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WORKING PAPER

Why Labels Fail: Fraud on a Market for Credence Goods with Unobservable Skill Heterogeneity among Experts

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Asymmetries of information and uncertainty about product quality are often a central issue for customers. It is particularly the case in the construction market, where firms act as experts providing both the diagnosis and the technical solutions to their clients, who are usually unable to assess them. As this sector is becoming more and more prevalent in the conversation on global warming and the energy efficiency gap, it is important to understand how its unique characteristics call for carefully designed policies. This article presents a credence-good model with skill heterogeneity among experts meant to replicate key features of the sector, focusing on maintenance and retrofit services. In particular, the equilibrium is characterized by a low and unique market price, and skilled firms cannot distinguish themselves from their unskilled counterparts. This setup is then used to contrast the efficiency of two public policy tools implemented to make construction markets more efficient: human capital development investments and quality labels. Analytical results indicate that the latter may impact over-treatment, but does not affect the level of under-treatment in equilibrium, while increasing the number of skilled firms is always efficient to increase customer satisfaction. Under this model's assumptions, if the goal is to ensure the proper renovation of the building stock, labels miss the mark.

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Executive summary

This paper aims to replicate the behaviors observed on many European countries' construction sectors in order to contrast different policy design. They are highly atomistic and competitive, yet customers are often unhappy with the quality of service they get, despite important regulation efforts. As the energy retrofit of the building stock has become a central objective of the post-Covid European economic recovery plan, it is more important than ever to gain more insight into the nature and drivers of fraudulent behavior. European states have turned to two main policy tools to ensure a baseline quality of retrofits: investing in workers' skill development or setting up certifications to signal high-quality firms to consumers. The UK and several Eastern European governments have focused on human capital development, through apprenticeship founding or increased educational resources. Quality labels are the main tool used in countries like France, where certifications are implemented and offered by the state or independent organizations like Effinergie or Qualibat. These certifications have been around for years, yet many firms are still unlabeled - and customer satisfaction has not drastically increased. The credence good model developed in this paper aims to contrast these two policy objectives in a simple manner, through the analysis of comparative statics and alternative specifications.

The core assumption of the model presented in this paper is that consumers do not have the practical skills to assess their maintenance and renovation needs, nor to address them. It is also assumed to be impossible to distinguish between skilled and unskilled firms prior to interacting with them. As they limit their search, it gives unskilled firms the opportunity to be active on the market with a positive profit. The model is then extended to include labeled firms - that is, skilled firms whose type is known with certainty by the customer. Not all skilled firms get labeled, which matches real-world behaviors. Not all skilled firms have an incentive to get the label: some of them may already have a well-established reputation and do not have enough a reason to get the label, others may have arrived on the market too recently to know labels exist. The equilibria described in this paper successfully replicate key features of many European countries' construction sectors, that is: low and non-discriminating resale prices and the persistence of fraud in equilibrium despite intense competitive pressure. The three main takeaways are that (1) at the equilibrium, firms offer their services at their reserve prices and customers cannot distinguish unskilled and skilled contractors (2) maximum overtreatment and undertreatment coexist in this market equilibrium, and (3) labels are in most cases inefficient to push unskilled firms out of the market.

These theoretical results can provide some insight as to why such public policies have not been successful in undermining fraud and boost the energy gains that are supposedly achievable in the residential sector. The variable that seems to play the most important role in deterring fraud is the share of skilled firms, which gives ground to policies aiming to enhance expertise in the sector. Their impact may not be as immediate as labels', but they would have a more decisive impact on undertreatment. Using French insurance data, the NGO *Agence Qualité Bâtiment* found that the total compensation paid to households amounted to approximately 847 million euros in 2020. It has increased every year by 5.9\% on average since 2011, which was when the RGE label was introduced. The same year, the state spent 5.4 billion euros to found 629 635 apprenticeship contracts, 11% of which were in the construction sector. Using the mean cost, it adds up to 594 million euros spent on apprenticeships in the sector - 70\% of the cost of defects and malfunctions. From a social welfare perspective, improving professional training for contractors could be a better allocation of resources.

1. INTRODUCTION

The procurement of goods and services in a context of asymmetries of information is a well-studied problem in economics. The way imperfect information affects market outcomes, welfare and equilibrium behaviors has been largely documented in the case of public good provision, healthcare and labor (Laffont and Tirole 1993). The procurement of services by private consumers has been less extensively studied, yet there are some specific aspects deserving more attention. Credence good problems arise whenever consumers cannot evaluate the information given to them with certainty: they have to *trust* the agent or institution providing them with it. Contrary to experience goods, this is usually still true *ex post*, as customers cannot distinguish between an expert who provided the wrong solution and a product failure despite the expert's best efforts (Gottschalk 2018). Fraud arises whenever an expert seller has incentives to misrepresents the consumer's needs to increase their profits. It is a well-documented and frequent problem in real-world credence markets, whether it be detours taken by taxi drivers (Tang 2020), unnecessary car repairs (Rasch and Waibel 2018) or superfluous prescription drugs (Gottschalk, Mimra, and Waibel 2020). These case studies raise two connected questions: what drives experts' dishonesty and how can public policies be implemented to prevent it?

This paper attempts to replicate the dynamics of European countries' construction sectors, which are highly atomistic and competitive, yet customers are often unhappy with the quality of service they get (OECD 2010). It aims to understand why these inefficiencies persist in equilibrium in such a competitive environment, despite important regulation efforts. In particular, European states have turned to two main policy tools: investing in workers' skill development or setting up certifications to signal high-quality firms to consumers. The UK and several Eastern European governments have focused on human capital development, through apprenticeship founding or increased educational resources (ECSO 2020). Labels are the main tool used in countries like France, where many certifications are implemented and offered by the state, unions (eg. the Qualirecycle BTP label offered by the FFB union) or independent organizations like Effinergie or Qualibat, which specialize in certifying the quality of different aspects of the construction process. We focus on the labeling of quality products, even though it should be mentioned that labeling could also be used as a way to reveal low-quality ones (Baksi and Bose 2007).These different certifications have been around for years, but many firms do not see the point in getting them - and customer satisfaction has not drastically increased (ECSO 2018). This model aims to contrast these two policy objectives in a simple manner, through the analysis of comparative statics and alternative specifications. As the energy retrofit of the building stock has become a central objective of the post-Covid European economic recovery plan, it is more important than ever to gain more insight into the nature and drivers of fraudulent behavior, and on the conditions under which labels may or may not be effective.

The core assumption of the model presented in this paper is that consumers do not have the practical skills to assess their maintenance and renovation needs, nor to address them. It is also impossible to distinguish between skilled and unskilled firms prior to interacting with them. As they limit their search, it gives unskilled firms the opportunity to be active on the market with a positive profit. The model is then extended to include labeled firms - that is, skilled firms whose type is known with certainty by the customer. Not all skilled firms get labeled, which matches real-world behaviors. Not all skilled firms have an incentive to get the label: some of them may already have a well-established reputation and do not have enough a reason to get the label, others may have arrived on the market too recently to know labels exist (ECSO 2018). The main results are that (1) at the equilibrium, firms offer their services at their reserve prices and customers cannot distinguish unskilled and skilled contractors (2) maximum overtreatment and undertreatment coexist in this market equilibrium, and (3) labels are in most cases inefficient to push unskilled firms out of the market. Comparative statics are used to discuss the different outcomes and the effect of some key variables. A literature review is presented in section 2, followed by a simple version of the model in section 3 and the general setup in section 4. Finally, different label specifications are reviewed in section 5 and section 6 concludes.

2. Literature review

The procurement of construction and maintenance services poses problems because households are not experts. Contractors are usually in charge of both the diagnosis of the problem and the implementation of a solution, as they are supposed to know what works best given the current characteristics of the dwellings – to maximize energy efficiency improvements for instance. Renovations are typical credence goods, as defined by Darby and Karni (1973). The consumer acts as a principal relying on the expertise of agents to determine what they need. The present literature review highlights previous work related to producer heterogeneity, starting with empirical findings supporting its importance before reviewing the different ways theoretical contributions have accounted for it. Models estimating the impact of labels will also be briefly discussed, even though they are based on a different set of assumptions. A more general review of past credence good models can be found in Balafoutas and Kerschbamer (2020).

Recent papers in econometrics and behavioral economics provide evidence on factors driving firms to defraud their customers, exposing how heterogeneity among experts can drastically influence market outcomes[1]. The first large-scale lab experiment was carried out by Dulleck, Kerschbamer, and Sutter (2011), who tested some of the main theoretical results on credence goods. Their findings particularly highlighted how unobservable supplier heterogeneity could have a significant impact on the market's efficiency, despite being greatly overlooked in the literature. Drawing from a field study on auto repair shops, Rasch and Waibel (2018) found that financial pressure and lack of reputational concerns were strong drivers of fraudulent behaviors, which is also consistent with the theory. Interestingly, they showed that overcharging becomes more likely in a more competitive environment, and that high-skill firms were less prone to overcharge. Agarwal, Liu, and Prasad (2019) conducted an experiment focusing on unobservable diagnosis effort, and found that market efficiency did not increase when consumers could obtain another expert's opinion, but that further information acquired through personal research made the equilibrium level of fraud drop[2]. It provided interesting insights for many credence goods markets where experts operate with little or no diagnosis fee but fraud still occurs in equilibrium - which is typically the case for the construction industry. Their results somehow contradict those of Mimra, Rasch, and Waibel (2016), who found that the availability of a costly second opinion induced a 40% fall in the equilibrium level of overtreatment if search costs were sufficiently low. A key difference between the two papers is that diagnosis is not costly for experts in Mimra, Rasch, and Waibel (2016), providing them with fewer incentives to defraud customers. Tang (2020) is one of the few papers relying on real-world data to study overtreatment, in the form of detours taken by taxi divers.

[1] Kerschbamer and Sutter (2017) provide a good overview of the related literature relying on lab experiments or field studies.

[2] Their experimental design is based on augmented version of the sequential model found in Pesendorfer and Wolinsky (2003) that allows consumers to either seek a secondary opinion from another expert *ex post* or to look for information themselves *ex ante* after the diagnosis.

His results suggest significant disparities in taxi drivers' propensity to cheat depending on their cultural background, and that these behaviors persist over time. It provided empirical support to the idea that there are types of firms that are more or less likely to defraud consumers, but this dimension has not yet been detailed in past theoretical contributions.

