N°2023 – 04 • MARCH 2023

WORKING PAPER

Energy storage and the direction of technical change

Mohamed BAHLALI[1*, 2*]

We study how storage technologies drive the direction of technical change in the energy sector. To that end, we extend the model of endogenous innovation from Acemoglu et al. (2012) by adding a storage factor in the production function. Clean and dirty inputs are assumed to be perfect substitutes, but clean energy is intermittent and needs to be backed up by dirty energy or storage. We show that (i) without intervention, as long as energy storage is expensive, innovation is pushed to the dirty sector -- and all the more when renewable energy is cheap (ii) a policy aiming at phasing out dirty energy as quickly as possible should allocate, at each time, the research to the less advanced sector between clean energy and storage. These results suggest that governments should now prioritize funding research in storage technologies, in order to accelerate the energy transition and shift innovation away from fossil fuels.

JEL CODES: O33; O44; Q30; Q54; Q55; Q58

1* LEDa, Université Paris-Dauphine - PSL, Place du Maréchal de Lattre de Tassigny, Paris, France
2* Climate Economics Chair, Paris

Email address:
mohamed.bahlali@chaireeconomieduclimat.org

KEYWORDS

Energy storage | Renewable energy | Innovation | Public policy

Executive summary

Decarbonizing energy, including through the deployment of renewable technologies, is a priority to mitigate climate change. In this context, the promotion of green innovation constitutes an important public policy lever, contributing to reducing the cost of these technologies. There has been rich economic literature, both theoretical and empirical, about the relationships between innovation, public policy and the energy transition.

The model of environmental directed technical change by Acemoglu, Aghion, Bursztyn and Hemous (hereafter AABH) has led to abundant research. It describes the production of a unique final good from two inputs, a clean one and a dirty one. The authors show the existence of a virtuous path dependency of clean innovation: more innovation today would contribute to more innovation in the future. As a consequence, sustainable growth could be achieved with temporary taxes/subsidies aiming at redirecting innovation from fossil toward renewable technologies - once done, clean innovation would then increase by itself indefinitely.

In this paper, we show that this result no longer holds when renewables intermittency is taken into account. To that purpose, we extend AABH framework by adding a storage factor in the production function. In our model, clean and dirty factors are assumed to be perfect substitutes, but clean energy is intermittent and needs to be backed up by dirty energy or storage.

We show that high storage costs are conducive to dirty innovation, and all the more when renewables are cheap. Indeed, as long as storage is expensive, fossil fuels keep being used as a necessary back-up to renewables. In that case, high dirty energy prices make innovations in this sector more profitable. Thus, contrary to AABH conclusions, we find that redirecting innovation towards clean energy is not enough to ensure a sustainable growth path: storage technologies must also be developed to steer innovation away from fossil fuels.

Within this framework, we also study the problem of the social planner. Which of the renewable, fossil or storage research sectors should be fostered? The answer is not obvious at first sight, because each one has some benefits and drawbacks: renewable energy is clean but intermittent, dirty energy is flexible but polluting, storage provides clean flexibility but does not produce any energy. We show that at the social optimum, there should be no innovation in the dirty sector: all the research should be allocated to clean and storage technologies. It is difficult to say explicitly in what proportions. However, if the objective is to minimize the period during which dirty energy is used, then innovation should always be directed towards the less advanced sector between clean energy and storage.

In 2021, renewables were cheaper than fossil fuels, but storage was still expensive: the cost of gas combined cycle electricity was ranging between 45$ and 74$/MWh, the cost of utility-scale solar PV electricity between 28$ and 37$/MWh, and the cost of wholesale storage between 165$ and 296$/MWh. These data, together with our model, suggest that governments should now prioritize research in the storage sector, in order to accelerate the energy transition and shift innovation away from fossil fuels.

.

1 Introduction

Decarbonising energy, including through the development of renewable technologies, is a priority to control climate change. In this context, the promotion of green innovation constitutes an important public policy lever, contributing to reducing the cost of these technologies. There has been rich economic literature, both theoretical and empirical, about the relationships between innovation, public policy and the energy transition (Popp (2019)).

The model of environmental directed technical change by Acemoglu et al. (2012) (hereafter AABH) has become a reference and lead to abundant research. It describes the production of a unique final good from two inputs, a clean one and a dirty one. When the two inputs are substitutes, the authors show the existence of a virtuous path dependency of clean innovation: more innovation today would contribute to more innovation in the future. As a consequence, sustainable growth could be achieved with temporary taxes/subsidies aiming at redirecting innovation from fossil toward renewable technologies - once done, clean innovation would then increase by itself indefinitely.

In this paper, we show that this result no longer holds when renewables intermittency is taken into account. To that purpose, we extend AABH framework by adding a storage factor in the production function. In our model, clean and dirty factors are assumed to be perfect substitutes, but clean energy is intermittent and needs to be backed up by dirty energy or storage.

We show that high storage costs are conducive to dirty innovation, and all the more when renewables are cheap. Indeed, as long as storage is expensive, fossil fuels keep being used as a necessary back-up to renewables. In that case, high dirty energy prices make innovations in this sector more profitable. Thus, contrary to AABH conclusions, we find that redirecting innovation towards clean energy is not enough to ensure a sustainable growth path: storage technologies must also be developed to steer innovation away from fossil fuels.

Within this framework, we also study the problem of the social planner. Which of the renewable, fossil or storage research sectors should be fostered? The answer is not obvious at first sight, because each one has some benefits and drawbacks: renewable energy is clean but intermittent, dirty energy is flexible but polluting, storage provides clean flexibility but does not produce any energy. We show that at the social optimum, there should be no innovation in the dirty sector: all the research should be allocated to clean and storage technologies. It is difficult to say explicitly in what proportions. However, if the objective is to minimize the period during which dirty energy is used, then innovation should always be directed towards the less advanced sector between clean energy and storage.

The paper is organized as follows. We describe the model in Section 2, characterize the laissez-faire equilibrium in Section 3, and the social optimal allocation in Section 4. Section 5 concludes.

2 Model

The model builds on Acemoglu et al. (2012). A final good is produced competitively using clean and dirty energy inputs. Clean energy is intermittent and needs to be backed up by dirty energy or storage. They are, in turn, produced from a continuum of intermediate machines that scientists can improve through innovation.

2.1 Final good production

The final (numéraire) output is produced competitively using clean and dirty energy, Y_c and Y_d, according to the following production function

$$Y_t = (Y_{ct} + Y_{dt})^\kappa \tag{1}$$

where $\kappa \in (0,1)$. Clean and dirty inputs are assumed to be perfect substitutes, but clean energy is intermittent. Dirty energy and/or storage are needed in back-up to get a smooth supply, hence the following constraint

$$Y_{ct} \leq \rho Y_{dt} + Y_{st} \tag{2}$$

where Y_{st} is the use of energy storage and ρ is a technical parameter. A representative final good producer therefore earns the profit:

$$\Pi_t^{FG} = (Y_{ct} + Y_{dt})^\kappa - p_{ct}Y_{ct} - p_{dt}Y_{dt} - p_{st}Y_{st} \tag{3}$$

and solves

$$\max_{Y_{ct}, Y_{dt}, Y_{st}} \Pi_t^{FG} \tag{4}$$

under the constraint (2), where p_{jt} is the price of the input in sector $j \in \{c, d, s\}$.

2.2 Clean, dirty and storage inputs production

Each input Y_j (with $j \in \{c, d, s\}$) is produced using labor L and a continuum of sector-specific intermediates:

$$Y_{jt} = L_{jt}^{1-\alpha} \int_0^1 A_{jit}^{1-\alpha} x_{jit}^\alpha \, \mathrm{d}i \tag{5}$$

where $\alpha \in (0,1)$, A_{jit} is the productivity of intermediate of type $i \in [0,1]$ used in sector $j \in \{c, d, s\}$, and x_{jit} is the quantity of this intermediate. Total labour is normalised to 1:

$$L_{ct} + L_{dt} + L_{st} = 1 \tag{6}$$

A representative input producer therefore earns the profit:

$$\Pi_{jt}^I = p_{jt} L_{jt}^{1-\alpha} \int_0^1 A_{jit}^{1-\alpha} x_{jit}^\alpha \, \mathrm{d}i - w_t L_{jt} - \int_0^1 p_{jit} x_{jit} \, \mathrm{d}i \tag{7}$$

and solves

$$\max_{L_{jt}, x_{jit}} \Pi_{jt}^I \tag{8}$$

under the constraint (6), where p_{jit} is the price of the intermediate machine i in sector j and w_t is the wage.

