau^x, \tau^m)}{\partial \tau^m}\Big] + \Big[p_{L2}\frac{\partial n(\tau^x, \tau^m)}{\partial \tau^m} + \frac{\partial p_{L2}}{\partial \tau^m} n(\tau^x, \tau^m)\Big].$$

Using a similar argument as in the export tax case, we obtain

$$\lim_{\tau^m \to 0} \frac{\partial \delta_1}{\partial \tau^m} \;=\; \frac{\partial p_{L2}}{\partial \tau^m} n(\tau^x, \tau^m) > 0. \tag{12}$$

Provided that the conditions stated in the proposition is satisfied, expressions (11) and (12) are true for every realisation of shocks. Therefore, they hold in expectation.

We omit the proof of the second part of the proposition for brevity. The same line of reasoning as above shows that country 2 is always worse off relative to the case where there are no taxes, i.e. $\delta_2(\tau^x, \tau^m) < \delta_2(0,0)$. Moreover, sunk costs may completely eliminate the remaining advantage of linking so that $\delta_2(\tau^x, \tau^m) < 0$.

To prove the third part of the proposition we evaluate the change in aggregate economic advantage of linking for small export and import taxes. We consider the export tax case first.

$$\begin{aligned}\frac{\partial \Delta}{\partial \tau^x} \;&=\; B_1'(\bar{q} - n(\tau^x, \tau^m), \theta_1)\Big[\frac{-\partial n(\tau^x, \tau^m)}{\partial \tau^x}\Big] + B_2'(\bar{q} + n(\tau^x, \tau^m), \theta_2)\Big[\frac{\partial n(\tau^x, \tau^m)}{\partial \tau^x}\Big] \\ &=\; \frac{-\partial n(\tau^x, \tau^m)}{\partial \tau^x}\Big[B_1'(\bar{q} - n(\tau^x, \tau^m), \theta_1) - B_2'(\bar{q} + n(\tau^x, \tau^m), \theta_2)\Big] < 0 \end{aligned} \tag{13}$$

When taxes are zero there is no price wedge under linking and marginal benefits are equal, e.g. $B_1'(\bar{q} - n(0,0), \theta_1) = B_2'(\bar{q} + n(0,0), \theta_2)$. With taxes $B_1' = p_{L1} < p_L$ and $B_2' = p_{L2} > p_L$. Thus, the difference in marginal benefits is negative and the entire expression is negative.

Similarly, for the import tax case

$$\begin{aligned}\frac{\partial \Delta}{\partial \tau^m} \;&=\; B_1'(\bar{q} - n(\tau^x, \tau^m), \theta_1)\Big[\frac{-\partial n(\tau^x, \tau^m)}{\partial \tau^m}\Big] + B_2'(\bar{q} + n(\tau^x, \tau^m), \theta_2)\Big[\frac{\partial n(\tau^x, \tau^m)}{\partial \tau^m}\Big] \\ &=\; \frac{\partial n(\tau^x, \tau^m)}{\partial \tau^m}\Big[B_1'(\bar{q} - n(\tau^x, \tau^m), \theta_1) - B_2'(\bar{q} + n(\tau^x, \tau^m), \theta_2)\Big] < 0 \end{aligned} \tag{14}$$

where the first component is negative and the difference in marginal benefits is positive. Consequently the entire expression is negative.

Since (13) and (14) hold for every realization of shocks, they also hold in expectation.