The first theoretical papers focused on sequential search principal-agent setups, in which the service provided can be of two types (high or low cost), which cannot be observed by the principal interacting with homogeneous agents (Pitchik and Schotter 1987). Fraud usually comes in the form of overcharging, meaning an expert charging the high-cost service but actually implementing the low-cost solution. In most models, introducing competition restores Bertrand efficiency despite the information asymmetries, and there is no fraud in equilibrium. For instance, Wolinsky (1993) detailed how separating equilibria could emerge depending on the search-cum-diagnosis cost and showed that competition could lead to efficiency in this case. Another way to look at this issue is to focus on diagnosis, in the spirit of Darby and Karni (1973): consumers know what kind of service is provided, but they ignore whether they needed that level of service in the first place, as the diagnosis phase lies with the firm. Competition might also lead to efficient outcomes when there are large economies of scope between diagnosis and repair, as price signals can successfully reveal firm incentives to consumers (Emons 1997). Despite being the most extensively studied case, overcharging is not the only form of fraud on credence good markets, as Dulleck and Kerschbamer (2006) pointed out. Using a synthetic model to compare various inefficiencies and types of fraudulent behaviors, they compared previous models' outcomes and highlighted the relative importance of assumptions regarding the firm's liability or the *ex post* verifiability of the goods' type. They derived two main results. First, under liability or verifiability undertreatment cannot be an equilibrium behavior since the customer will notice their utility of zero. Equilibrium overtreatment is however possible, since a consumer with a minor problem derives the same utility when it is fixed, whatever solution the firm used. Second, overtreatment strictly dominates overcharging from the firm's perspective when the solutions have increasing costs. They further discussed the potential implications of producer heterogeneity, while stressing that it is a missing dimension in previous contributions to the literature. These models indeed differ in the nature of the information asymmetry, but they all rely on identical firms facing consumers of different types, while in reality experts do not have the same competence, nor the same propensity to defraud customers.

Very few papers focus explicitly on producer heterogeneity. There have been some attempts to introduce differences among experts in their choice of diagnosis effort, which is assumed to be unobservable by consumers. In the model developed by Pesendorfer and Wolinsky (2003), consumers do not know their own type, which can only be observed by firms conditionally on a costly design effort. The main focus of their model is to take into account customers' efforts to gather several opinions. Contrary to the problem studied in this paper, the information asymmetry emerges because firms privately choose their level of effort, but they are otherwise homogeneous. Consumers can sequentially sample several firms until their recommendations match to discover their own types. Dulleck, Gong, and Li (2015) extended the model with sequential bidding by contractors: consumers shortlist a finite number of firms, which then individually choose their effort levels and bid on the price and on the design in an auction. They assumed consumer chooses the firm offering the right design at the lowest price and compensates the others at a fixed design fee. Sampling two firms is enough to restore Bertrand competition and to incentivize contractors to provide a high effort level - or at least to do so with a positive probability. Non-degenerate fixed price equilibria exist only under certain conditions on the cost of effort and the search cost, which must remain small relative to the value customers give to the project. A crucial aspect remaining overlooked was that firms may be heterogeneous in their skills, namely their design and implementation costs. Using a similar setup but allowing for an endogenous diagnosis price, Alger and Salanié (2006) generated an equilibrium with overtreatment, as they let the design fee allowed to be set below cost. All firms still adopt the same pricing behavior in equilibria and are assumed to have the same capacity to solve the issue, meaning inefficiencies are only due to moral hazard. In other words, firms' lack of actual skill does not come into play - they *choose* to defraud their customers, despite being able to fix their issue.

Another class of credence good models introduced firm heterogeneity by assuming experts can be of two types, skilled or unskilled, which are observable by customers. Glazer and McGuire (1996) made the first contribution to this line of work, comparing safe sellers who can always solve the issue, and cheaper but risky sellers who may solve it depending on its seriousness - which customers cannot appreciate. They found that price competition is enough to ensure that risky sellers do not serve customers whose problems they cannot fix. Similarly, Emons (2000) focused on the price and quality choice of sellers who can only imperfectly diagnose the issue when entering a market where safe experts operate. He showed that product differentiation can be used by risky sellers to loosen price competition on the market, but it is not their most profitable option in equilibrium. Complementary results can be found in Bouckaert and Degryse (2000), whose model included experts who are able to fix the issue and non-experts fixing it only with a positive probability. An equilibrium with price differentiation arises only when the probability of successful repair using the non-expert's technology is small enough. Dulleck and Kerschbamer (2009) also developed a framework with experts who can perform a costly diagnosis, and discounters who can offer the same quality at a lower price but cannot tell the consumer if their problem is severe or simple. In this setting, experts can defraud consumers by over-treating them, but consumers may also defraud experts by going to a discounter to obtain the quality recommended by the expert. This may result in equilibrium undertreatment, as experts can cheat with a positive probability to keep consumers imperfectly informed. Overall, these models find efficient equilibria because consumers can discriminate firms *ex ante*, hence they optimally choose to take a risk or not.

Up to this point, skill heterogeneity has hence been introduced either by assuming homogeneous firms could decide on their unobservable effort level, or by having observable firm types regarding their skill level. The main novelty of this paper is to introduce unobservable firm types and to focus on equilibrium undertreatment. To do so, the verifiability and liability assumptions are lifted. The customer may have an easy or complex issue, which they cannot observe, and some of the firms they face are not able to produce the correct diagnosis. It is a bid setup, in which firms compete in prices and diagnosis to execute a task for a consumer. This model focuses explicitly on firm heterogeneity and goes further than previous work specifying firm types in that consumers are not able to discriminate firms *ex ante*. It is an appropriate setup to discuss the conditions under which labels may be effective. A similar approach was developed by Bonroy and Constantatos (2008), with a firm producing a high-end product at a higher marginal cost than a low-quality producer, but they focused on customers' beliefs, which is not the main subject matter here. They showed that as labels increase cost, they may reduce the quality producers' market share. There are also conditions under which they proved the existence of vicious effects, as labels increase competition in prices. These findings are in line with our partial labeling assumption, as there are many barriers preventing skilled firms from getting certifications on the quality of their products. Also relying on consumers' heterogeneous beliefs and preferences, Baksi, Bose, and Xiang (2017) found that even if labeling can sometimes improve social welfare, it always leads to a decrease in high-end producers' profits if customers over-estimate the quality of intermediate products. This is again in line with our results and provides more ground for the assumption that some skilled firms will refuse to get a costly quality certification.

3. Simple setup

3.1. One consumer facing two firms

Let us first examine a very simple setup as a way to introduce the main variables of the general model. Consider a market with one consumer and two heterogeneous firms *vis-à-vis* their skill levels. They have a hard-fix issue $\overline{c}$ with probability $\mu \in [0,1]$ - meaning they have an easy-fix issue with probability $1-\mu$. Firm with skill $\overline{\beta}$ can solve both types of issue, while firm with skill $\underline{\beta}$ can only solve easy problems. The consumer derives utility V net of the price if the problem is fixed; they can never observe firms' skills, nor can they diagnose their own issue. Firms observe their own skill level and can diagnose the consumer's issue before setting up their selling price.

Firm $\underline{\beta}$ will always state that the problem is an easy-fix ($\underline{c}$) and offer a price p, while firm $\overline{\beta}$ can offer the correct diagnosis and propose prices $\overline{p}$ or $\underline{p}$ accordingly. For simplicity, assume firms can solve the issues at no cost: the skill difference stems solely from the fact that the $\underline{\beta}$ firm cannot perform a diagnosis of the customer's needs. Overtreatment is not ruled out, as the $\overline{\beta}$ firm can choose to misreport a $\underline{c}$ issue as a $\overline{c}$ issue with probability $\eta \in [0,1]$. It may be profitable to do so if $\overline{p} \geq \underline{p}$, hence the value of η is set by the skilled firm before setting their price. Figure 1 displays the game in its extensive form, with the resulting diagnosis and payoffs for the two firms and the consumer - dotted lines represent information asymmetry.

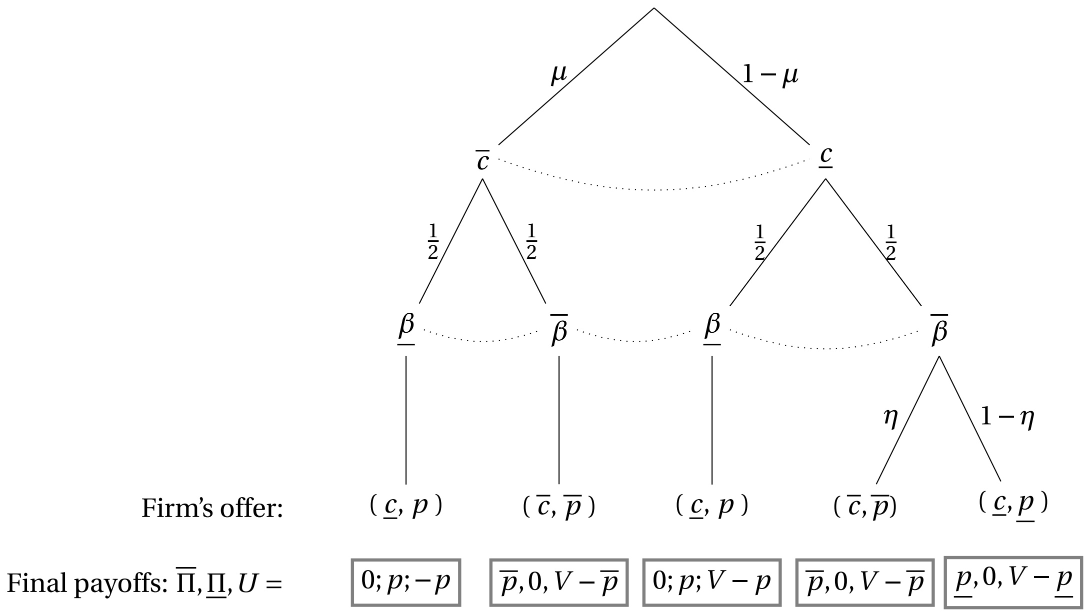


Figure 1: One-shot game

Proposition 1. *In equilibrium the skilled firm optimally sets $\eta^* = 1$ and only charge $\overline{p}^* = V$, and the unskilled* *firm's optimal price is $\underline{p}^* = (1-\mu)V$.*

Proof. Assume $\eta < 1$. The consumer gets a $\overline{c}$ or a $\underline{c}$ diagnosis with probabilities $\mathbb{P}(c = \overline{c}) = \frac{\mu+\eta(1-\mu)}{2}$ and $\mathbb{P}(c = \underline{c}) = \frac{1+(1-\eta)(1-\mu)}{2}$ respectively. If the diagnosis is $\overline{c}$, they know they are facing the skilled firm and will get V with certainty, even if they had a $\underline{c}$ issue. Hence $U(\overline{c}) = V - \overline{p}$. They agree to pay price $\overline{p}$ if and only if their utility is positive, that is if $\overline{p} \leq V$. The skilled firm maximizes its profits by setting $\overline{p}^* = V$.