2.3 Intermediate machines producers

Intermediates are supplied by monopolistically competitive firms at constant cost of ψ. Without loss of generality, we set $\psi := \alpha$. Producer of machine i earns the profit:

$$\Pi_{jit}^M = p_{jit} x_{jit} - \Psi x_{jit} \tag{9}$$

and solves

$$\max_{p_{jit}} \Pi^M_{jit} \tag{10}$$

Market clearing for the final good implies

$$C_t = Y_t - \psi\left(\int_0^1 x_{cit}\,\mathrm{d}i + \int_0^1 x_{dit}\,\mathrm{d}i + \int_0^1 x_{sit}\,\mathrm{d}i\right) \tag{11}$$

2.4 Scientists

There is a continuum of scientists in $(0,1)$. At the beginning of each period, each scientist decides in which sector j she will carry out her research. She is then randomly allocated to one machine i in this sector, and is successful in innovation with probability $n_j \in (0,1)$, where innovation increases the quality of the machine by a factor $1+\gamma$. She then obtains a one-period patent and becomes the entrepreneur for the current period in the production of machine i. Let s_{jt} be the number of scientists in sector j at time t. Total number of scientists is normalised to 1:

$$s_{ct} + s_{dt} + s_{st} = 1 \tag{12}$$

The expected profit of a scientist allocated to machine i in sector j is therefore

$$\Pi^S_{jt} = \eta_j \int_0^1 \Pi^M_{jit}\,\mathrm{d}i \tag{13}$$

where $A_{jt} = \int_0^1 A_{jit}\,\mathrm{d}i$ is the average productivity of machines in sector j, which evolves according to the following dynamics

$$A_{jt} = (1 + \gamma\eta_j s_{jt})A_{jt-1} \tag{14}$$

Scientists rationally choose the sector with highest expected revenue, i.e. for any $k \in \{c,d,s\}$

$$s_{kt} > 0 \iff \Pi^S_{kt} \geq \Pi^S_{jt} \;\; \forall j \in \{c,d,s\} \tag{15}$$

2.5 Environment

Environmental quality, denoted by $S_t \in [0, \overline{S}]$, evolves according to the following dynamics[1]

$$S_{t+1} = -\xi Y_{dt} + (1+\delta)S_t \tag{16}$$

where the parameter ξ measures the rate of environmental degradation resulting from the production of dirty inputs, and δ is the rate of "environmental regeneration".

We say that there is an "environmental disaster" at time t if $S_t = 0$.

[1] When the right hand side is higher than $\overline{S}$, we set $S_{t+1} = \overline{S}$; and when it is negative, we set $S_{t+1} = 0$

3 The Laissez-Faire equilibrium

In this section, we study the equilibrium without policy intervention. After solving the problem of the final good producers and characterizing the expected profit of a scientist in each sector, we study the direction of technical change. We show that if the cost of storage is too high, innovation will always head towards the dirty sector, even if clean energy is cheaper.

Definition 3.1. *A laissez-faire equilibrium is a sequence of demands for inputs Y_{jt}, demands for machines x_{jit}, labour demands L_{jt}, research allocations s_{jt}, input prices p_{jt} and machine prices p_{jit} solution of the system formed by (4), (8), (10) under the constraints (2), (6), (11), (12), (14), (16) and condition (15), and where the wage w_t and prices p_{jt} respectively clear the labour and input markets.*

We first solve the problem of the final good producers, i.e. solving (4) under constraint (2). As a result, we obtain in the following Lemma the equilibrium demand for inputs.[2]

Lemma 3.1. *At the laissez-faire equilibrium:*

1. *If $p_{ct} > p_{dt}$, then $\kappa Y_{dt}^{\kappa-1} = p_{dt}$, $Y_{ct} = 0$, $Y_{st} = 0$.*
2. *If $p_{ct} < p_{dt}$, then:*
   (a) *if $p_{st} > \frac{p_{dt}-p_{ct}}{1+\rho}$, then $\kappa(Y_{dt}+Y_{ct})^{\kappa-1} = \frac{p_{dt}+\rho p_{ct}}{1+\rho}$, $Y_{ct} = \rho Y_{dt}$ and $Y_{st} = 0$.*
   (b) *if $p_{st} < \frac{p_{dt}-p_{ct}}{1+\rho}$, then $Y_{dt} = 0$, $\kappa Y_{ct}^{\kappa-1} = p_{ct} + p_{st}$ and $Y_{st} = Y_{ct}$.*
3. *If $p_{ct} = p_{dt}$, then $Y_{st} = 0$, $Y_{dt} > 0$ and $0 \leq Y_{ct} \leq \rho Y_{dt}$.*

When renewables are more expensive than fossil fuels, the final good is only produced by dirty energy. When renewables become cheaper, as long as storage remains expensive, renewables are backed by fossil fuels. When storage technologies become, in turn, cheap enough, production becomes clean and storage is used to compensate for renewables intermittency.

Now, applying optimality conditions to (8) and (10) under their respective constraints gives equilibrium prices and quantities of machines as

$$x_{jit} = p_{jt}^{1/(1-\alpha)} L_{jt} A_{jit} \tag{17}$$

$$p_{jit} = \psi^2/\alpha = \alpha \tag{18}$$

as well as the relationship between input prices and knowledge

$$\frac{p_{jt}}{p_{kt}} = \left(\frac{A_{jt}}{A_{kt}}\right)^{-(1-\alpha)} \tag{19}$$

which indicates that the most technologically advanced sector is the one with the lowest price. Combining (17) and (18), we get the equilibrium profits of machine producers

$$\Pi_{jit}^{M} = \alpha(1-\alpha) p_{jt}^{1/(1-\alpha)} L_{jt} A_{jit} \tag{20}$$

[2]All the proofs of this paper are available in the supplementary material.

and therefore the expected profit of scientists engaged in research in sector j

$$\Pi_{jt}^{S} = \eta_j \int_0^1 \Pi_{jit}^{M}\, \mathrm{d}i = \eta_j (1+\gamma)(1-\alpha)\alpha p_{jt}^{1/(1-\alpha)} L_{jt} A_{jt-1} \tag{21}$$

The scientists' profits ratios between sectors $j,k \in \{c,d,s\}$ are then given by

$$\frac{\Pi_{jt}^{SC}}{\Pi_{kt}^{SC}} = \frac{\eta_j}{\eta_k}\left(\frac{p_{jt}}{p_{kt}}\right)^{1/(1-\alpha)} \frac{L_{jt}}{L_{kt}}\frac{A_{jt-1}}{A_{kt-1}} \tag{22}$$

The higher this ratio, the more profitable is R&D directed towards technology j compared to technology k. We find again the three incentive effects on innovation revealed by AABH: a price effect, a market size effect and a direct productivity effect. As highlighted by Lemma 3.1, the market size depends on the relative prices of dirty energy, clean energy and storage. Consequently, R&D profits also depend on these prices and are given by the following Lemma.

Lemma 3.2. *At the laissez-faire equilibrium:*

1. *If* $p_{ct} > p_{dt}$*, then* $\Pi_{ct}^{S} = 0$ *and* $\Pi_{st}^{S} = 0$.
2. *If* $p_{ct} < p_{dt}$*, then:*
   (a) *If* $p_{st} < \frac{p_{dt}-p_{ct}}{1+\rho}$*, then* $\Pi_{dt}^{S} = 0$ *and*

$$\frac{\Pi_{ct}^{S}}{\Pi_{st}^{S}} = \frac{\eta_c}{\eta_s}\left(\frac{1+\gamma\eta_c s_{ct}}{1+\gamma\eta_s s_{st}}\right)^{-(2-\alpha)}\left(\frac{A_{ct-1}}{A_{st-1}}\right)^{-(1-\alpha)} \tag{23}$$

   (b) *If* $p_{st} > \frac{p_{dt}-p_{ct}}{1+\rho}$*, then* $\Pi_{st}^{S} = 0$ *and*

$$\frac{\Pi_{ct}^{S}}{\Pi_{dt}^{S}} = \rho\frac{\eta_c}{\eta_d}\left(\frac{1+\gamma\eta_c s_{ct}}{1+\gamma\eta_d s_{dt}}\right)^{-(2-\alpha)}\left(\frac{A_{ct-1}}{A_{dt-1}}\right)^{-(1-\alpha)} \tag{24}$$

Main outcome of this Lemma is that innovation will always favor *the less technologically advanced sector* used in the production. This means that the price effect (that encourages innovation towards the less advanced sector) is more important than the market size effect (that encourages innovation towards the most advanced sector). AABH comes to the opposite conclusion, when the clean and dirty inputs are substituable. This is precisely because we assumed that clean energy is intermittent: as opposed to AABH, here clean and dirty energy (as well as clean energy with storage) are not substituable factors. They actually behave as complementary factors due to the intermittency constraint (2).

For example, assume that $p_{ct} < p_{dt}$ and $p_{st} > \frac{p_{dt}-p_{ct}}{1+\rho}$. In this case, clean energy is cheap but storage is expensive. By Lemma 3.1, both clean and dirty energy will be used in the production: the two sectors share the same market size. As a consequence, innovation will go the sector where price is the highest: the dirty one.

Thus, if the cost of storage is too high, innovation will always head towards the dirty sector, even if clean energy is cheaper. But the story does not end there. With innovation, the price of dirty energy will fall, eventually below that of clean energy, pushing it out of the production of the final good. Environment will deteriorate, leading to an environmental disaster.

This dynamics is expressed in the Proposition 3.2 hereafter. Let us denote by s^*_{ct}, s^*_{dt} and s^*_{st} the laissez-faire allocation of scientists. Consider the following assumption

Assumption 3.1. $0 < \rho\eta_c/\eta_d < 1$.

The following Proposition first states that an environmental disaster occurs if the dirty sector is initially more technologically advanced than the clean sector.

