Complications in Cooperating when Players are Asymmetric: Theory and Experimental Evidence

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Example 1: East Texas, early 1930s

- challenging conditions for oil firms
  - firms of differing sizes
  - weak demand, “over supply”
  - attempts to restrict production (“dancing partners”) invalidated by courts
- Majors lobby for quota system to prop up prices
  - small firms resist
  - regular violations of quotas, typically by independents
  - leads to movement for “field unitization”
    - one operator, firms allocated shares of field production
    - moderate success
    - ongoing resistance from small firms

Example 2: OPEC, early 1980s

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country	quota	output	quota	output
Saudi Arabia	7650	6961	5000	4951
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- Iran cheats on its quota in 1982
  - then gets a bigger quota in 1983
- Venezuela cheats on its quota in 1982
  - then gets a bigger quota in 1983
  - cheats again in 1983
- Saudi's quota is reduced from 1982 to 1983 (and again in 1984)
- after prices collapse in 1986 quota system is formalized
  - quotas based on reserves, capacity
  - limited ability to prop up prices

Major themes: IO

1. Forming a cooperative agreement is more difficult when players are asymmetric
   - differences in technology
   - differences in quality of inputs
   - boils down to differences in costs
2. smaller (higher cost) firm more likely to defect
3. ultimate effect: asymmetric cartels appear to be largely ineffective

Example 3: Climate negotiations

- Kyoto: emission reductions from Annex I (developed) countries only
  - motivated by equity concerns
  - pushback from some large countries
- Copenhagen: bilateral discussions between US and China as means of pushing discussion forward
- Paris agreement: INDCs
  - all countries propose reductions
  - sense that much of the heavy lifting is done by developed ("large"?) countries

Questions: climate application

1. Is forming an IEA more difficult when countries are asymmetric?
   - differences in technical skills
   - differences in assets
   - differences in exposure to climate damages
   - can boil down to differences in abatement costs (*i.e.*, benefits from emissions)
2. smaller (lower net benefit) country more likely to defect?
3. ultimate effect: asymmetric IEAs appear to be largely ineffective?

My goals in this paper

- To investigate these conjectures
  - what do equilibria look like in asymmetric games?
    - without social considerations (baseline)
    - with social considerations
  - which type of player seems more likely to be sticking point?
- analyze experimental evidence

Repeated game – emissions

- Players: countries 1, 2
  - e_i = country i's emissions; $E = e_1 + e_2$ = global emissions
  - asymmetric emission benefits / abatement costs
    - net benefit: $b_i = MB_i - MC_i$
    - $b_1 - b_2 \equiv c \geq 0$ (symmetric: $c = 0$, asymmetric: $c > 0$)
- common marginal damage from emissions, dE
  - payoff for firm i in period t: $\pi_{it} = [b_i - dE_t]e_{it} = [b_i - de_{jt}]e_{it} - d(e_{it})^2$
- common discount factor δ
- one-shot Nash equilibrium emissions: $e_i^N = \frac{2b_i - b_j}{3d}$

Suppose each country plays the grim strategy:

choose e_i^c if both players have chosen e_k^c in all previous periods $t \geq 0$
otherwise choose e_i^N

Repeated play: Cournot duopoly

- Players: firms 1, 2
  - each firm has constant *MC*
    - $MC_1 = 0, MC_2 = c$ (symmetric: $c = 0$, asymmetric: $c > 0$)
- homogenous good, linear inverse demand: $p = a - bQ$
  - payoff for firm i in period t:

$$\pi_{it} = [a - c_i - bQ]q_{it} = [a - c_i - bq_{jt}]q_{it} - b(q_{it})^2$$

- common discount factor δ
- one-shot Cournot output:

$$q_i^N = \frac{a - 2c_1 + c_j}{3b}; \quad \pi^N = b(q_i^N)^2$$

Quasi-cooperative outcome

Suppose each firm plays the grim strategy:

choose x_i^c if both players have chosen x^c in all previous periods $t \geq 0$

otherwise choose x_i^N

- two subgames of note:
  1. subgames where no player has deviated in any previous period
  2. subgames where ≥ 1 player has deviated in some previous period
- subgame class 2 satisfied trivially
- note that the entire game falls into subgame class 1
  - demonstrating action rules yield a NE implies SPNE