Facing a $\underline{c}$ diagnosis, the consumer has the following expected utility depending on the type of firm and their actual problem[3]:

$$\mathbb{E}(U|c = \underline{c}, \eta < 1) = \frac{1}{2}\Big(V - \overline{p}\Big) + \frac{1}{2}\Big(\mu(0-\underline{p}) + (1-\mu)(V-\underline{p})\Big) = \frac{2-\mu}{2}V - \frac{1}{2}\overline{p} - \frac{1}{2}\underline{p}$$

As they don't know what firm they are facing, they cannot distinguish $\underline{p}$ from $\overline{p}$. Let $\tilde{p} \in \{\overline{p}, \underline{p}\}$ be the price as perceived by the consumer when choosing to accept a $\underline{c}$ diagnosis or not. Their expected utility becomes $\mathbb{E}(U|c = \underline{c}, \eta < 1) = \frac{2-\mu}{2}V - \tilde{p}$, thus the maximum prices firms can set are $\underline{p} = \overline{p} = \frac{2-\mu}{2}V$.

It is straightforward that $\overline{p}^* \geq \underline{p} \quad \forall \mu \in [0,1]$, which implies that the skilled firm's optimal lying strategy is $\eta^* = 1$, meaning the $\overline{\beta}$ firm always offer the $\overline{c}$ diagnosis. As a result, $\mathbb{P}(c = \overline{c}) = \mathbb{P}(c = \underline{c}) = \frac{1}{2}$. It does not affect the $\overline{c}$ diagnosis case, but if the customer receives a $\underline{c}$ diagnosis they now have the following expected utility:

$$\mathbb{E}(U|c = \underline{c}, \eta = 1) = \mu(0-\underline{p}) + (1-\mu)(V-\underline{p}) = (1-\mu)V - \underline{p}$$

To maximize its profits while keeping the customer's utility non-negative, the unskilled firm has to set $\underline{p}^* = (1-\mu)V$. $\square$

As the skilled firm specializes in equilibrium, the maximum price the unskilled firm can set is lower than what they could charge if the customer had some uncertainty on the firm's type when getting a $\underline{c}$ diagnosis. It is a rather intuitive result, as the diagnosis carries more information when firms specialize - and the customer is less willing to pay for an unskilled firm's services. One can also note that if $\mu = 0$, they know they cannot have a hard-fix issue and do not take any risk accepting a $\underline{c}$ diagnosis. As a consequence, all firms could set their selling price at V. It is also worth mentioning that as long as the customer's valuation is positive, all firms have an incentive to be active on the market because their expected profits will be positive as well, as equilibrium payoffs are given by:

$$\begin{aligned}
\mathbb{E}(U) &= \tfrac{1}{2}(V - \overline{p}^*) + \tfrac{1}{2}(\mu(0-\underline{p}^*) + (1-\mu)(V-\underline{p}^*)) = 0 \\
\mathbb{E}(\overline{\Pi}^*) &= \tfrac{1}{2}\overline{p}^* = \tfrac{1}{2}V \\
\mathbb{E}(\underline{\Pi}^*) &= \tfrac{1}{2}\underline{p}^* = \tfrac{1-\mu}{2}V
\end{aligned}$$

Both firms are active on the market as long as $\mu < 1$ and $V > 0$. The skilled firm's profits are higher than that of their unskilled counterpart, which is driven by the fact that they can sell at a higher retail price along with the $\overline{c}$ diagnosis. In particular, if $\mu = 0$, meaning the consumer always has and easy-fix problem, both firms

[3] See Appendix 10 for the derivation of these probabilities.

have the same expected profits as prices equalize. In this equilibrium, the customer has to deal with both potential overtreatment and undertreatment, which are due respectively to the skilled firm misreporting and the unskilled firm's incapacity to diagnose a $\overline{c}$ issue. There is no *ex-post* uncertainty about the drawn firm's type in equilibrium as each type specializes in one treatment, but the consumer only learns their own type if they dealt with the $\underline{\beta}$ firm: either the issue is fixed and they deduce it was a $\underline{c}$ one, or it is not fixed and they learn it was a $\overline{c}$ one. The persistence of some uncertainty is a realistic result, as renovation and maintenance services are typically very hard for the customer to evaluate. The persistence of the issue can be noticed, but it is impossible to tell if the solution was appropriate once the problem is fixed. Taking a very concrete example, if your boiler stops working and a plumber comes to change it, you will be able to tell that you have hot water or not afterwards, but you cannot really know if a full replacement was necessary: perhaps a more complex repair of some parts could have sufficed. It is typically quite difficult to assess the overall quality of a renovation outcome as a non-expert.

3.2. Extending to J firms

Before turning to the general model, let us review the impact of having more than two firms on the equilibrium outcomes. Assume now that the consumer faces $J > 2$ firms on the market. Denote $\delta \in [0,1]$ the share of $\overline{\beta}$-type firms among them. Let the consumer draw one firm at random and decides to buy or not from the $(c, \tilde{p})$ offer they face.

Proposition 2. *The equilibrium behaviors are unchanged : skilled firms' optimal lying strategy is* $\eta^* = 1$ *and they only charge* $\overline{p}^* = V$. *Unskilled firms' optimal price is* $p^* = (1-\mu)V$.

Proof. Let $\eta < 1$. We now have $\mathbb{P}(c = \overline{c}) = \delta\big(\mu + \eta(1-\mu)\big)$ and $\mathbb{P}(c = \underline{c}) = 1 - \delta + \delta(1-\eta)(1-\mu)$. Facing a $\overline{c}$ diagnosis, the customer still gets $U(\overline{c}) = V - \overline{p}$ with certainty so the firm's optimal price remains $\overline{p}^* = V$.

If the customer receives a $\underline{c}$ diagnosis, their expected utility depends on their probability of having a hard-fix issue and on the share of skilled firms:

$$\begin{aligned}
\mathbb{E}(U|c = \underline{c}, \eta < 1) &= \delta(V - \underline{p}) + (1-\delta)\big(\mu(0-p) + (1-\mu)(V-p)\big) \\
&= \big(1 - \mu(1-\delta)\big)V - \delta\underline{p} - (1-\delta)p
\end{aligned}$$

Following the same reasoning as in proposition 1, let $\tilde{p} \in \{\underline{p}, p\}$, which means the customer's utility becomes $\mathbb{E}(U|c = \underline{c}, \eta < 1) = \big(1 - \mu(1-\delta)\big)V - \tilde{p}$. The maximum prices firms can set are thus $\underline{p} = p = \big(1 - \mu(1-\delta)\big)V$.

As $\overline{p}^* \geq \underline{p} \quad \forall \mu, \delta \in [0,1]$, skilled firms' optimal strategy remains $\eta^* = 1$. As a consequence, the diagnoses' probabilities become $\mathbb{P}(c = \overline{c}) = \delta$ and $\mathbb{P}(c = \underline{c}) = 1 - \delta$, but the consumer's utility is only affected in case of a $\underline{c}$ diagnosis:

$$\mathbb{E}(U|c = \underline{c}, \eta = 1) = \mu(0-p) + (1-\mu)(V-p)$$

The maximum price unskilled firms can set while maintaining non-negative utility is $p^* = (1-\mu)V$ □

Consequently, equilibrium payoffs for the consumers and each type of firm are:

$$
\begin{aligned}
\mathbb{E}(U) &= \delta(V-\overline{p}^*)+(1-\delta)(\mu(0-p^*)+(1-\mu)(V-p^*)) \quad = 0\\
\mathbb{E}(\overline{\Pi}^*) &= \tfrac{1}{J}\overline{p}^* = \tfrac{1}{J}V\\
\mathbb{E}(\underline{\Pi}^*) &= \tfrac{1}{J}p^* = \tfrac{1-\mu}{J}V
\end{aligned}
$$

Increasing the number of firms on the market does not increase the customer's utility if they interact with only one firm: it remains 0 in equilibrium. Firms' individual profits are lower since $J > 2$, but it is only due to a lower probability of being drawn by the consumer. The share of same-type firm does not affect any of the payoffs and this equilibrium remains characterized by maximum over-treatment and under-treatment. The main takeaway here is that whatever the number and types of firms on the market, if the customer consults only one, set prices will remain at their highest level because firms do not feel any competitive pressure. It seems more relevant to consider that the customer meets several firms, as it is both a way to reduce uncertainty and to increase their equilibrium expected utility, as shown in the following section.