Proposition 3.1. *Assume*

$$(1+\gamma\eta_c)A_{c0} < A_{d0} \tag{25}$$

Then $s^*_{st} = 0$, $s^*_{ct} = 0$ and $s^*_{dt} = 1$ for all t. There is an environmental disaster.

Now, the next Proposition states that an environmental disaster occurs if the storage sector is not technologically advanced enough, even if the clean sector is more advanced than the dirty one.

Proposition 3.2. *Assume (3.1) holds, and*

$$\begin{cases} A_{d0} < A_{c0} \\ A_{s0} < \left[\frac{A_{d0}^{-(1-\alpha)} - A_{c0}^{-(1-\alpha)}}{(1+\rho)(1+\gamma\eta_s)}\right]^{-\frac{1}{1-\alpha}} \end{cases} \tag{26}$$

Then $s^*_{st} = 0$ for all t, $s^*_{ct} \to 0$ and $s^*_{dt} \to 1$ in finite time. There is an environmental disaster.

On the contrary, if the clean and storage sectors are well enough developed, innovation and production move away from the dirty sector and an environmental disaster is avoided. This is expressed in Proposition 3.3.

Proposition 3.3. *Assume*

$$\begin{cases} (1+\gamma\eta_d)A_{d0} < A_{c0} \\ A_{s0} \geq \left[\frac{(1+\gamma\eta_d)A_{d0}^{-(1-\alpha)} - A_{c0}^{-(1-\alpha)}}{1+\rho}\right]^{-\frac{1}{1-\alpha}} \end{cases} \tag{27}$$

Then, $s^*_{dt} = 0$ for all t. Environment regenerates in finite time and there is no disaster.

To summarize, Propositions 3.1, 3.2 and 3.3 emphasize the key role of storage in achieving a fully clean innovation and production system. When storage is not well developed, all innovation ends up going to the dirty sector. Thus, contrary to AABH conclusions, we find that redirecting innovation towards clean energy is not enough to ensure a sustainable growth path: storage technologies must also be developed to steer innovation away from fossil fuels.

4 The Social Optimal Allocation

In this section, we study the socially optimal allocation. We show that the optimal policy shoud combine a (temporary) carbon tax with a R&D subsidy to the clean and storage sectors. We then study how the research should be allocated between these two sectors.

Assume that a representative household has the following intertemporal utility

$$\sum_{t=0}^{\infty} \frac{u(C_t, S_t)}{(1+r)^t} \tag{28}$$

where C_t is consumption, S_t is environmental quality and r is the discount rate. We assume that the function u is twice differentiable, increasing and jointly concave in both variables, and satisfies

$$\lim_{C\to 0} \frac{\partial u(C,S)}{\partial C} = \infty, \quad \lim_{S\to 0} \frac{\partial u(C,S)}{\partial C} = \infty, \quad \text{and} \quad \lim_{C\to 0} u(C,S) = -\infty$$

Definition 4.1. *The social optimal allocation is a dynamic path that maximises intertemporal utility (28) over final good quantity Y_t, consumption C_t, input quantities Y_{jt}, machines quantities x_{jit}, labour allocation L_{jt}, scientists allocation s_{jt}, environmental quality S_t and machines productivities A_{jit} ; according to the constraints (1), (2), (5), (6), (11), (12) and (16).*

Let denote by λ_t, μ_t^1, μ_{jt}, λ_{jt} and ω_{t+1} the respective Lagrange multipliers of constraints (1), (2), (5) and (16). The value $\widehat{p}_{jt} := \lambda_{jt}/\lambda_t$ may be considered as the shadow price of input j at time t. The following Lemma gives the optimal use of clean, dirty and storage inputs.

Lemma 4.1. *Let $\widehat{p}^*_{dt} := \widehat{p}_{dt} + \omega_{t+1}\xi/\lambda_t$. At the social optimum:*

- *When $\widehat{p}^*_{dt} < \widehat{p}_{ct}$, then $Y_{ct} = 0$, $Y_{st} = 0$ and $\kappa Y_{dt}^{\kappa-1} = \widehat{p}^*_{dt}$*
- *When $\widehat{p}^*_{dt} > \widehat{p}_{ct}$ and $\widehat{p}_{st} > \frac{\widehat{p}^*_{dt} - \widehat{p}_{ct}}{1+\rho}$, then $Y_{st} = 0$, $\kappa[(1+\rho)Y_{dt}]^{\kappa-1} = \frac{\widehat{p}^*_{dt} + \rho\widehat{p}_{ct}}{1+\rho}$ and $Y_{ct} = \rho Y_{dt}$*
- *When $\widehat{p}^*_{dt} > \widehat{p}_{ct}$ and $\widehat{p}_{st} < \frac{\widehat{p}^*_{dt} - \widehat{p}_{ct}}{1+\rho}$, then $Y_{dt} = 0$, $\kappa Y_{ct}^{\kappa-1} = \widehat{p}_{ct} + \widehat{p}_{st}$ and $Y_{st} = Y_{ct}$.*

Moreover:

$$\widehat{p}_{dt} = \kappa(Y_{ct} + Y_{dt})^{\kappa-1} + \rho\widehat{p}_{st} - \frac{\omega_{t+1}\xi}{\lambda_t} + \frac{\mu_{dt}}{\lambda_t} \tag{29}$$

$$\widehat{p}_{ct} = \kappa(Y_{ct} + Y_{dt})^{\kappa-1} - \widehat{p}_{st} + \frac{\mu_{ct}}{\lambda_t} \tag{30}$$

Equalities (29) and (30) highlight the externalities relative to the use of each input. The use of dirty energy leads to a negative environmental externality ($-\frac{\omega_{t+1}\xi}{\lambda_t}$) and a positive flexibility externality ($+\rho\widehat{p}_{st}$). The use of clean energy leads to negative flexibility externality ($-\widehat{p}_{st}$). Flexibility constraints are already internalized by the final good producers, through the condition (2). Therefore, as in AABH, we obtain that a carbon tax is enough to achieve socially optimal final good production:

$$\tau_t := \frac{\omega_{t+1}\xi}{\lambda_t \widehat{p}_{dt}} \tag{31}$$

Now, let us focus on the optimal scientists allocation. Which research sector needs to be fostered? Each one has some benefits and drawbacks: dirty energy increases production and flexibility but is polluting, renewable energy is clean but intermittent, storage provides clean flexibility but is not productive. The following Proposition states that at the social optimum, there is no research in the dirty sector, and production of the dirty input vanishes in finite time.

Proposition 4.1. *Assume that the discount rate r is low enough. At the social optimum, there is no research in the dirty sector. All dirty production tends to zero in finite time. The long-run consumption level is*

$$C_t = \alpha^{-\frac{\alpha}{1-\alpha}} \kappa^{\frac{1}{1-\kappa\alpha}} (1-\kappa\alpha) \left[A_{ct}^{-(1-\alpha)} + A_{st}^{-(1-\alpha)} \right]^{-\frac{1}{1-\kappa\alpha}}$$

Thus, at the social optimum, environment regenerates in finite time. As a consequence, the optimal carbon tax is temporary. Indeed, as in AABH,[3] we can show that as soon as $S_t = \overline{S}$, $\omega_t = 0$, hence $\tau_t = 0$.

Moreover, all the research is allocated to the clean and storage sectors. In what proportions? It is actually difficult to explicitly obtain the optimal innovation policy that maximises the intertemporal utility (28). However, it is still possible to determine the innovation policy that leads to the fastest transition from dirty to clean energy.

By Proposition 4.1, at the social optimum, $s_{dt} = 0$. Thus $s_{st} = 1 - s_{ct}$. The optimal innovation policy can therefore be reduced to the only choice of s_{ct}. Let us simply write $s_{ct} = s_t$. We also assume that the sequence $(s_t)_{t\geq 0}$ belongs to $\{0,1\}^{\mathbf{N}}$.

In the presence of a carbon tax, for any innovation policy $s := (s_t)_{t\geq 0}$,

$$T(s) := \min \left\{ T \geq 0,\ \forall t \geq T,\ A_{ct}^{-(1-\alpha)} + (1+\rho) A_{st}^{-(1-\alpha)} \leq (1+\tau_t)[(1+\gamma\eta_d)A_{d0}]^{-(1-\alpha)} \right\}$$

is, by Proposition 3.3, the duration needed to phase out dirty energy. Let

$$T^* = \min_{(s_t)\in\{0,1\}^{\mathbf{N}}} T(s)$$

be the shortest of these durations.

The following Proposition 4.2 and its Corollary 4.1 state that T^* can be achieved with an innovation policy that allocates, at each time, the research to the less advanced sector between clean energy and storage.

To simplify the proof of this Proposition, we make the following assumptions.

Assumption 4.1. $\eta_c = \eta_s := \eta$.

Assumption 4.2. $A_{c0} > A_{s0}$ *and there exists* $n \in \mathbb{N}$ *such that*

$$A_{c0} = (1+\rho)^{-1/(1-\alpha)}(1+\gamma\eta)^n A_{s0}$$

and

$$A_{c0}^{-(1-\alpha)} + (1+\rho)(1+\gamma\eta)^{-(1-\alpha)n} A_{s0}^{-(1-\alpha)} > (1+\tau_n)[(1+\gamma\eta_d)A_{d0}]^{-(1-\alpha)}$$

The first assumption states that the success rate of research is the same in the two sectors. The second assumption states that the clean sector is initially more advanced than the storage sector and, having reached the same level of technological development, both are still not competitive enough against the dirty sector.