Repeated game analyzed

- Suppose player j uses grim strategy
  - if player i follows grim strategy she will pick x_i^c,
    - earns payoffs of π_i^c this period, return to same subgame next period
    - hence payoffs of π_i^c next period, and so the period after, and after that...
  - therefore the PDV of following the grim strategy is $V_i^c = \frac{\pi_i^c}{1-\delta}$
  - if deviate to e_i^d, get one-time gain of π_i^d, Cournot/Nash profits π_i^N thereafter
  - so PDV of deviation is $V_i^d = \pi_i^d + \frac{\delta}{1-\delta}\pi_i^N$
- require $V_i^c \geq V_i^d$ (**incentive constraint**)
  - common to focus on most cooperative regime:

$$\pi_i^c = (1-\delta)\pi_i^d + \delta\pi_i^N$$

- in LQ structure, $V_i^c = V_i^d$ induces quadratic relation b/w x_i^c and x_j^c

Penal code strategy

- In period 1, and $t > 1$, if neither firm defected in period $t-1$, firm $i = 1,2$ chooses the (cooperative action) x_i^c
- Should one player defect in period t, players switch to the punishment phase in period $t+1$. [“repentance” action, x_k^r; “punishment” action, x_m^p]
  - k is deviator in t, and m is punisher
- If both players carry through with the punishment phase in period τ, play reverts to the cooperative phase in $\tau + 1$.

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- conditions for SPNE:

$$\pi_i^c(x_j^c)(1+\delta) \geq \pi_i^d(x_j^c) + \delta\pi_i^r(x_i^r, x_j^p);$$

$$\pi_i^r(x_i^r, x_j^p) + \delta\pi_i^c(x_j^c) \geq \pi_i^d(x_j^p) + \delta\pi_i^r(x_i^r, x_j^p);$$

$$\pi_i^p(x_i^p, x_j^r) + \delta\pi_i^c(x_j^c) \geq \pi_i^d(x_j^r) + \delta\pi_i^r(x_i^r, x_j^p).$$

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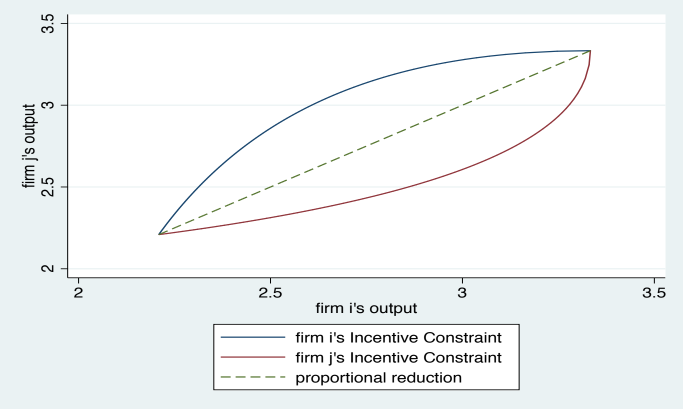
$$\pi_i^p(x_i^p, x_j^r) + \delta\pi_i^c(x_j^c) \geq \pi_i^d(x_j^r) + \delta\pi_i^r(x_i^r, x_j^p).$$