4. General model

4.1. Setup

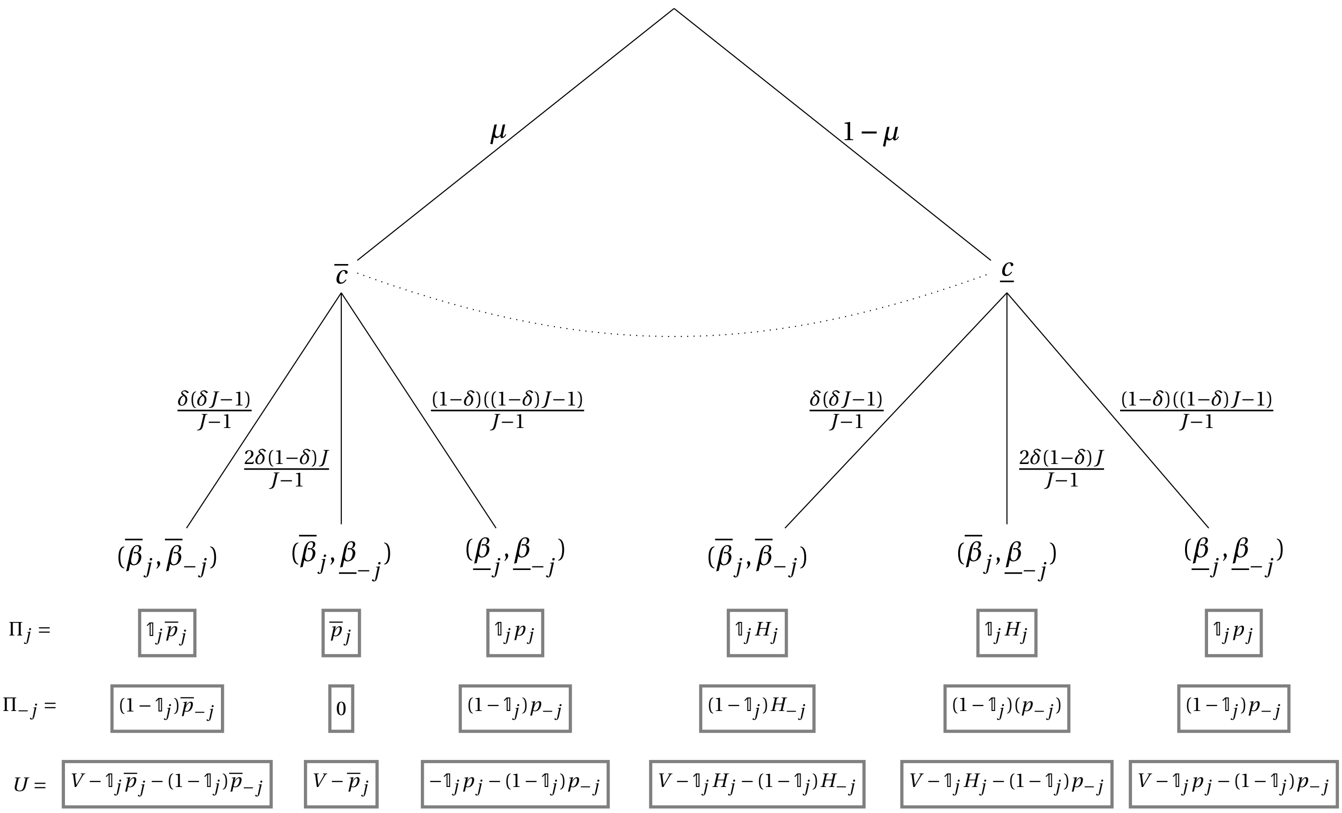


Figure 2: One-shot game with two firms drawn

Assume now that the consumer simultaneously draws two firms, and then compares their prices and diagnoses before making a buy-or not decision. Firms set their prices above a threshold $k > 0$ without knowing what kind of firm they are competing against. This threshold is set for computational reasons, but it could be interpreted as a minimum resale price to cover fixed costs - capital investments for instance. Each skilled firm j has a lying policy, meaning they choose to over-treat their client with probability η_j. Whatever the offers they are facing, the customer always goes for the lowest price when the two firms offer the same diagnosis. When facing a conflicting diagnosis, they go for the $(\overline{c}, \overline{p})$ one.

Figure 2 displays the game in its extensive form. Denote $\mathbb{1}_j$ the indicator function equal to 1 if the customer chooses firm j and 0 otherwise, and let H_j be the linear combination $\eta_j \overline{p}_j + (1-\eta_j)\underline{p}_j$. The computation of total probabilities is detailed in figure 11 in the Appendix.

4.2. Equilibrium lying strategy and prices

Assume for now that all firms are active on the market, in order to determine the equilibrium prices and lying policy before reviewing their participation conditions.

Proposition 3. *In equilibrium, skilled firms' optimal choice is to set* $\eta^* = 1$ *and* $\underline{p}^* = \overline{p^*} = k$.

Proof. From a skilled firm's perspective, let us first consider a customer with a $\overline{c}$ issue. In this case, a skilled firm's expected profits are:

$$\mathbb{E}(\overline{\Pi}_j | c = \overline{c}) = \frac{\delta J - 1}{J-1} \mathbb{1}_j \overline{p}_j + \frac{(1-\delta)}{J-1} \overline{p_j}$$

They get the deal with certainty if they are matched with an unskilled firm (with probability $\frac{(1-\delta)J}{J-1}$), but if they are matched with another skilled firm they only win over the customer if they price is lower than their competitor's (which occurs with probability $\frac{\delta J-1}{J-1}$). Hence for any $\overline{p}_j, \overline{p}_{-j} \in [k, +\infty[$ such that $\overline{p}_j \geq \overline{p}_{-j}$, there is a small $\epsilon > 0$ such that setting $\overline{p}_j - \epsilon < \overline{p}_{-j}$ would yield higher profits for firm j. This Bertrand mechanism drives skilled firms' prices to k, as undercutting becomes impossible. The customer would in this case randomize, which means that skilled firms' profits become:

$$\mathbb{E}(\overline{\Pi}_j | c = \overline{c}) = \frac{\delta J - 1}{J-1} \times \frac{k}{2} + \frac{(1-\delta)J}{J-1} k = \frac{J(2-\delta)-1}{J-1} \frac{k}{2}$$

Figure 3 illustrates this mechanism for $J = 1000$, $\delta = 0.5$ and $k = 50$, drawing firm j's isoprofit lines in the (p_{-j}, p_j) referential. The top left graph depicts situation where initially $\overline{p}_j > \overline{p}_{-j}$, the top right graph the case where both prices are equal and strictly above k initially and the bottom one displays the equilibrium situation. It is clear in the two top graphs that setting a price slightly below their competitor's is always optimal when prices are set strictly above k.

If the customer has a $\underline{c}$ issue, a skilled firm j simultaneously chooses its lying strategy η_j and its resale prices $\overline{p}_j$ and $\underline{p}_j$. Depending on their competitor's type, their expected profits are:

$$\begin{cases} \mathbb{E}(\overline{\Pi}_j | c = \underline{c}, \beta_{-j} = \overline{\beta}) & = & \eta_j \big(\eta_{-j} \mathbb{1}_j \overline{p}_j + (1-\eta_{-j})\overline{p}_j\big) + (1-\eta_j)\big(\eta_{-j} \times 0 + (1-\eta_{-j})\mathbb{1}_j \underline{p}_j\big) \\ \mathbb{E}(\overline{\Pi}_j | c = \underline{c}, \beta_{-j} = \underline{\beta}) & = & \eta_j \overline{p}_j + (1-\eta_j)\mathbb{1}_j \underline{p}_j \end{cases}$$

If a skilled firm j is matched with an unskilled firm and chooses not to misreport with a positive probability (meaning $\eta_j < 1$), the same Bertrand mechanism described previously drives $\underline{p}_j$ and p_{-j} down to k, which means the customer randomizes. Hence their expected profits become:

$$\mathbb{E}(\overline{\Pi}_j | c = \underline{c}, \beta_{-j} = \underline{\beta}) = \eta_j \overline{p}_j + (1-\eta_j)\frac{k}{2}$$

As $\frac{\partial \mathbb{E}(\overline{\Pi}_j | \underline{c}, \underline{\beta}_{-j})}{\partial \eta_j} = \overline{p}_j - \frac{k}{2}$ and $\overline{p}_j \geq k$, it is straightforward that their expected profits conditional on the customer having a $\overline{c}$ issue and being drawn with an unskilled firm are an increasing function of η_j, whatever the value of $\overline{p}_j$. Their optimal choice is to set $\eta_j(\underline{\beta}_{-j}) = 1$.

Turning to the case where they are matched with another skilled firm, if both firm may misreport with a positive probability the Bertrand mechanism described beforehand implies that $\underline{p}_j = \overline{p}_j = k$ for all skilled firms. Their expected profits in that case become:

$$\mathbb{E}(\overline{\Pi}_j | c = \underline{c}, \beta_{-j} = \overline{\beta}) = \eta_j \big(\eta_{-j}\frac{k}{2} + (1-\eta_{-j})k\big) + (1-\eta_j)(1-\eta_{-j})\frac{k}{2}$$

Hence $\frac{\partial \mathbb{E}(\underline{\Pi}_j | \underline{c}, \underline{\beta}_{-j})}{\partial \eta_j} = \frac{k}{2}$, which is always positive. It implies that their optimal choice is to set $\eta_j(\overline{\beta}_{-j}) = 1$, which further means that $\eta^* = 1$ is their equilibrium lying policy.

Finally, let us show that $\overline{p}_j^* = k \quad \forall j$ when the customer has a $\overline{c}$ issue. Whatever their competitor's type, a skilled firm's expected profits under their optimal lying strategy are given by:

$$\mathbb{E}(\overline{\Pi}_j | c = \overline{c}) = \frac{\delta J - 1}{J-1}\mathbb{1}_j \overline{p}_j + \frac{(1-\delta)J}{J-1}\overline{p}_j$$

This is equivalent to their expected profits when the customer has a $\underline{c}$ issue, so following the same steps their optimal choice is $\overline{p}_j^* = k$. □

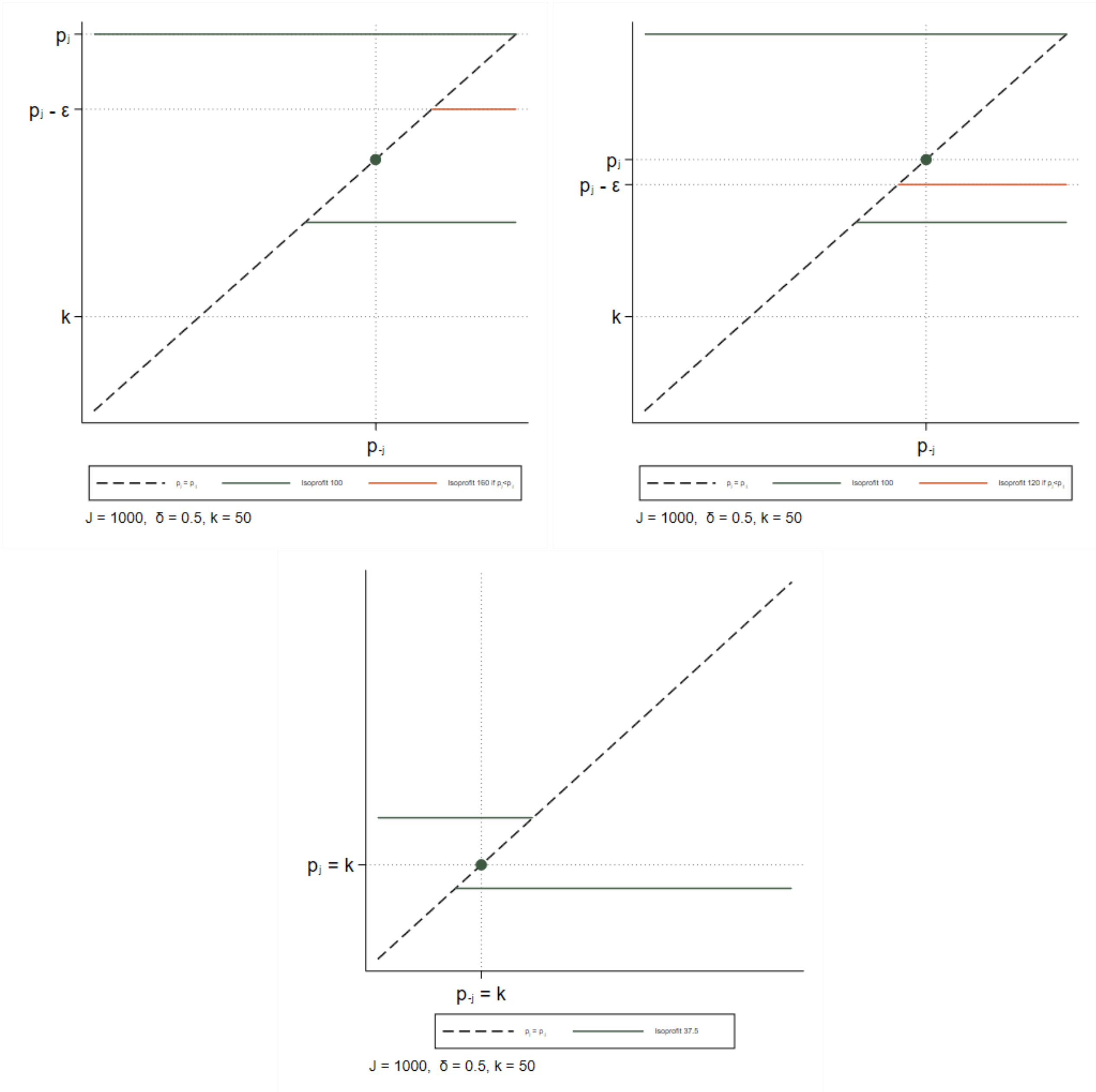
Sampling two firms is enough to restore some competitive pressure, which leads to lower prices but has no effect on overtreatment. As prices are at their lowest, and increasing them would only decrease expected profits, over-treatment is the only dimension they can play on to differentiate themselves from unskilled firms in the eye of the customer. Low and uniform prices are in line with what can be observed on the market and the incapacity to put a skill premium on prices matches common complaints made by professional organizations. Turning to unskilled firms, they also lower their resale price in reaction to potential competitors.