[3] Proof of Proposition 5, equality A.8.

Proposition 4.2. *The shortest duration to phase out dirty energy, T^*, can be achieved with the following innovation policy* ($\triangle$)*:*

$$s_t^{\triangle} = 1, \quad if \quad (1+\gamma\eta)A_{ct} \leq (1+\rho)^{-\frac{1}{1-\alpha}} A_{st}$$
$$s_t^{\triangle} = 0, \quad if \quad A_{ct} \geq (1+\rho)^{-\frac{1}{1-\alpha}} (1+\gamma\eta) A_{st}$$
$$s_t^{\triangle} = 0 \ or \ 1, \quad if \quad A_{ct} = A_{st}$$

We can obtain a simpler expression of the optimal policy, if γ and η are small.

Corollary 4.1. *If γ and η are small, the optimal innovation policy of Proposition 4.2 can be approximated by*

$$s_t^{\triangle} = 1, \quad if \quad A_{ct} \leq (1+\rho)^{-\frac{1}{1-\alpha}} A_{st}$$
$$s_t^{\triangle} = 0, \quad if \quad A_{ct} \geq (1+\rho)^{-\frac{1}{1-\alpha}} A_{st}$$
$$s_t^{\triangle} = 0 \ or \ 1, \quad if \quad A_{ct} = A_{st}$$

In 2021, renewables were cheaper than fossil fuels, but storage was still expensive: the cost of gas combined cycle electricity was ranging between 45$ and 74$/MWh, the cost of utility-scale solar PV electricity between 28$ and 37$/MWh, and the cost of wholesale storage between 165$ and 296$/MWh (Lazard (2021a), Lazard (2021b)). These data, together with our model, suggest that governments should now prioritise research in the storage sector, in order to accelerate the energy transition and shift innovation away from fossil fuels.

5 Conclusion

Our model emphasizes the key role of storage technologies in successfully achieving the energy transition. As long as storage is expensive, renewables must be backed up by fossil fuels, which end up capturing all the innovation. The regulator can counteract this effect and reach the social optimum by implementing a carbon tax and subsidising the research in the clean and storage sectors. It is difficult to explicitly determine in which proportions, and we leave this question for a future research. However, if the objective of the social planner is to minimize the transition period during which fossil fuels are used, then innovation should always be directed towards the less advanced sector between clean energy and storage.

References

Daron Acemoglu, Philippe Aghion, Leonardo Bursztyn, and David Hemous. The Environment and Directed Technical Change. *The American Economic Review*, 102(1):131–166, 2012.

Lazard. Lazard's levelized cost of energy analysis - version 15.0, 2021a.

Lazard. Lazard's levelized cost of storage analysis - version 7.0, 2021b.

David Popp. Environmental policy and innovation: a decade of research. 2019.

Proofs

Proof of Lemma 3.1

The proof is a long but straightforward application of the KKT conditions. Let us write the Lagrangian

$$\mathcal{L}(Y_{ct}, Y_{st}, Y_{dt}, \mu_t^1, \mu_t^c, \mu_t^s) = \Pi_t^{FG} - \mu_t^1(Y_{ct} - \rho Y_{dt} - Y_{st}) + \mu_t^c Y_{ct} + \mu_t^d Y_{dt} + \mu_t^s Y_{dt} \tag{32}$$

where μ_t^1, μ_t^c, μ_t^s and μ_t^d are the respective Lagrange multipliers of constraints $Y_{ct} \leq \rho Y_{dt} + Y_{st}$, $Y_{ct} \geq 0$, $Y_{st} \geq 0$ and $Y_{dt} \geq 0$. Applying KKT conditions yields:

$$\kappa(Y_{ct} + Y_{dt})^{\kappa-1} - p_{ct} - \mu_t^1 + \mu_t^c = 0 \tag{33}$$
$$\kappa(Y_{ct} + Y_{dt})^{\kappa-1} - p_{dt} + \rho\mu_t^1 + \mu_t^d = 0 \tag{34}$$
$$-p_{st} + \mu_t^1 + \mu_t^s = 0 \tag{35}$$
$$Y_{ct} - \rho Y_{dt} - Y_{st} = 0 \quad or \quad \mu_t^1 = 0 \tag{36}$$
$$Y_{ct} = 0 \quad or \quad \mu_t^c = 0 \tag{37}$$
$$Y_{dt} = 0 \quad or \quad \mu_t^d = 0 \tag{38}$$
$$Y_{st} = 0 \quad or \quad \mu_t^s = 0 \tag{39}$$

1st case $\mu_t^1 = 0$.

- 1st subcase $\mu_t^s = 0$. Then $p_{st} = 0$. This means that storage is free, we set this case aside.
- 2nd subcase $\mu_t^s > 0$. Then $p_{st} > 0$ and $Y_{st} = 0$.
  - 1st subsubcase $\mu_t^c = 0$ and $\mu_t^d > 0$. Then $p_{dt} > p_{ct}$, $Y_{dt} = 0$, $\kappa Y_{ct}^{\kappa-1} = p_{ct}$ and $Y_{st} = 0$. This solution is impossible because it does not respect the constraint (2).
  - 2nd subsubcase $\mu_t^c > 0$ and $\mu_t^d = 0$. Then $p_{dt} < p_{ct}$, $Y_{ct} = 0$, $\kappa Y_{dt}^{\kappa-1} = p_{dt}$ and $Y_{st} = 0$. This solution corresponds to the case 1 of the Lemma.
  - 3rd subsubcase $\mu_t^c = 0$ and $\mu_t^d = 0$. Then $p_{dt} = p_{ct}$, $\kappa(Y_{ct} + Y_{dt})^{\kappa-1} = p_{c/dt}$ and $Y_{st} = 0$. This solution is the limit of case 1 of the Lemma.

2nd case $\mu_t^1 > 0$.

Then $Y_{ct} = \rho Y_{dt} + Y_{st}$.

- 1st subcase $\mu_t^s = 0$. Then $p_{st} = \mu_t^1$.
  - 1st subsubcase $\mu_t^c = 0$, $\mu_t^d > 0$. Then $p_{dt} - p_{ct} = (1+\rho)p_{st} + \mu_t^d$, therefore $p_{st} < \frac{p_{dt} - p_{ct}}{1+\rho}$. Besides, $Y_{dt} = 0$, $\kappa Y_{ct}^{\kappa-1} = p_{ct} + p_{st}$, and $Y_{st} = Y_{ct}$. This solution corresponds to the case 2-(b) of the Lemma.
  - 2nd subsubcase $\mu_t^c > 0$, $\mu_t^d = 0$. Then $p_{dt} - p_{ct} = (1 + \rho)p_{st} - \mu_t^c$. Besides, $Y_{ct} = 0$, $\kappa Y_{dt}^{\kappa-1} = p_{dt} - \rho p_{st}$ and $Y_{st} = -\rho Y_{dt}$. This solution is impossible.

- 3rd subsubcase $\mu_t^c = 0$, $\mu_t^d = 0$. Then $p_{dt} - p_{ct} = (1+\rho)p_{st}$, and $\kappa(Y_{ct} + Y_{dt})^{\kappa-1} = p_{ct} + p_{st}$. This solution is the limit of case 2-(b) of the Lemma.

- 2nd subcase $\mu_t^s > 0$. Then $p_{st} = \mu_t^1 + \mu_t^s$ and $Y_{st} = 0$.
  - 1st subsubcase $\mu_t^c = 0$ and $\mu_t^d > 0$. Then $p_{dt} - p_{ct} = (1+\rho)\mu_t^1 + \mu_t^d$, $Y_{dt} = 0$, $\kappa Y_{ct}^{\kappa-1} = p_{ct}$ and $Y_{st} = 0$. This solution is impossible because it does not respect the constraint (2).
  - 2nd subsubcase $\mu_t^c > 0$ and $\mu_t^d = 0$. Then $p_{dt} - p_{ct} = (1+\rho)\mu_t^1 - \mu_t^c$, $Y_{ct} = 0$, $\kappa Y_{dt}^{\kappa-1} = p_{dt} - \rho\mu_t^1$ and $Y_{st} = 0$. This solution is impossible, because we should have $Y_{ct} = \rho Y_{dt} + Y_{st}$.
  - 3rd subsubcase $\mu_t^c = 0$ and $\mu_t^d = 0$. Then $p_{dt} - p_{ct} = (1+\rho)\mu_t^1$, therefore $p_{st} > \frac{p_{dt}-p_{ct}}{1+\rho}$. Besides, $\kappa(Y_{ct} + Y_{dt})^{\kappa-1} = p_{ct} + \mu_t^1$, $Y_{ct} = \rho Y_{dt}$ and $Y_{st} = 0$. This solution corresponds to the case 2-(a) of the Lemma.