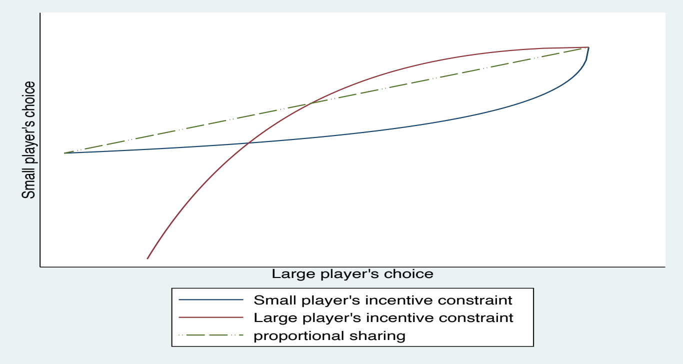
- $\Leftrightarrow \underline{\delta} = \frac{\Delta_1^d(x_2^z)}{\Gamma(x_1^c, x_2^c, x_1^r, x_2^p)} = \frac{\Delta_2^d(x_1^z)}{\Gamma(x_1^c, x_2^c, x_2^r, x_1^p)}, \quad z = c, r, p,$
  - $\Delta_i^d(x_j^z)$ is player i's gain from defecting, $\Gamma_i = \pi_i^c(x_1^c, x_2^c) - \pi_i^r(x_i^r, x_j^p)$

Collusive possibilities: symmetric players



- maximally effective cartel: equal (pro-rata) output reductions

Cooperative possibilities: asymmetric players



- maximally effective IEA: larger than equal (pro-rata) output reduction for H player
- implies greater share of cooperative gains goes to larger country
  - will smaller country accept smaller piece of pie?
  - analogy to ultimatum game?

A model with equity concerns

- denote large (small) player as 1 (2) $\Rightarrow \pi_1 > \pi_2$
- suppose

$$\pi_i = (\alpha_i - X)x_i, i = 1,2$$
$$U_i(\pi_i, \pi_j) = \pi_i - \gamma|\pi_i - \pi_j|, \text{ with } \gamma > 0$$

- then the players' utilities can be written as

$$U_1 = (1-\gamma)\pi_1 + \gamma\pi_2; \quad U_2 = (1+\gamma)\pi_2 - \gamma\pi_1$$

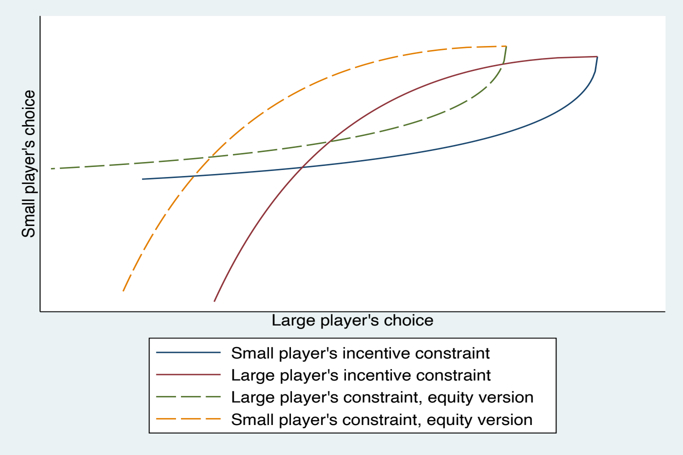
$\Rightarrow$ reaction functions shift to

$$x_1 = \frac{\alpha_1}{2} - \left(\frac{1}{2(1-\gamma)}\right)x_2 \qquad \text{(pivots in)}$$
$$x_2 = \frac{\alpha_2}{2} - \left(\frac{1}{2(1+\gamma)}\right)x_1 \qquad \text{(pivots out)}$$

- pushes NE towards smaller x_1, bigger x_2

Cooperative possibilities: asymmetric players, equity

- similar effect is induced on quasi-cooperative play
  - incentive constraints shift left (and slightly up)
  - substantially improved prospects for smaller player



Theory results

Generalization of equity model:

$$U_i(\pi_i, \pi_j) = \pi_i + \lambda_i \pi_j,$$

where we expect $\lambda_2 < 0 < \lambda_1$

Proposition: Introducing social preferences, via $\lambda_2 < 0$, tightens firm 2's incentive constraint when firms play the grim strategy or penal code strategy.