Proposition 4. *In equilibrium, unskilled firms' optimal choice is to set $p^* = k$.*

Proof. Since skilled firms set $\eta^* = 1$, unskilled firms' expected profits are given by:

$$\mathbb{E}(\underline{\Pi}) = \frac{2}{J}\frac{(1-\delta)J - 1}{J-1}\mathbb{1}_j p_j$$

As their profits are 0 whenever they set $p_j > p_{-j}$, the Bertrand mechanism previously described implies directly that $p_j^* = k$. □



Source: Author's computations.
Note: Simulated results assuming $J = 1000$, $\delta = 0.5$ and $k = 50$.

Figure 3: Skilled firms' isoprofit lines when facing a $\overline{c}$ customer.

Unskilled firms set the lowest price possible, not to align with skilled firms, but because of the competition with same-type firms. Contrary to previous work on credence goods, they do not try to pass as skilled firms - they can't -, yet competition drives all prices to their minimum level. Both kinds of firms are fully specialized in equilibrium, so the customer can derive their types based on the diagnosis received. It does not however lift all the uncertainty, especially if the share of unskilled firms is high, as discussed thereafter.

4.3. Equilibrium payoffs and participation conditions

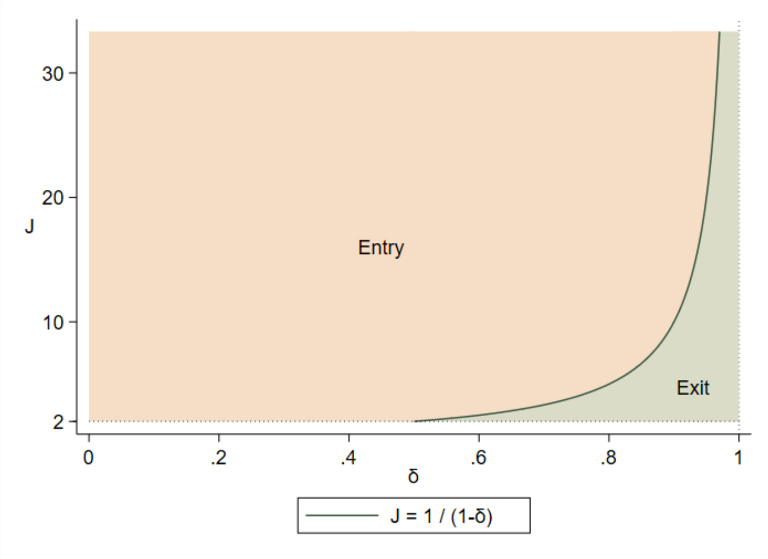
The consumer's expected utility depends on the number of firms (J), the price k set by all firms, the share of $\overline{\beta}$ firms (δ) and their probability to have a $\overline{c}$ issue (μ).

$$\begin{aligned}
\mathbb{E}(U^*) &= \Big(1-\tfrac{(1-\delta)((1-\delta)J-1)}{J-1}\Big)(V-k)+\tfrac{(1-\delta)((1-\delta)J-1)}{J-1}\Big(\mu(0-k)+(1-\mu)(V-k)\Big)\\
&= V\Big(1-\tfrac{(1-\delta)((1-\delta)J-1)}{J-1}+(1-\mu)\tfrac{(1-\delta)((1-\delta)J-1)}{J-1}\Big)-k\\
&= V\Big(1-\mu\tfrac{(1-\delta)((1-\delta)J-1)}{J-1}\Big)-k
\end{aligned}$$

Their participation condition is hence $k \leq V\Big(1-\mu\frac{(1-\delta)((1-\delta)J-1)}{J-1}\Big)$, meaning they only enter the market if k is low enough to compensate for the risk they take when accepting a $\underline{c}$ offer. Analyzing this result from another angle, it is consistent with the fact that housing renovations can be postponed but become more and more necessary over time (meaning V would tend to increase). Discomfort due to bad insulation might be tolerable the first years after buying and moving into a new dwelling, but will eventually have to be addressed. Households typically take some time before turning a renovation idea into a reality because of a multitude of decision barriers (see for instance, Azizi, Nair, and Olofsson 2019), which would be consistent with an increasing valuation as time passes. This model does not allow to study this time dimension, but it would be an interesting development. Turning to firms' expected payoffs:

$$\begin{aligned}
\mathbb{E}(\underline{\Pi}^*) &= \tfrac{2}{J}\tfrac{(1-\delta)J-1}{J-1}\tfrac{1}{2}k\\
&= \tfrac{(1-\delta)J-1}{J(J-1)}k\\
\mathbb{E}(\overline{\Pi}^*) &= \tfrac{2}{J}\Big(\tfrac{\delta J-1}{J-1}\tfrac{1}{2}+\tfrac{(1-\delta)J}{J-1}\Big)k\\
&= \tfrac{1}{J(J-1)}\Big(\delta J-1+2(1-\delta)J\Big)k\\
&= \tfrac{1}{J(J-1)}\Big(J(2-\delta)-1\Big)k
\end{aligned}$$

As firms always offer the same diagnosis independently of the customer's actual problem, μ does not impact their profits. The only parameters affecting their payoffs in equilibrium are the total number of firms and the share of same-type firms, which is determined by δ. For a strictly positive k, the participation conditions of skilled and unskilled firms are respectively $J \geq \frac{1}{1-2\delta}$ and $J \geq \frac{1}{1-\delta}$. As $J > 2$, skilled firms always have an incentive to be active on the market and unskilled firms always make a positive profit if $\delta \leq \frac{1}{2}$. If less than half the firms are skilled, the unskilled are always active (Figure 4). Unskilled firms may thus not enter the market if more than half the active firms are skilled, depending on the value of J. Interestingly, a higher overall number of firms makes it easier for them to make a positive profit in equilibrium, as it increases their chance to be drawn with another $\underline{\beta}$-type firm.



Source: Author's computations.

Figure 4: Unskilled firms' entry condition

4.4. Comparative statics

The model does not allow to study dynamic changes, but comparative statics provide insight on the impact of the market parameters on final outcomes. Unsurprisingly, equilibrium profits of both types of firms are increasing in the market price k. More interestingly, they both decrease and in the same magnitude as δ increases. From a skilled firm's perspective, an increase in δ means an increase in competition, as it becomes more probable to be drawn with another skilled firm and only make the sale with a 50% chance. Similarly, as unskilled firms can only win over the customer when they are matched with another unskilled firm, increasing the share of skilled firms implies a drop in their equilibrium profits.

$$\frac{\partial \mathbb{E}(\overline{\Pi}^*)}{\partial \delta} = \frac{\partial \mathbb{E}(\underline{\Pi}^*)}{\partial \delta} = \frac{-1}{J-1}k$$

The derivatives of equilibrium profits with respect to J are given by:

$$\begin{aligned} \frac{\partial \mathbb{E}(\overline{\Pi}^*)}{\partial J} &= \frac{1}{J^2(1-J)^2}\Big(-(2-\delta)J^2+2J-1\Big)k \\ \frac{\partial \mathbb{E}(\underline{\Pi}^*)}{\partial J} &= \frac{1}{J^2(J-1)^2}\Big(-(1-\delta)J^2+2J-1\Big)k \end{aligned}$$

Skilled firms' equilibrium profits are strictly decreasing function of J, as $-(2-\delta)J^2+2J-1 \leq 0 \quad \forall\, \delta \in [0,1]$. It is strictly decreasing for all $\delta < 1$, and the magnitude of the effect decreases as δ gets closer to 0. A higher number of skilled firms on the market increases their chance of being drawn with another skilled firm and decreases their individual chance of being drawn, both impacting their profits negatively. The effect of J on unskilled firm's profits is more ambiguous, as more firms on the market decreases their individual probability of being drawn but also increases their chance of being matched with another unskilled firm depending on the value of δ.

Proposition 5. *An increase in J is beneficial for unskilled firms' profits in equilibrium if and only if* $J \leq \frac{1+\sqrt{\delta}}{1-\delta}$

and $\delta \geq \frac{1}{4}$.

Proof. From the previous equation, $\frac{\partial \mathbb{E}(\underline{\Pi}^*)}{\partial J} \geq 0 \iff -(1-\delta)J^2 + 2J - 1 \geq 0$.