Proof of Lemma 3.2

If $p_{ct} > p_{dt}$, the result follows directly from Lemma 3.1. Assume that $p_{ct} < p_{dt}$, and that $p_{st} < \frac{p_{dt}-p_{ct}}{1+\rho}$. By Lemma 3.1, $\Pi_{dt}^S = 0$ and, by equations (5) and (17), we have

$$\frac{Y_{ct}}{Y_{st}} = \frac{L_{ct}}{L_{st}} \left(\frac{p_{ct}}{p_{st}}\right)^{\frac{\alpha}{1-\alpha}} \frac{A_{ct}}{A_{st}} \tag{40}$$

Using $Y_{ct} = Y_{st}$ by Lemma 3.1 and equations (14), (19) and (22), we get

$$\frac{\Pi_{ct}^S}{\Pi_{st}^S} = \frac{\eta_c}{\eta_s} \left(\frac{1+\gamma\eta_c s_{ct}}{1+\gamma\eta_s s_{st}}\right)^{-(2-\alpha)} \left(\frac{A_{ct-1}}{A_{st-1}}\right)^{-(1-\alpha)} \tag{41}$$

which is the desired result. A similar proof applies for the case where $p_{st} > \frac{p_{dt}-p_{ct}}{1+\rho}$.

Proof of Proposition 3.1

$(1+\gamma\eta_c)A_{c0} < A_{d0}$ implies $A_{c1} < A_{d1}$ which, in turn, implies $p_{c1} > p_{d1}$ and therefore $\Pi_{c1}^S = 0$ and $\Pi_{s1}^S = 0$. Then, $s_{c1}^* = 0$, $s_{s1}^* = 0$ and $s_{d1}^* = 1$. As a result, $A_{c1} = A_{c0}$ and $(1+\gamma\eta_c)A_{c1} = (1+\gamma\eta_c)A_{c0} < A_{d0} < A_{d1}$. By repetition, $s_{ct}^* = 0$, $s_{st}^* = 0$ and $s_{dt}^* = 1$ for all t.

Proof of Proposition 3.2

Proving that $s_{st}^* = 0$ for all t is very similar to the proof of Proposition 3.1.

Let us show that $s_{ct}^* \to 0$ in finite time. Assume that (26) holds not only at time zero, but also for some time $t-1$. The function

$$\frac{\Pi_{ct}^S}{\Pi_{dt}^S}(s_{ct}) = \rho\frac{\eta_c}{\eta_d} \left(\frac{1+\gamma\eta_c s_{ct}}{1+\gamma\eta_d(1-s_{ct})}\right)^{-(2-\alpha)} \left(\frac{A_{ct-1}}{A_{dt-1}}\right)^{-(1-\alpha)}$$

is decreasing in s_{ct}. As $A_{dt-1} < A_{ct-1}$, we have $\frac{\Pi^S_{ct}}{\Pi^S_{dt}}\left(s_{ct} = \frac{\eta_d}{\eta_c+\eta_d}\right) < 1$. Thus, if $s_{ct} = \frac{\eta_d}{\eta_c+\eta_d}$, $\Pi^S_{ct} < \Pi^S_{dt}$. Therefore, by condition (15), the laissez-faire allocation of scientists is such that $s^*_{ct} < \eta_d/(\eta_c + \eta_d)$.

If

$$\frac{\Pi^S_{ct}}{\Pi^S_{dt}}(s_{ct} = 0) < 1$$

i.e.

$$\rho\frac{\eta_c}{\eta_d}(1 + \gamma\eta_d)^{2-\alpha}\left(\frac{A_{ct-1}}{A_{dt-1}}\right)^{-(1-\alpha)} < 1$$

then $s^*_{ct} = 0$ is the unique equilibrium.

Otherwise, by the intermediate value theorem, there exists $s^{(1)}_{ct} \in \left[0, \frac{\eta_d}{\eta_c+\eta_d}\right]$ such that

$$\frac{\Pi^S_{ct}}{\Pi^S_{st}}\left(s^{(1)}_{ct}\right) = 1$$

Define $s^{(2)}_{ct}$ such that

$$\left(1 + \gamma\eta_c s^{(2)}_{ct}\right) A_{ct-1} = \left(1 + \gamma\eta_d(1 - s^{(2)}_{ct})\right) A_{dt-1}$$

i.e. $p_{ct} = p_{dt}$. When $s_{ct} < s^{(2)}_{ct}$, then $p_{ct} < p_{dt}$ and $\Pi^S_{ct} = 0$.

If $\frac{\eta_c}{\eta_d}\rho < \frac{A_{dt-1}}{A_{ct-1}}$, then $s^{(1)}_{ct} < s^{(2)}_{ct}$ and $s^*_{ct} = 0$ is the unique equilibrium. Otherwise, there are two possible equilibria: $s^*_{ct} = 0$ or $s^*_{ct} = s^{(1)}_{ct}$. Let us show that in any case, $s^*_{ct} \to 0$ in finite time. Let $\left(s^*_{c\phi(t)}\right)$ be the subsequence of scientist allocations such that $s^*_{c\phi(t)} = s^{(1)}_{ct}$ for all t. The subsequence $\left(s^*_{c\phi(t)}\right)$ is nondecreasing because for all t, $\frac{\Pi^S_{c\phi(t)}}{\Pi^S_{d\phi(t)}} = 1$ while the sequence $\left(\frac{A_{d\phi(t-1)}}{A_{c\phi(t-1)}}\right)$ increases. Indeed, since $s^{(1)}_{ct} \in \left[0, \frac{\eta_d}{\eta_c+\eta_d}\right]$, we have $\frac{1+\gamma\eta_c s^{(1)}_{ct}}{1+\gamma\eta_d(1-s^{(1)}_{ct})} > 1$, and therefore

$$\frac{A_{d\phi(t)}}{A_{c\phi(t)}} = \frac{1 + \gamma\eta_c s^{(1)}_{ct}}{1 + \gamma\eta_d(1 - s^{(1)}_{ct})}\frac{A_{d\phi(t)-1}}{A_{c\phi(t)-1}} > \frac{A_{d\phi(t)-1}}{A_{c\phi(t)-1}} \geq \frac{A_{d\phi(t-1)}}{A_{c\phi(t-1)}}$$

The subsequence $\left(s^*_{c\phi(t)}\right)$ is also bounded from above, by $\eta_d/(\eta_c + \eta_d)$. Therefore, it converges. Denote by $s^{(1)}_c$ its limit. If $s^{(1)}_c < \eta_d/(\eta_c + \eta_d)$, then $\frac{A_{dt}}{A_{ct}}$ tends to infinity (because $\frac{A_{dt}}{A_{ct}} = \frac{1+\gamma\eta_c s_{ct}}{1+\gamma\eta_d(1-s_{ct})}\frac{A_{dt-1}}{A_{ct-1}}$) and $s^*_{ct} \to 0$ in finite time. If $s^{(1)}_c = \eta_d/(\eta_c + \eta_d)$, then

$$\rho\frac{\eta_c}{\eta_d}\left(\frac{1 + \gamma\eta_c s^{(1)}_c}{1 + \gamma\eta_d\left(1 - s^{(1)}_c\right)}\right)^{-(2-\alpha)}\left(\frac{A_{ct-1}}{A_{dt-1}}\right)^{-(1-\alpha)} = 1$$

and

$$\frac{1 + \gamma\eta_c s^{(1)}_c}{1 + \gamma\eta_d\left(1 - s_c\right)} = 1$$

Therefore

$$\frac{A_{dt-1}}{A_{ct-1}} \to \left(\rho\frac{\eta_c}{\eta_d}\right)^{-\frac{1}{1-\alpha}} > 1$$

thanks to assumption (3.1). This implies that s^*_{ct} tends to 0 in finite time.

Proof of Lemma 4.1

The proof is a long but straightforward application of the KKT conditions. The proof follows the same arguments as in Lemma 3.1. Recall that μ^c_t, μ^d_t, μ^s_t, λ_t, λ^j_t and ω_{t+1} denote the respective Lagrange multipliers of $Y_{ct} \le \rho Y_{dt} + Y_{st}$, $Y_{ct} \ge 0$, $Y_{st} \ge 0$, $Y_{dt} \ge 0$, (1), (5) and (16). We also denote by $\widehat{p}_{jt} := \lambda^j_t/\lambda_t$ the shadow price of input j at time t.

Applying KKT conditions yields:

$$\lambda_t\kappa(Y_{ct}+Y_{dt})^{\kappa-1} - \lambda^c_t - \mu^1_t + \mu^c_t = 0 \tag{42}$$

$$\lambda_t\kappa(Y_{ct}+Y_{dt})^{\kappa-1} - \lambda^d_t - \omega_{t+1}\xi + \rho\mu^1_t + \mu^d_t = 0 \tag{43}$$

$$-\lambda^s_t + \mu^1_t + \mu^s_t = 0 \tag{44}$$

$$Y_{ct} - \rho Y_{dt} - Y_{st} = 0 \quad or \quad \mu^1_t = 0 \tag{45}$$

$$Y_{ct} = 0 \quad or \quad \mu^c_t = 0 \tag{46}$$

$$Y_{dt} = 0 \quad or \quad \mu^d_t = 0 \tag{47}$$

$$Y_{st} = 0 \quad or \quad \mu^s_t = 0 \tag{48}$$

1st case $\mu^1_t = 0$.