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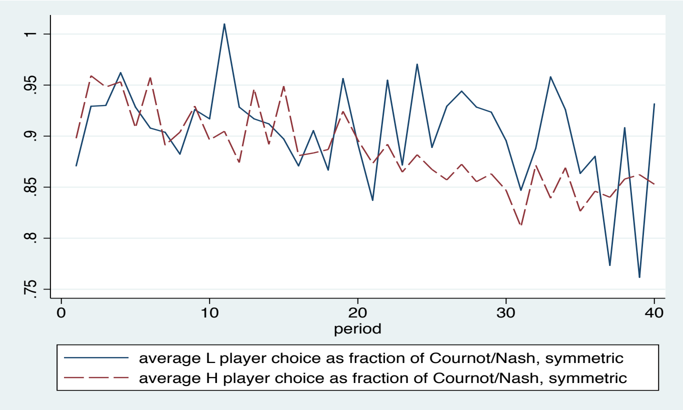
Proposition: Introducing social preferences via $\lambda_1 > 0$ loosens firm 1's incentive constraint when firms play the grim strategy or penal code strategy.

Experimental design

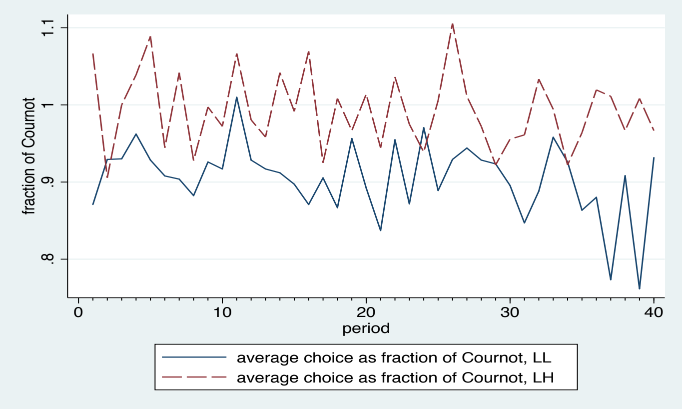
- two market structures, each has $a = 4, b = \frac{1}{24}$
  1. *symmetric* design: $c_i = 0$
  2. *asymmetric* design: $c_1 = 0, c_2 = \frac{1}{2}$
- Cournot/Nash equilibrium outputs: $q_i^N = 32$ (symmetric); $q_1^N = 36, q_2^N = 24$ (asymmetric)
- profits presented to subjects via *payoff tables*
  - profit from various (integer) output combinations shown in matrix form
- all experimental sessions ran at least 35 periods
  - random termination rule (continuation $p = .8$)
- six experimental sessions
  - three symmetric sessions: 38 subjects (19 pairs) made choices for between 35 and 46 periods
  - three asymmetric sessions: 50 subjects (25 pairs) made choices for between for 36 to 46 periods

Experimental results 1: symmetric firms



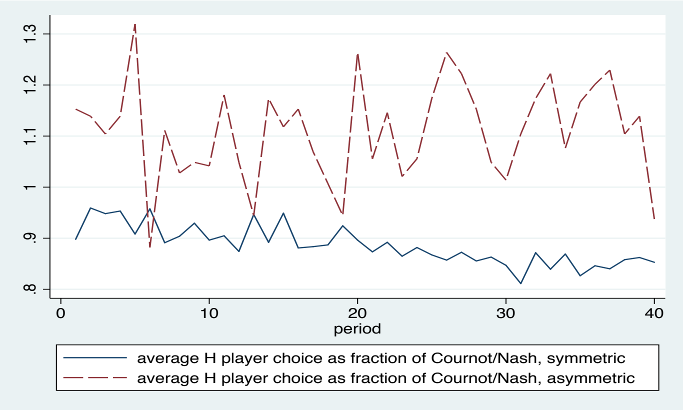
- substantial reductions below Cournot/Nash eq'm output

Experimental results 2: symmetric vs. asymmetric (L)