Denote f the function defined on $]2,+\infty[$ by $f(x) = -(1-\delta)x^2 + 2x - 1$, with $\delta \in [0,1]$. Its determinant is given by:

$$\Delta = 4 - 4(1-\delta) = 4\delta$$

If $\delta = 0$ then $\Delta = 0$ and $f(x) > 0 \forall x > 2$, as the polynomial has a single root in $x = 1$. If $\delta > 0$, f has two roots, denoted x_1 and x_2, defined as functions of $\delta \in [0,1[$:

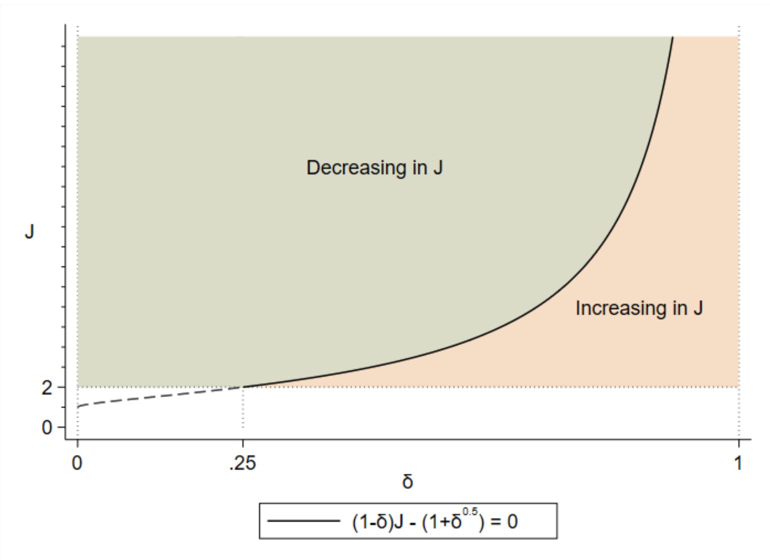
$$\begin{cases} x_1 & = & \frac{-2-\sqrt{\delta}}{-2(1-\delta)} & = & \frac{1+\sqrt{\delta}}{1-\delta} \\ x_2 & = & \frac{-2+\sqrt{\delta}}{-2(1-\delta)} & = & \frac{1-\sqrt{\delta}}{1-\delta} \end{cases}$$

As x_2 is a strictly decreasing function of δ and is equal to 1 when $\delta = 0$, $f(x) > x_2 \quad \forall x > 2$. It follows that $f(x)$ is positive when $x_1 > 2$ and $x \in]2, x_1]$ and negative otherwise. It can also be noted that x_1 is a strictly increasing function of δ and that $x_1 = 2 \iff \delta = \frac{1}{4}$, meaning that for $\delta \leq \frac{1}{4}$, f(x) is always negative.

Overall, the sign of the derivative of $\mathbb{E}(\underline{\Pi}^*)$ with respect to J is either positive of negative depending on the value of J and δ:

- If $\delta \leq \frac{1}{4}$ or $J \geq \frac{1+\sqrt{\delta}}{1-\delta}$, then $\mathbb{E}(\underline{\Pi}^*) \leq 0$.
- If $\delta \geq \frac{1}{4}$ and $J \leq \frac{1+\sqrt{\delta}}{1-\delta}$, then $\mathbb{E}(\underline{\Pi}^*) \geq 0$.

□



Source: Author's computations.

Figure 5: Sign of $\frac{\partial \mathbb{E}(\underline{\Pi}^*)}{\partial J}$ depending on the value of J and δ

Figure 5 illustrates these two cases. When there are a lot of skilled firms and a few firms overall, the negative impact of an increase in J on the probability of being drawn $\frac{2}{J}$ is more than compensated by the increase in the probability of being drawn with another unskilled firm. This result suggests that if a substantial number of firms operate on the market, focusing on the development of competence could be an effective way to make unskilled firms less profitable - and in turn increase customer's satisfaction and overall effectiveness of housing retrofit measures. This idea has been long prevalent among European policy makers, for instance with the emphasis on apprenticeships during François Hollande's presidency in France. The objective was to train young aspiring workers by making them work directly with experienced professionals for long periods of time, to ensure skill transmission beyond theoretical courses.

The customer's expected utility in equilibrium is composed of two terms: their instant utility if they always got V when they paid k, minus an extra cost $\lambda = \mu V(1-\delta)\frac{(1-\delta)J-1}{J-1}$, which stems from the uncertainty linked to the information asymmetry. It is straightforward that $\mathbb{E}(U)$ is a decreasing function of k and λ. The value of λ tends towards 0 when δ is close to 1: in the extreme case where there are only $\overline{\beta}$ firms on the market, there is no extra cost due to information asymmetries. Conversely, if $\delta = 0$ then $\lambda = \mu V$, meaning that if there are no skilled firms on the market the net loss to consumer is their valuation V of getting the issue fixed times the probability of having a $\overline{c}$ issue. It is also notable that $\lim_{\mu \to 0} \lambda = 0$ since firm types do not matter if the customer cannot have a $\overline{c}$ issue. There are however some non-linearities in the sign of λ depending on the model parameters μ, δ and J.

$$\begin{aligned} \frac{\partial \mathbb{E}(U)}{\partial \mu} &= -\frac{V(1-\delta)}{J-1}\big((1-\delta)J-1\big) \\ \frac{\partial \mathbb{E}(U)}{\partial \delta} &= -\frac{\mu V}{J-1}\big(1-2(1-\delta)J\big) \end{aligned}$$

These imply that $\frac{\partial \mathbb{E}(U)}{\partial \mu} \leq 0 \iff \delta \leq \frac{J-1}{J}$ and $\frac{\partial \mathbb{E}(U)}{\partial \delta} \geq 0 \iff \delta \leq \frac{2J-1}{2J}$. In other words, if δ is low enough, an increase in the share of skilled firms always leads to an increase in utility, and an increase in the probability of having a $\overline{c}$ issue has a detrimental effect. Let $\delta_\mu = \frac{J-1}{J}$ and $\delta_\delta = \frac{2J-1}{2J}$. They both tend towards 1 when J tends towards $+\infty$, meaning that these effects are always true whatever the value of δ when there are an infinity of firms of the market. They are both increasing functions of J, and as $J > 2$, it is straightforward that:

$$\begin{cases} \delta_\mu > \frac{1}{2} \\ \delta_\delta > \frac{3}{4} \end{cases}$$

The following graph sums up how λ evolves with respect to μ and δ, depending on these two thresholds. For simplicity, assume that $\delta \leq \frac{J-1}{J}$ henceforth.

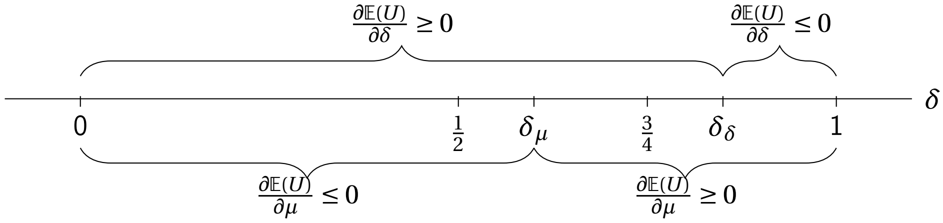


Figure 6: Sign of the change in the customer's expected utility following a change in μ or δ

As $\frac{\partial \mathbb{E}(U)}{\partial J} = -\frac{\mu\delta(1-\delta)}{(J-1)^2}V$, an increase in J implies a strict decrease in the customer's utility in equilibrium as long as $\delta \in]0,1[$. It has no impact if there is only one type of firm on the market, in other words if either $\delta = 0$ or $\delta = 1$, or if the customer cannot have a serious issue, ie. $\mu = 0$. The higher the original number of firms J, the lesser the negative impact of an extra firm on the market on their utility, and the effect tends towards zero when there are infinitely many firms on the market. This is a somewhat counterintuitive result: consumers do not benefit from having too many different suppliers. This is a direct implication of the fact that they limit their search instead of investigating a large number of firms. It is a restrictive assumption, but it is consistent with reported customer behavior on the market. Having an expert assess a renovation need takes time, and a lot of consumers will choose to consult only a few firms.

The main takeaways from this model are that in the market equilibrium prices are low and skilled firms do not have an incentive to be truthful, as they compete with one another. The unique price result directly stems from the assumption that all firms have the same reserve prices. Introducing firm-specific reservation prices does not alter the main results, nor does it change the comparative statics analysis for the most part, as shown in Appendix C. If the model were extended to N consumers as a repeated game, this could however be useful to reproduce the price dispersion observed in real-world markets (Grandclément et al. 2018). Equilibrium fraud comes both in the form of overtreatment and undertreatment, the latter being more worrisome in the context of energy-efficiency renovations. Overtreatment is a misallocation of resources but does not hinder the final objective, which is to reduce the energy consumption of the residential sector. On the contrary, undertreatment undermines this goal and has been the main target of regulators.

5. Extensions

5.1. Introducing partial labeling of skilled firms

Labels seem like a natural solution to the undertreatment issue, as they are meant to restore information on firm types. As previously discussed, quality labels have been implemented in the construction sector to reassure customers on either the process, the materials used, the skills of the workers, etc. Despite this profusion of options, it is noticeable that they are not widespread among firms - the energy-efficiency certification "*Reconnu garant de l'environnement*" (RGE)[4] for instance accounts for roughly 15% of firms in France. Another key aspect of these labels is that they remain relatively unknown to the general public. In order to assess the efficiency of these labels, the previous model is extended, first assuming the customer would always choose a labeled firm over any other, and then in the more realistic hypothesis that they would be indifferent between a labeled firm and another one offering the same diagnosis at the same price.

Keeping the same framework, let a certain share of skilled firms decide to get a label, which reveals their type to the consumer. This is equivalent to introducing a third type in the previous game, denoted $\overline{\beta}^l$. Assume these firms are known to be always truthful in their diagnosis, so if the consumer faces an offer by a labeled firm against an offer by any other type, they buy from the labeled firm. If they draw two labeled firms, they buy from the cheapest one. Let $\delta = \delta^l + \delta^u$, where δ^l is the share of labeled skilled firms and δ^u is the share of unlabeled skilled firms. Figure 7 displays the game in its extensive form and Figure 12 in the Appendix provides details on the computation of total probabilities.

[4]It can be translated to "Recognised Environmental Guarantor".

Proposition 6. *In equilibrium, there a unique market price set at k, and all unlabeled skilled firms set* $\eta^* = 1$.