- 1st subcase $\mu^s_t = 0$. Then $\lambda^s_t = 0$. This means that storage has no cost, we set this case aside.
- 2nd subcase $\mu^s_t > 0$. Then $\lambda^s_t > 0$ and $Y_{st} = 0$.
  - 1st subsubcase $\mu^c_t = 0$ and $\mu^d_t > 0$. Then $\lambda^d_t + \omega_{t+1}\xi > \lambda^c_t$, $Y_{dt} = 0$, $\kappa Y_{ct}^{\kappa-1} = \hat{p}_{ct}$ and $Y_{st} = 0$. This solution is impossible because it does not respect the constraint (2).
  - 2nd subsubcase $\mu^c_t > 0$ and $\mu^d_t = 0$. Then $\hat{p}_{dt} + \omega_{t+1}\xi/\lambda_t < \hat{p}_{ct}$, $Y_{ct} = 0$, $\kappa Y_{dt}^{\kappa-1} = \hat{p}_{dt} + \omega_{t+1}\xi/\lambda_t$ and $Y_{st} = 0$. This solution corresponds to the case 1 of the Lemma.
  - 3rd subsubcase $\mu^c_t = 0$ and $\mu^d_t = 0$. Then $\hat{p}_{dt} + \omega_{t+1}\xi/\lambda_t = \hat{p}_{ct}$, $\kappa(Y_{ct}+Y_{dt})^{\kappa-1} = \hat{p}_{ct} = \hat{p}_{dt} + \omega_{t+1}\xi/\lambda_t$ and $Y_{st} = 0$. This solution is the limit of case 1 of the Lemma.

2nd case $\mu^1_t > 0$.

Then $Y_{ct} = \rho Y_{dt} + Y_{st}$.

- 1st subcase $\mu^s_t = 0$. Then $\lambda^s_t = \mu^1_t$.
  - 1st subsubcase $\mu^c_t = 0$, $\mu^d_t > 0$. Then $\lambda^d_t + \omega_{t+1}\xi - \lambda^c_t = (1+\rho)\lambda^s_t + \mu^d_t$, therefore $\hat{p}_{st} < \frac{\hat{p}_{dt}\omega_{t+1}\xi/\lambda_t - \hat{p}_{ct}}{1+\rho}$. Besides, $Y_{dt} = 0$, $\kappa Y_{ct}^{\kappa-1} = \hat{p}_{ct} + \hat{p}_{st}$, and $Y_{st} = Y_{ct}$. This solution corresponds to the case 2-(b) of the Lemma.

- 2nd subsubcase $\mu_t^c > 0$, $\mu_t^d = 0$. Then $\lambda_t^d + \omega_{t+1}\xi - \lambda_t^c = (1+\rho)\lambda_t^s - \mu_t^c$. Besides, $Y_{ct} = 0$, $\kappa Y_{dt}^{\kappa-1} = \hat{p}_{dt} + \omega_{t+1}\xi/\lambda_t - \rho\hat{p}_{st}$ and $Y_{st} = -\rho Y_{dt}$. This solution is impossible (because Y_{st} is negative).
- 3rd subsubcase $\mu_t^c = 0$, $\mu_t^d = 0$. Then $\lambda_t^d + \omega_{t+1}\xi - \lambda_t^c = (1+\rho)\lambda_t^s$, and $\kappa(Y_{ct}+Y_{dt})^{\kappa-1} = \hat{p}_{ct} + \hat{p}_{st}$. This solution is the limit of case 2-(b) of the Lemma.

• 2nd subcase $\mu_t^s > 0$. Then $\lambda_t^s = \mu_t^1 + \mu_t^s$ and $Y_{st} = 0$.

- 1st subsubcase $\mu_t^c = 0$ and $\mu_t^d > 0$. Then $\lambda_t^d + \omega_{t+1}\xi - \lambda_t^c = (1+\rho)\mu_t^1 + \mu_t^d$, $Y_{dt} = 0$, $\kappa Y_{ct}^{\kappa-1} = \hat{p}_{ct}$ and $Y_{st} = 0$. This solution is impossible because it does not respect the constraint (2).
- 2nd subsubcase $\mu_t^c > 0$ and $\mu_t^d = 0$. Then $\lambda_t^d + \omega_{t+1}\xi - \lambda_t^c = (1+\rho)\mu_t^1 - \mu_t^c$, $Y_{ct} = 0$, $\kappa Y_{dt}^{\kappa-1} = \hat{p}_{dt} + \omega_{t+1}\xi/\lambda_t - \rho\mu_t^1$ and $Y_{st} = 0$. This solution is impossible, because we should have $Y_{ct} = \rho Y_{dt} + Y_{st}$.
- 3rd subsubcase $\mu_t^c = 0$ and $\mu_t^d = 0$. Then $\lambda_t^d + \omega_{t+1}\xi - \lambda_t^c = (1+\rho)\mu_t^1$, therefore $\hat{p}_{st} > \frac{\hat{p}_{dt} + \omega_{t+1}\xi/\lambda_t - \hat{p}_{ct}}{1+\rho}$. Besides, $\kappa(Y_{ct} + Y_{dt})^{\kappa-1} = \hat{p}_{ct} + \mu_t^1$, $Y_{ct} = \rho Y_{dt}$ and $Y_{st} = 0$. This solution corresponds to the case 2-(a) of the Lemma.

Proof of Proposition 4.1

The sketch of the proof is as follows. First, we show that at the social optimum, production of the final good is unbounded (Lemma 5.1). As a consequence, the use of dirty input goes to zero in finite time: otherwise, there would be an environmental disaster (Lemma 5.4). This implies that research in the dirty sector goes to zero in finite time (Lemma 5.5). We finally derive the long-term consumption level (Lemma 5.6).

Lemma 5.1. *At the social optimum, (Y_t) is not bounded.*

Proof. Consider an allocation where (Y_t) is bounded. Then (C_t) is bounded. Let $\overline{C}$ be a majorant of (C_t). Assume that in this allocation, $A_{s0} \leq A_{c0}$ (the opposite case can be treated in the same way).

Now, let "$\cdot^a$" be an alternative allocation such that A_{st}^a catches up with A_{ct}^a in finite time. Let $\widehat{t}$ such that $A_{s\widehat{t}}^a = A_{c\widehat{t}}^a$.

For all $t \geq \widehat{t}$, let $L_{st}^a = 1/2$, $L_{ct}^a = 1/2$, $L_{dt}^a = 0$, $s_{st}^a = 1/2$, $s_{ct}^a = 1/2$, $s_{dt}^a = 0$, $x_{sit}^a = 1$, $x_{cit}^a = 1$, $x_{dit}^a = 0$.

Then, for all $t > \widehat{t}$, we have $Y_{dt}^a = 0$ and $Y_{ct}^a = Y_{st}^a$. Moreover, since $A_{ct}^a, A_{st}^a \to \infty$ (because $s_{st}^a = 1/2$, $s_{ct}^a = 1/2$ for all $t \geq \widehat{t}$), we have $Y_{ct}^a, Y_{st}^a \to \infty$. As a result, $S_t^a \to \overline{S}$ in finite time and $C_t^a \to \infty$.

Let t_0 such that for all $t \geq t_0$, $S_t^a = \overline{S}$ and $C_t^a > \overline{C}$. Then, for all $t \geq t_0$,

$$u(C_t^a, S_t^a) - u(C_t, S_t) > u(C_t^a, \overline{S}) - u(\overline{C}, \overline{S}) > 0$$

by monotonicity of $u(\cdot, \overline{S})$. Therefore

$$\begin{aligned} W^a - W &= \sum_{t=0}^{\infty} \frac{1}{(1+r)^t}[u(C_t^a, S_t^a) - u(C_t, S_t)] \\ &\leq \sum_{t=0}^{t_0} \frac{1}{(1+r)^t}[u(C_t^a, S_t^a) - u(C_t, S_t)] + \frac{1}{(1+r)^{t_0}} \sum_{t=t_0}^{\infty} \frac{1}{(1+r)^{t-t_0}}[u(C_t^a, \overline{S}) - u(\overline{C}, \overline{S})] \\ &\underset{r \to 0}{\longrightarrow} \infty \end{aligned}$$

Thus, there exists $r_0 > 0$ such that $W_{r_0}^a - W_{r_0} > 0$. The alternative allocation "$\cdot^a$" is better than the initial one. As a consequence, at the social optimum, (Y_t) is unbounded. □

Lemma 5.2. *The socially optimal productions of clean, dirty, and storage inputs are given by*

- *If* $\widehat{p}_{dt}^* < \widehat{p}_{ct}$, *then* $Y_{ct} = 0$, $Y_{st} = 0$ *and* $Y_{dt} = \alpha^{-\frac{\alpha}{1-\alpha}} \left[\kappa Y_{dt}^{\kappa-1} - \frac{\omega_{t+1}\xi}{\lambda_t}\right]^{\frac{\alpha}{1-\alpha}} A_{dt}$
- *If* $\widehat{p}_{dt}^* > \widehat{p}_{ct}$ *and* $\widehat{p}_{st} > \frac{\widehat{p}_{dt}^* - \widehat{p}_{ct}}{1+\rho}$, *then* $Y_{st} = 0$, $Y_{dt} = \alpha^{-\frac{\alpha}{1-\alpha}} \kappa^{\frac{1}{1-\kappa\alpha}} (1+\rho)^{\frac{\kappa\alpha}{1-\kappa\alpha}} \left[A_{dt}^{-(1-\alpha)} + \rho A_{ct}^{-(1-\alpha)}\right]^{-\frac{1}{1-\kappa\alpha}}$ *and* $Y_{ct} = \rho Y_{dt}$
- *If* $\widehat{p}_{dt}^* > \widehat{p}_{ct}$ *and* $\widehat{p}_{st} < \frac{\widehat{p}_{dt}^* - \widehat{p}_{ct}}{1+\rho}$, *then* $Y_{dt} = 0$, $Y_{ct} = \alpha^{-\frac{\alpha}{1-\alpha}} \kappa^{\frac{1}{1-\kappa\alpha}} \left[A_{ct}^{-(1-\alpha)} + A_{st}^{-(1-\alpha)}\right]^{-\frac{1}{1-\kappa\alpha}}$ *and* $Y_{st} = Y_{ct}$.