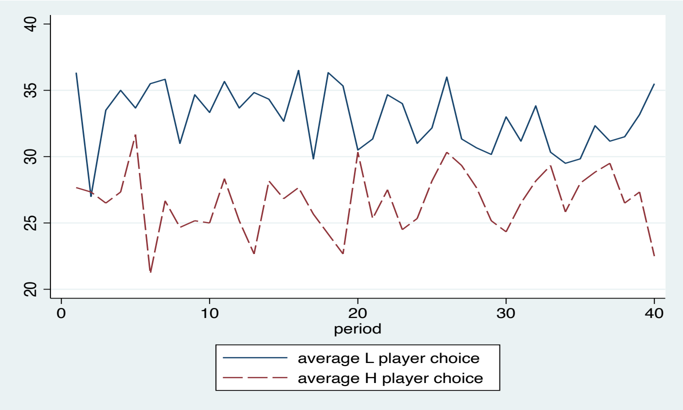


- asymmetric markets far less collusive than symmetric markets
  - virtually no reduction below Cournot/Nash eq'm output

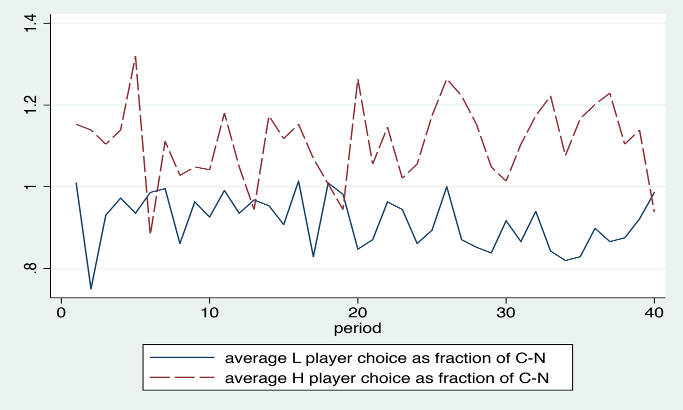
Experimental results 3: symmetric vs. asymmetric (H)



Experimental results 4A: asymmetric firms (levels)



Experimental results 4B: asymmetric firms (pct. C-N)



- theory: H players should accept larger than pro-rata output reductions
- results: inconsistent with these predictions
  - L players: substantial reductions below Cournot/Nash eq'm output
  - H players: one-shot best-reply to L player output?

Econometric model

- unbalanced panel
  - over-weighting observations from sessions that ran longer?
- truncate at period 35 $\Rightarrow$ create balanced panel
- allow for play using "dynamic reaction functions"

$$q_{it} = \varphi_{0h} + \mu_{1h} q_{it-1} + \mu_{2h} q_{it-2} + \mu_{3h} q_{it-3} + \nu_{1h} q_{jt-1} + \nu_{2h} q_{jt-2} + \nu_{3h} q_{jt-3}$$

- where $h = L$ (respectively, H) if player i is low (respectively, high) cost
- k indexes the players' subject pair
- compactly:

$$q_{it} = \varphi_{i0} + \Sigma_{n=1}^{3} \mu_{nh} q_{i,t-n} + \Sigma_{n=1}^{3} \nu_{nh} q_{j,t-n} + \omega_{kt} + \eta_{it}$$

  - individual-specific fixed effects (via φ_{i0})
  - pair-specific variance (*i.e.*, random effects, via ω_{kt}^2)
  - individual-specific residual, η_{it}, is assumed to be white noise
  - estimate w/robust standard errors (clustered at the subject pair level)

Long-run outcomes

- suppose subjects in asymmetric structure play (q_L^*, q_H^*) for ≥ 4 periods

$$q_L^* = \varphi_{0L} + \mu_{1L} q_L^* + \mu_{2L} q_L^* + \mu_{3L} q_L^* + \nu_{1L} q_H^* + \nu_{2L} q_H^* + \nu_{3L} q_H^*, \quad (1)$$
$$q_H^* = \varphi_{0H} + \mu_{1H} q_H^* + \mu_{2H} q_H^* + \mu_{3H} q_H^* + \nu_{1H} q_L^* + \nu_{2H} q_L^* + \nu_{3H} q_L^*. \quad (2)$$