Proof. This proof essentially follows that of Proposition 3 and 4. Nothing changes for unlabeled skilled firms and unskilled firms, except that unlabeled skilled firms cannot win over the customer if they are drawn with a labeled firm, hence in equilibrium they all set their prices to k and $\eta^* = 1$ remains an optimal strategy. Facing a $\underline{c}$ customer, a labeled skilled firm's profits are given by:

$$\mathbb{E}(\overline{\Pi}_j^l | c = \underline{c}) = \frac{\delta^l J - 1}{J-1} \times \mathbb{1}_j \times \underline{p}_j^l + \frac{(1-\delta^l)J}{J-1} \times \underline{p}_j^l$$

The same Bertrand mechanism described previously applies: for any price set by an opposite labeled firm such that $k < \underline{p}_{-j}^l \leq \underline{p}_j^l$, there is a strictly positive ϵ such that setting $\underline{p}_j^l = \underline{p}_j^l - \epsilon$ generates a strictly higher profit. As it pushes prices down, in equilibrium all labeled firms set $\underline{p}^l = k$. If the customer has a $\overline{c}$ issue, their profits take the same form as in the $\underline{c}$ case, except they set a price $\overline{p}^l$. Following the same reasoning, it is straightforward that $\overline{p}^l = k$ in equilibrium. □

Equilibrium payoffs are given by:

$$\begin{aligned}
\mathbb{E}(\underline{\Pi}^*) &= \tfrac{2}{J}\Big(\tfrac{(1-\delta)J-1}{J-1}\tfrac{k}{2} + \big(1 - \tfrac{(1-\delta)J-1}{J-1}\big)\times 0\Big) &&= \tfrac{(1-\delta)J-1}{J(J-1)}k \\
\mathbb{E}(\overline{\Pi}^{u*}) &= \tfrac{2}{J}\Big(\tfrac{\delta^l J}{J-1}\times 0 + \tfrac{\delta^u J-1}{J-1}\tfrac{1}{2}k + \tfrac{(1-\delta)J}{J-1}k\Big) &&= \tfrac{k}{J(J-1)}\Big(J\big(2-\delta-\delta^l\big)-1\Big) \\
\mathbb{E}(\overline{\Pi}^{l*}) &= \tfrac{2}{J}\Big(\tfrac{\delta^l J-1}{J-1}\tfrac{k}{2} + \big(1 - \tfrac{\delta^l J-1}{J-1}\big)k\Big) &&= \tfrac{k}{J(J-1)}\Big(J(2-\delta^l)-1\Big) \\
\mathbb{E}(U^*) &= \tfrac{(1-\delta)((1-\delta)J-1)}{J-1}(V(1-\mu)-k) + \big(1 - \tfrac{(1-\delta)((1-\delta)J-1)}{J-1}\big)(V-k) &&= V\Big(1-\mu(1-\delta)\tfrac{(1-\delta)J-1}{J-1}\Big) - k
\end{aligned}$$

Because of the competition among them, labeled skilled firms cannot set higher prices, even if the customer always favor them against any other unlabeled firm. It could be one of the causes undermining the appeal of certifications for skilled firms, providing ground for the assumption that only some of them would go through this process. Labeling is also inefficient to prevent overtreatment and to push unskilled firms out of the market, since their profits only depend on the overall share of skilled firms. Hence, if the share of skilled firms δ remains the same, it does not increase the consumer's equilibrium utility. There is less uncertainty for the consumer, who will learn their own type whenever drawing an unskilled firms or at least one labeled firm, but it does not translate into a higher utility level.

In fine, on a market where unskilled firms would be otherwise active, introducing a type-revealing label will not push them out but it will affect unlabeled skilled firms' profits. They are strictly lower in this setup if $\delta^l > 0$, and they do not always have an incentive to enter the market. Their expected equilibrium profits are positive if and only if $J \geq \frac{1}{2(1-\delta^l)-\delta^u}$. As $\delta \leq \frac{3}{4} \implies \frac{1}{2(1-\delta^l)-\delta^u} \leq 2$, their equilibrium expected profits are positive as long as $\delta \leq \frac{3}{4}$ or $J \geq \frac{1}{2(1-\delta^l)-\delta^u}$. They are negative if and only if $\delta > \frac{3}{4}$ and $J \leq \frac{1}{2(1-\delta^l)-\delta^u}$. Figure 8 displays unlabeled skilled firms' entry condition, plotting J as a function of δ^l for given levels of δ. It can be noted that everything else equal, the higher δ is, the more firms need to be on the market for unlabeled firms to expect positive profits in equilibrium. Similarly, given any $\delta > \frac{3}{4}$, an increase in δ^l implies a more constraining entry condition.

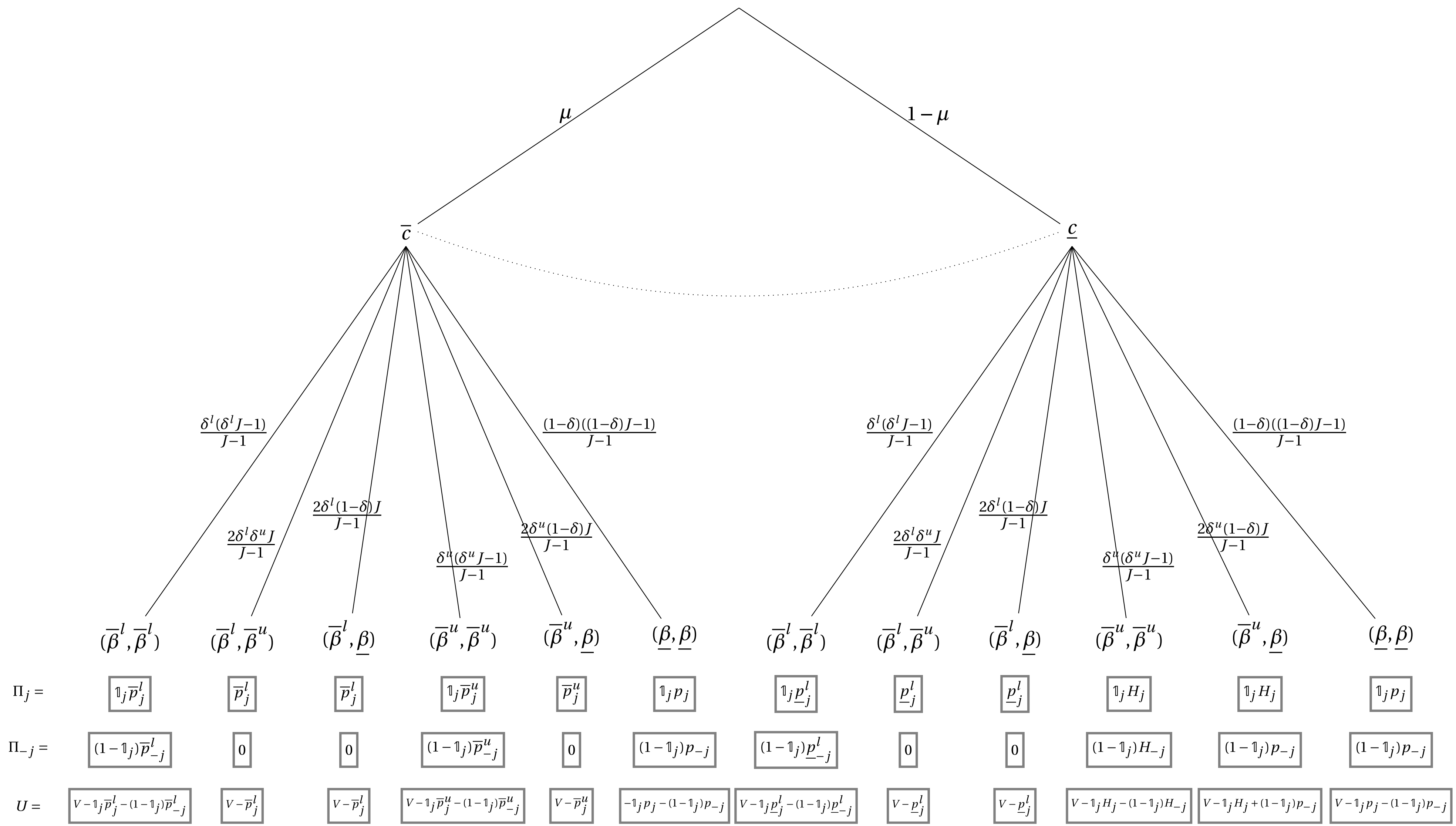
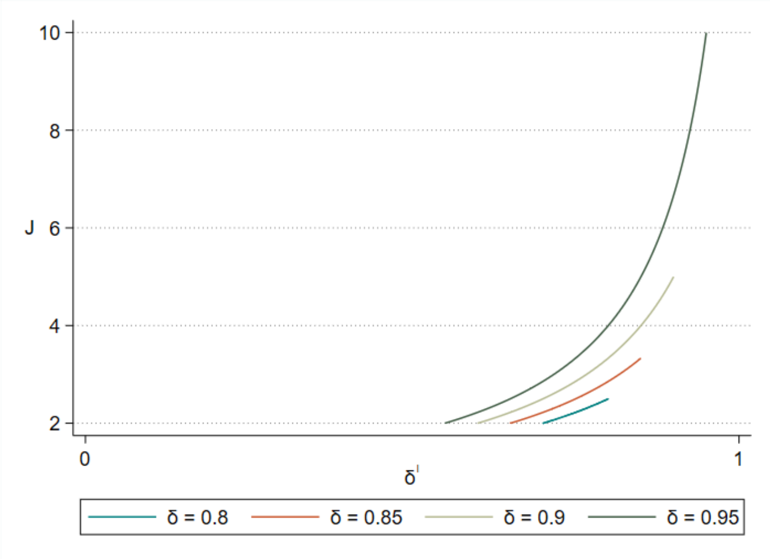


Figure 7: Game with labeled skilled firms

The main effect of labels is hence to increase the competitive pressure on skilled firms, by reducing their chances to win the customer over. If the share of labeled firms remains quite low, however, it won't be enough of a reason to push them out of the market.



Source: Author's computations.

Figure 8: Unlabeled skilled firms' entry condition

5.2. Partial labeling with a less inclined consumer

As labels are not always well known outside of the professional spheres, it could be unlikely that the customer would systematically choose a certified firm. Consider the same setup with labels, except that consumers do not always go for the labeled firm if they are facing a $(\overline{\beta}^l, \overline{\beta}^u)$ pair or a $(\overline{\beta}^l, \underline{\beta})$ one offering the same diagnosis. As shown thereafter, there is still one market price set at k, as this result is driven by the perspective of being drawn with same-type firms, but the equilibrium profits of labeled and unlabeled skilled firms will be affected.