Proof. At the social optimum, we still have, for $j \in \{c, d, s\}$ and $i \in [0, 1]$

$$x_{jit} = \left(\frac{\widehat{p}_{jt}}{\alpha}\right)^{\frac{1}{1-\alpha}} A_{jit} L_{jt} \tag{49}$$

and

$$\frac{\widehat{p}_{jt}}{\widehat{p}_{kt}} = \left(\frac{A_{jt}}{A_{kt}}\right)^{-(1-\alpha)} \tag{50}$$

Combining (5) with (49) allows to get

$$Y_{jt} = L_{jt} \widehat{p}_{jt}^{\frac{\alpha}{1-\alpha}} A_{jt} \tag{51}$$

Combining (6), (50), (51) and the results of Lemma 4.1 allows to get the results in each case. □

Let us call the first case ($\widehat{p}_{dt}^* < \widehat{p}_{ct}$) "regime 1", the second case ($\widehat{p}_{dt}^* > \widehat{p}_{ct}$ and $\widehat{p}_{st} > \frac{\widehat{p}_{dt}^* - \widehat{p}_{ct}}{1+\rho}$) "regime 2" and the third case ($\widehat{p}_{dt}^* > \widehat{p}_{ct}$ and $\widehat{p}_{st} < \frac{\widehat{p}_{dt}^* - \widehat{p}_{ct}}{1+\rho}$) "regime 3".

Lemma 5.3. *At the social optimum,* $A_{ct} \to \infty$ *and* $A_{st} \to \infty$.

Proof. For $j \in \{c, d, s\}$, the sequences (A_{jt}) are nondecreasing, so either they converge, or they tend to infinity. Let us examine the different cases.

- **1st case** $A_{dt} \to \infty$. Then, there exists $t_0 > 0$ such that for all $t \geq t_0$, $\widehat{p}_{dt}^* > \widehat{p}_{ct}$; otherwise, there would be an environmental disaster (because we would have $Y_{dt} \to \infty$). The only regimes compatible with the social optimum are the regimes 2 and 3.

– **1st subcase** $A_{ct} \to a_c \in \mathbb{R}$. Then, by Lemma 5.2, (Y_{ct}), (Y_{dt}) and (Y_{st}) are bounded in the regimes 2 and 3, whatever the behaviour of the sequence (A_{st}). Therefore, (Y_t) is bounded. This contradicts Lemma 5.1. As a consequence, this subcase is impossible.

– **2nd subcase** $A_{ct} \to \infty$. If $A_{st} \to a_s \in \mathbb{R}$, the regime 3 would be bounded. Since (Y_t) is not bounded, there would exist $M > 1$ and t_1 such that in the regime 2, $Y_{t_1} > M(1+\rho)(1+\delta)\overline{S}/\xi$. But in this case, $Y_{dt_1} = \frac{1}{1+\rho} Y_{t_1} > (1+\delta)\overline{S}/\xi$ and there would be an environmental disaster. Thus, $A_{st} \to \infty$.

- **2nd case** $A_{dt} \to a_d \in \mathbb{R}$. Then the regime 1 is bounded.

– **1st subcase** $A_{ct} \to a_c \in \mathbb{R}$. Then the regimes 2 and 3 are also bounded, whatever the behaviour of the sequence (A_{st}). This subcase contradicts Lemma 5.1 and is therefore not possible.

– **2nd subcase** $A_{ct} \to \infty$. If $A_{st} \to a_s \in \mathbb{R}$, then the regimes 2 and 3 would also be bounded, in contradiction with Lemma 5.1. As a consequence, $A_{st} \to \infty$.

Conclusion: we have $A_{ct} \to \infty$ and $A_{st} \to \infty$. □

Lemma 5.4. *At the social optimum, $Y_{dt} \to 0$ in finite time.*

Proof. From Lemma 5.3, we know that at the social optimum, $A_{ct} \to \infty$ and $A_{st} \to \infty$.

- **1st case** $A_{dt} \to \infty$. Then there exists $t_0 > 0$ such that for all $t \geq t_0$, $\widehat{p}^*_{dt} > \widehat{p}_{ct}$ (regimes 2 and 3), because otherwise there would be an environmental disaster. Besides, as

$$\alpha^{-\frac{\alpha}{1-\alpha}} \kappa^{\frac{\alpha}{1-\kappa\alpha}} (1+\rho)^{\frac{\kappa\alpha}{1-\kappa\alpha}} \left[A_{dt}^{-(1-\alpha)} + \rho A_{ct}^{-(1-\alpha)} \right]^{-\frac{1}{1-\kappa\alpha}} \underset{t\to\infty}{\longrightarrow} \infty$$

there exists $t_1 > 0$ such that for all $t \geq t_1$, $\widehat{p}_{st} < \frac{\widehat{p}^*_{dt} - \widehat{p}_{ct}}{1+\rho}$ (regime 3). Indeed, otherwise, by Lemma 5.2, there would be an environmental disaster. Thus, for all $t \geq t_1$, $Y_{dt} = 0$.

- **2nd case** $A_{dt} \to a_d$. Then since

$$\frac{\widehat{p}_{ct}}{\widehat{p}_{dt}} = \left(\frac{A_{ct}}{A_{dt}} \right)^{-(1-\alpha)}$$

there exists t_0 such that for all $t \geq t_0$,

$$\widehat{p}_{ct} \leq \widehat{p}_{dt}$$

Besides, there exists $t_1 > t_0$, for all $t \geq t_1$, $A_{ct}^{-(1-\alpha)} + (1+\rho)A_{st}^{-(1-\alpha)} \leq A_{dt}^{-(1-\alpha)}$, i.e.

$$\widehat{p}_{st} \leq \frac{\widehat{p}_{dt} - \widehat{p}_{ct}}{1+\rho} \leq \frac{\widehat{p}^*_{dt} - \widehat{p}_{ct}}{1+\rho}$$

Therefore, for all $t \geq t_1$, we are in the regime 3, and $Y_{dt} = 0$.

Conclusion: $Y_{dt} \to 0$ in finite time. □

Lemma 5.5. *At the social optimum, for all t, $s_{dt} = 0$, if the discount rate is low enough.*

Proof. By contradiction, let us assume that there exists t_0 such that $s_{dt_0} \neq 0$. The optimal policy implies that there exists t_1 such that for all $t \geq t_1$, $Y_{dt} = 0$ and $S_t = \overline{S}$.

Consider an alternative policy "$\cdot^a$" such that for all $t \leq t_1$,

$$\begin{aligned}
&s^a_{dt} = 0,\ s^a_{ct} = s_{ct} + \tilde{s}_t,\ s^a_{st} = s_{st} + (s_{dt} - \tilde{s}_t)\ ; \\
&L^a_{dt} = 0,\ L^a_{ct} = 1/2,\ L^a_{st} = 1/2\ ; \\
&x^a_{dit} = 0,\ x^a_{cit} = 1,\ x^a_{sit} = \left[\int {A^a_{cit}}^{1-\alpha}\ di / \int {A^a_{sit}}^{1-\alpha}\ di\right]^{1/\alpha}
\end{aligned}$$

with

$$\tilde{s}_t = \frac{(1+s_{st})^{-1}}{(1+s_{ct})^{-1} + (1+s_{st})^{-1}} \cdot s_{dt}$$

and, for all $t > t_1$, we keep the same policy as the initial one. With this alternative policy, for all t, $Y^a_{ct} = Y^a_{st}$ and $Y^a_{dt} = 0$.