- define $\tilde{\mu}_h = \mu_{1h} + \mu_{2h} + \mu_{3h}; \tilde{\nu}_h = \nu_{1h} + \nu_{2h} + \nu_{3h}, h + L, H$
- solving the system of equations (1)–(2) yields:

$$q_L^* = \frac{\varphi_{0L}(1 - \tilde{\mu}_H) + \tilde{\nu}_L \varphi_{0H}}{(1 - \tilde{\mu}_L)(1 - \tilde{\mu}_H) - \tilde{\nu}_L \tilde{\nu}_H}, \quad (3)$$
$$q_H^* = \frac{\varphi_{0L} \tilde{\nu}_H + \varphi_{0H}(1 - \tilde{\mu}_L)}{(1 - \tilde{\mu}_L)(1 - \tilde{\mu}_H) - \tilde{\nu}_L \tilde{\nu}_H}. \quad (4)$$

- interpret these as equilibrium (steady state) outputs

Regression results notation

- within a given treatment (LL, LH, HH) create vectors for each player i:
  - x_{ht-s} = i's choice in period $t-s, s=1,2,3$
  - y_{ht-s} = i's rival's choice in period $t-s, s=1,2,3$
- stack these vectors to get regressors
  - x_{h1} is the vector for once-lagged own choices by $h=L,H$ subjects
  - y_{h1} is the vector for once-lagged rival's choices by $h=L,H$ subjects
  - similarly for twice-, thrice-lagged choices

Regression Results

reg'r	LH (N=1600)	LL (N=1216)	HH (N=1386)
x_{L1}	-0.264***	-0.327***	
x_{L2}	0.151	0.026	
x_{L3}	-0.04	-0.131***	
y_{L1}	0.253**	0.210***	
y_{L2}	-0.033	-0.022	
y_{L3}	0.075	0.103**	
x_{H1}	-0.05		0.103
x_{H2}	0.1		0.502***
x_{H3}	-0.059		-0.048
y_{H1}	0.182***		0.101
y_{H2}	-0.141***		-0.003
y_{H3}	0.029		0.044**
constant	21.064***	42.757***	7.328***
Q_L^*	33.21	29.22	—
Q_H^*	24.51	—	24.35

Inferring γ

- suppose these values are proportional to NE (based on some value of γ)
  - as if pro-rata reductions
- gives a relation $f(\gamma)$ for Q_L/Q_H
- compare to $R \equiv Q_L^*/Q_H^* \Rightarrow \gamma^*$
- then infer $\mu^* = Q_i^*/Q_i^N(\gamma^*)$

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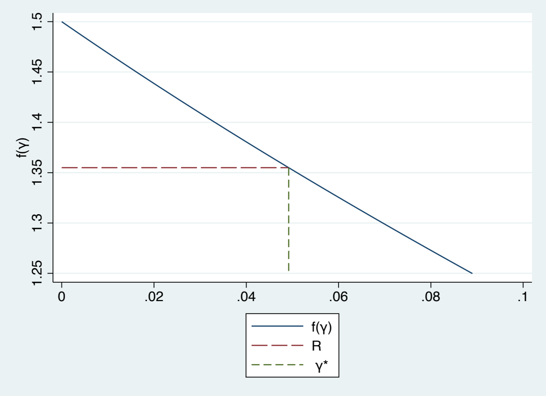

Conclusion

- empirical evidence suggests quasi-cooperative play is undercut when players' payoffs are asymmetric
  - commonly, 'smaller' players are source of friction
- in quasi-cooperative equilibrium of conventional model, gains from cooperation to large player are commonly less than for small player
  - seems incompatible with empirical results above
- one possible resolution is that players exhibit **equity concerns**
- pushes one-shot equilibrium towards larger actions for small player (vs. standard model)
- enlarges scope for small player to benefit in quasi-cooperative outcome of repeated game...

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- estimated long-run choices can be inverted to give estimate of $\gamma = .0492$
- based on that estimate, pro-rata reductions from NE are only about 4%