Now,

$$W^a - W = \sum_{t=0}^{t_1} \frac{1}{(1+r)^t}[u(C^a_t, S^a_t) - u(C_t, S_t)] + \frac{1}{(1+r)^{t_1}} \sum_{t=t_1+1}^{\infty} \frac{1}{(1+r)^{t-t_1}}[u(C^a_t, \overline{S}) - u(C_t, \overline{S})]$$

But, for all $t \geq t_1$,

$$\begin{aligned}
C^a_t - C_t &= Y^a_t - Y_t \\
&= Y^{a\kappa}_{ct} - Y^{\kappa}_{ct} \\
&= L^{a\ \kappa}_{ct} \left(\int {A^a_{cit}}^{1-\alpha} {x^a_{cit}}^{\alpha} di\right)^{\kappa} - L^{\kappa}_{ct} \left(\int A^{1-\alpha}_{cit} x^{\alpha}_{cit} di\right)^{\kappa} \\
&= L^{\kappa}_{ct} \left(\int {A^a_{cit}}^{1-\alpha} x^{\alpha}_{cit} di\right)^{\kappa} - L^{\kappa}_{ct} \left(\int A^{1-\alpha}_{cit} x^{\alpha}_{cit} di\right)^{\kappa} \\
&= L^{\kappa}_{ct} \left(\int A^{1-\alpha}_{cit} x^{\alpha}_{cit} di\right)^{\kappa} \left[\left(\frac{\int {A^a_{cit}}^{1-\alpha} x^{\alpha}_{cit} di}{\int A^{1-\alpha}_{cit} x^{\alpha}_{cit} di}\right)^{\kappa} - 1\right] \\
&= Y^{\kappa}_{ct} \left[\left(\frac{\int {A^a_{cit}}^{1-\alpha} x^{\alpha}_{cit} di}{\int A^{1-\alpha}_{cit} x^{\alpha}_{cit} di}\right)^{\kappa} - 1\right] \\
&= Y_t \left[\left(\frac{\int {A^a_{cit}}^{1-\alpha} x^{\alpha}_{cit} di}{\int A^{1-\alpha}_{cit} x^{\alpha}_{cit} di}\right)^{\kappa} - 1\right]
\end{aligned}$$

The term $\frac{\int {A^a_{cit}}^{1-\alpha} x^{\alpha}_{cit} di}{\int A^{1-\alpha}_{cit} x^{\alpha}_{cit} di}$ is strictly higher than one because $A^a_{cit} \geq A_{cit}$ (since for all $t \geq 0$, $s^a_{ct} \geq s_{ct}$) and $A^a_{cit_0} > A_{cit_0}$ (because $s^a_{ct_0} > s_{ct_0}$ - since $s_{dt_0} \neq 0$). Besides, the sequence (Y_t) is unbounded from Lemma 5.1. It actually goes to infinity from Lemmas 5.2, 5.3 and 5.4. Thus, $Y_t \to \infty$. Therefore, $C^a_t - C_t \to \infty$, and then $u(C^a_t, \overline{S}) - u(C_t, \overline{S}) \to \infty$. As a result, $W^a - W \underset{r \to 0}{\longrightarrow} \infty$. Therefore, if the discount rate is low enough, the initial policy is not an optimal one. Conclusion: $s_{dt} = 0$ for all t. □

Lemma 5.6. *At the social optimum, let $t_0 > 0$ be such that for all $t > t_0$, $Y_{dt} = 0$ (regime 3). For all*

$t \geq t_0$, *we have*

$$C_t = \alpha^{-\frac{\alpha}{1-\alpha}} \kappa^{\frac{1}{1-\kappa\alpha}} (1-\kappa\alpha) \left[A_{ct}^{-(1-\alpha)} + A_{st}^{-(1-\alpha)} \right]^{-\frac{1}{1-\kappa\alpha}}$$

Proof. For all $t \geq t_0$, we have

$$\begin{aligned} C_t &= Y_{ct}^{\kappa} - \psi \int_0^1 x_{cit}\, di - \psi \int_0^1 x_{sit}\, di \\ &= Y_{ct}^{\kappa} - \psi \alpha^{-\frac{1}{1-\alpha}} \widehat{p}_{ct}^{\frac{\alpha}{1-\alpha}} A_{ct} L_{ct} - \psi \alpha^{-\frac{1}{1-\alpha}} \widehat{p}_{st}^{\frac{\alpha}{1-\alpha}} A_{st} L_{st} \\ &= Y_{ct}^{\kappa} - \psi \alpha^{-1} \widehat{p}_{ct} Y_{ct} - \psi \alpha^{-1} \widehat{p}_{st} Y_{st} \\ &= Y_{ct}^{\kappa} - \psi \alpha^{-1} (\widehat{p}_{ct} + \widehat{p}_{st}) Y_{ct} \\ &= Y_{ct}^{\kappa} - \psi \alpha^{-1} \kappa Y_{ct}^{\kappa-1} Y_{ct} \\ &= \alpha^{-\frac{\alpha}{1-\alpha}} \kappa^{\frac{1}{1-\kappa\alpha}} (1-\kappa\alpha) \left[A_{ct}^{-(1-\alpha)} + A_{st}^{-(1-\alpha)} \right]^{-\frac{1}{1-\kappa\alpha}} \end{aligned}$$

where we used equation (49) and Lemma 5.2. □

Proof of Proposition 4.2

First, T^* does exist and is a minimum because the set $\left\{T(s), s \in \{0,1\}^{\mathbf{N}}\right\}$ is non-empty, countable and minorized (by zero). Let s^* be an associated policy – i.e. such that $T(s^*) = T^*$.

Let us note, for all $t > 0$, $B_{ct} := A_{ct}^{-(1-\alpha)}$ and $B_{st} := A_{st}^{-(1-\alpha)}$. The dynamics of B_{ct} and B_{st} are

$$B_{ct} = (1 + \gamma\eta s_t)^{-(1-\alpha)} B_{ct-1} \tag{52}$$

$$B_{st} = (1 + \gamma\eta(1 - s_t))^{-(1-\alpha)} B_{st-1} \tag{53}$$

Since $s_t \in \{0, 1\}$, we have, for all $t > 0$

$$B_{ct} B_{st} = (1 + \gamma\eta)^{-(1-\alpha)} B_{ct-1} B_{st-1}$$

Therefore

$$B_{cT^*} B_{sT^*} = (1 + \gamma\eta)^{-(1-\alpha)T^*} B_{c0} B_{s0}$$

Then

$$B_{cT^*} + (1+\rho) B_{sT^*} = B_{cT^*} + \frac{(1+\rho)(1+\gamma\eta)^{-(1-\alpha)T^*} B_{c0} B_{s0}}{B_{cT^*}}$$

The function

$$F(B_{cT^*}) := B_{cT^*} + \frac{(1+\rho)(1+\gamma\eta)^{-(1-\alpha)T^*} B_{c0} B_{s0}}{B_{cT^*}}$$

admits a minimum for $B_{cT^*}^2 = (1+\rho)(1+\gamma\eta)^{-(1-\alpha)T^*} B_{c0} B_{s0} = (1+\rho) B_{cT^*} B_{sT^*}$, i.e. for $B_{cT^*} = (1+\rho) B_{sT^*}$.

The innovation policy ($\triangle$)

$$s_t^{\triangle} = 1, \quad if \quad (1+\rho) B_{st} \leq (1+\gamma\eta)^{-(1-\alpha)} B_{ct}$$

$$s_t^{\triangle} = 0, \quad if \quad (1+\gamma\eta)^{-(1-\alpha)} (1+\rho) B_{st} \geq B_{ct}$$

$$s_t^{\triangle} = 0 \ or \ 1, \quad if \quad B_{ct} = B_{st}$$

is such that, up to one iteration, $B_{cT^*}^{\triangle} = (1+\rho)B_{sT^*}^{\triangle}$. If we note $B_{jt}^{\triangle}$ and B_{jt}^* the dynamics respectively associated to $s^{\triangle}$ and s^*, we therefore have $B_{cT^*}^{\triangle} + (1+\rho)B_{sT^*}^{\triangle} \leq B_{cT^*}^* + (1+\rho)B_{sT^*}^*$, i.e. ${A_{cT^*}^{\triangle}}^{-(1-\alpha)} + (1+\rho){A_{sT^*}^{\triangle}}^{-(1-\alpha)} \leq {A_{cT^*}^*}^{-(1-\alpha)} + (1+\rho){A_{sT^*}^*}^{-(1-\alpha)}$

Conclusion: T^* can be achieved with the innovation policy ($\triangle$).

WORKING PAPER

PREVIOUS ISSUES

Improving the energy efficiency of the French residential sector: the role of the middlemen — **N°2023-03**
Esther RAINEAU-RISPAL

An equilibrium model of city with atmospheric pollution dispersion — **N°2023-02**
Mohamed BAHLALI, Quentin PETIT

Land allocation and the adoption of innovative practices in agriculture: a real option modelling of the underlying hidden costs — **N°2023-01**
Camille TEVENART, Marc BAUDRY, Edouard Civel

Droughts and deforestation: Does seasonality matter? — **N°2022-02**
Giulia VAGLIETTI, Philippe DELACOTE, Antoine LEBLOIS

The case for a Carbon Border Adjustment: Where do economists stand? — **N°2022-01**
Aliénor CAMERON, Marc BAUDRY

Les CCfDs au service du développement de l'hydrogène bas-carbone en Europe — **N°2021-09**
Corinne CHATON, Coline METTA-VERSMESSEN

Green innovation downturn: the role of imperfect competition — **N°2021-08**
Mohamed BAHLALI, René AÏD, Anna CRETI

An assessment of the European regulation on battery recycling for electric vehicles — **N°2021-07**
Quentin HOARAU, Etienne LORANG

Working Paper Publication Directors :

Marc Baudry, Philippe Delacote, Olivier Massol