Proposition 7. *In equilibrium, unlabeled skilled firms always set $\overline{p}^{u*} = \underline{p}^{u*} = k$.*

Proof. In this setup, an unlabeled skilled firm's expected profits when facing a $\overline{c}$ customer are the same as in the proof of Proposition 3:

$$\mathbb{E}(\overline{\Pi}^u | c = \overline{c}) = \frac{\delta^l J}{J-1} \mathbb{1}_j \overline{p}_j^u + \frac{\delta^u J - 1}{J-1} \mathbb{1}_j \overline{p}_j^u + \frac{(1-\delta)J}{J-1} \overline{p}_j^u = \frac{\delta J - 1}{J-1} \mathbb{1}_j \overline{p}_j^u + \frac{(1-\delta)J}{J-1} \overline{p}_j^u$$

When facing a $\underline{c}$ customer, their expected profits are also unchanged when they are matched with a same-type firm or an unskilled firm:

$$\begin{cases} \mathbb{E}(\overline{\Pi}^u | c = \underline{c}, \beta_{-j} = \overline{\beta}^u) & = & \eta_j \big(\eta_{-j} \mathbb{1}_j \overline{p}_j^u + (1-\eta_{-j}) \overline{p}_j^u\big) + (1-\eta_j)\big(\eta_{-j} \times 0 + (1-\eta_{-j}) \mathbb{1}_j \underline{p}_j\big) \\ \mathbb{E}(\overline{\Pi}^u | c = \underline{c}, \beta_{-j} = \underline{\beta}) & = & \eta_j \overline{p}_j^u + (1-\eta_j) \mathbb{1}_j \underline{p}_j^u \end{cases}$$

The optimal strategy in these cases does not change: all prices drop to k and $\eta^*(\overline{\beta}^u) = \eta^*(\underline{\beta}) = 1$. If they deal with a $\underline{c}$ customer and are drawn with a labeled firm however, their profits are given by:

$$\mathbb{E}(\overline{\Pi}^u | c = \underline{c}, \beta_{-j} = \overline{\beta}^l) = \eta_j \times 0 + (1-\eta_j)\mathbb{1}_j \underline{p}^u_j$$

It is straightforward that in this situation it is optimal to set $\underline{p}^u_j = k$ (and $\underline{p}^l_{-j} = k$). Their profits become $\mathbb{E}(\overline{\Pi}^u | c = \underline{c}, \beta_{-j} = \overline{\beta}^l) = (1-\eta_j)\frac{k}{2}$, implying $\frac{\partial \mathbb{E}(\overline{\Pi}^u | c = \underline{c}, \beta_{-j} = \overline{\beta}^l)}{\partial \eta_j} = -\frac{k}{2}$. As $\mathbb{E}(\overline{\Pi}^u | \underline{c}, \overline{\beta}^l)$ is a decreasing function of η_j, firms j's optimal lying strategy is $\eta^*_j(\overline{\beta}^l) = 0$.

Unlabeled skilled firms' equilibrium prices is always to set $\overline{p}^{u*} = \underline{p}^{u*} = k$, independently of their choice regarding their individual lying strategy η_j. □

It is not a surprising result, as prices are pushed down solely by the fact that the customer consults more than one firm, and by the uncertainty on the competitor's type. The robustness of this result is central, as intense competitive pressure on prices is a key feature of the construction market this model attempts to replicate. The optimal lying policy may however become truth-telling in this extension, depending on the share of labeled firms.

Proposition 8. *If $\delta^l > \frac{1}{2}$ or $J < \frac{1}{1-2\delta^l}$, $\eta^* = 0$ is the equilibrium strategy for labeled skilled firms. If $\delta^l < \frac{1}{2}$ and $J > \frac{1}{1-2\delta^l}$, their equilibrium strategy is $\eta^* = 1$. If $\delta^l \in [\frac{1}{4}, \frac{1}{2}]$ and $J = \frac{1}{1-2\delta^l}$, their equilibrium strategy is $\eta^* = \frac{1}{2}$.*

Proof. As firms do not know their competitor's type, they set the value of η depending on their overall profits:

$$\begin{aligned}
\mathbb{E}(\overline{\Pi}^{u*} | \eta = 1) &= \frac{2}{J}\Big(\frac{(1-\delta)J}{J-1}k + \frac{\delta^u J - 1}{J-1}\frac{k}{2} + \frac{\delta^l J - 1}{J-1}\big(\mu\frac{k}{2} + (1-\mu)\times 0\big)\Big) \\
&= \frac{k}{J(J-1)}\Big(J(2-\delta-(1-\mu)\delta^l) - 1\Big) \\
\mathbb{E}(\overline{\Pi}^{u*} | \eta = 0) &= \frac{2}{J}\Big(\frac{(1-\delta)J}{J-1}(\mu k + (1-\mu)\frac{k}{2}) + \frac{\delta J - 1}{J-1}\frac{k}{2}\Big) \\
&= \frac{k}{J(J-1)}\Big(J(1+\mu(1-\delta)) - 1\Big)
\end{aligned}$$

As $\mathbb{E}(\overline{\Pi}^{u*} | \eta = 1) - \mathbb{E}(\overline{\Pi}^{u*} | \eta = 0) = \frac{(1-\mu)k}{J-1}\big(1-\delta-\delta^l\big)$, lying is a dominant strategy if and only if $1-\delta \geq \delta^l$. Furthermore, $\delta \leq \frac{1}{2} \implies 1-\delta \geq \frac{1}{2}$, meaning $1-\delta \geq \delta \geq \delta^l$. Consequently, truth-telling is a dominant strategy if and only if $\delta \geq \frac{1}{2}$ and $\delta^l \geq 1-\delta$. It is strictly dominant if these inequalities are strict. In case of equality, the firm randomizes, setting $\eta = \frac{1}{2}$.

Let us review these three cases to determine the equilibria. First, let $1-\delta > \delta^l$ and assume all unlabeled skilled firms adopt the strategy $\eta^* = 1$ and set their prices to k. If one firm j were to deviate in this equilibrium by setting $\eta_j = \eta < 1$, their deviation profits would be:

$$\begin{aligned}
\mathbb{E}(\overline{\Pi}^{u*}_j | \eta^*_j = \eta, \eta^*_{-j} = 1) &= \frac{2}{J}\Big(\frac{\delta^l J}{J-1}\big(\mu\frac{k}{2} + (1-\mu)\eta\times 0 + (2-\mu)(1-\eta)\frac{k}{2}\big) + \frac{\delta^u J - 1}{J-1}\big(\mu\frac{k}{2} + (1-\mu)\eta\frac{k}{2} \\
&\quad + (1-\mu)(1-\eta)\times 0\big) + \frac{(1-\delta)J}{J-1}(\mu k + (1-\mu)\eta k + (1-\mu)(1-\eta)\frac{k}{2})\Big) \\
&= \frac{k}{J(J-1)}\Big(J\big(1+\mu+\eta(1-\mu) + \delta^l(1-\mu)(1-2\eta) - \delta\big) - \mu - \eta(1-\mu)\Big)
\end{aligned}$$

Let $\Delta\overline{\Pi}^u_{\eta^*=1} = \mathbb{E}(\overline{\Pi}^{u*}_j | \eta^*_j = \eta, \eta^*_{-j} = 1) - \mathbb{E}(\overline{\Pi}^{u*} | \eta^*_j = 1 \forall j)$, which simplifies to:

$$\begin{aligned}\Delta\overline{\Pi}^{u}_{\eta^*=1} &= \tfrac{k}{J(J-1)}\Big(J\big(1+\mu+\eta(1-\mu)+\delta^l(1-\mu)(1-2\eta)-\delta-2+\delta+(1-\mu)\delta^l\big)-\mu-\eta(1-\mu)+1\Big)\\ &= \tfrac{(1-\mu)(1-\eta)k}{J(J-1)}\Big(1-J(1-2\delta^l)\Big)\end{aligned}$$

As $1-\delta>\delta^l$ and $\delta>\delta^l$, it is straightforward that $\delta^l<\frac{1}{2}$, which implies that $1-2\delta^l\in]0,1[$. Furthermore, as $\eta<1$, $\Delta\overline{\Pi}^{u}_{\eta^*=1}>0 \iff 1-J(1-2\delta^l)>0$, which is equivalent to $J<\frac{1}{1-2\delta^l}$. In that case as the deviation profit is a decreasing function of η : $\frac{\partial\mathbb{E}(\overline{\Pi}^{u*}_j|\eta^*_j=\eta,\eta^*_{-j}=1)}{\partial\eta}=-\frac{(1-\mu)k}{J(J-1)}(1-J(1-2\delta^l))<0$, the optimal deviation strategy is to set $\eta^*_j=0$

Turning to the $1-\delta<\delta^l$ case, assume all unlabeled skilled firms adopt the strategy $\eta^*=0$ and set their prices to k. If one firm j were to deviate in this equilibrium by setting $\eta_j=\eta>0$, their deviation profits would be:

$$\begin{aligned}\mathbb{E}(\overline{\Pi}^{u*}_j|\eta^*_j=\eta,\eta^*_{-j}=0) &= \tfrac{2}{J}\Big(\tfrac{\delta^l J}{J-1}\big(\mu\tfrac{k}{2}+(1-\mu)\eta\times 0+(1-\mu)(1-\eta)\tfrac{k}{2}\big)+\tfrac{\delta^u J-1}{J-1}\big(\mu\tfrac{k}{2}+(1-\mu)\eta k\\ &\quad +(1-\mu)(1-\eta)\tfrac{k}{2}\big)+\tfrac{(1-\delta)J}{J-1}\big(\mu k+(1-\mu)\eta k+(1-\mu)(1-\eta)\tfrac{k}{2}\big)\Big)\\ &= \tfrac{k}{J(J-1)}\Big(J\big(1+\mu+\eta(1-\mu)-2\eta\delta^l(1-\mu)-\mu\delta\big)-1-(1-\mu)\eta\Big)\end{aligned}$$

It yields the following net deviation gain:

$$\begin{aligned}\Delta\overline{\Pi}^{u}_{\eta^*=0} &= \tfrac{k}{J(J-1)}\Big(J\big(1+\mu+\eta(1-\mu)-2\eta\delta^l(1-\mu)-\mu\delta-1-\mu(1-\delta)\big)-1-(1-\mu)\eta+1\Big)\\ &= \tfrac{\eta(1-\mu)k}{J-1}\Big(J(1-2\delta^l)-1\Big)\end{aligned}$$

As $\eta>0$, $\Delta\overline{\Pi}^{u}_{\eta^*=0}$ is always negative if $1-2\delta^l\le 0$, which is equivalent to $\delta^l\le\frac{1}{2}$. Alternatively, if $\delta^l<\frac{1}{2}$, the difference is strictly positive if and only if $J(1-2\delta^l)-1>0 \iff J>\frac{1}{1-2\delta^l}$. In that case, there are profitable deviations from the equilibrium, and as $\frac{\partial\mathbb{E}(\overline{\Pi}^{u*}_j|\eta^*_j=\eta,\eta^*_{-j}=0)}{\partial\eta}=\frac{(1-\mu)k}{J-1}(J(1-2\delta^l)-1)>0$, the optimal deviation strategy is to set $\eta^*_j=1$.

Finally if $1-\delta=\delta^l$ and all unlabeled skilled firms randomize, meaning $\eta^*=\frac{1}{2}$ and set their prices to k, their expected profits in equilibrium are given by:

$$\begin{aligned}\mathbb{E}(\overline{\Pi}^{u*}|\eta^*=\tfrac{1}{2}) &= \tfrac{2}{J}\Big(\tfrac{\delta^l J}{J-1}\big(\mu\tfrac{k}{2}+\tfrac{1-\mu}{2}\times 0+\tfrac{1-\mu}{2}\tfrac{k}{2}\big)+\tfrac{\delta^l J-1}{J-1}\big(\mu\tfrac{k}{2}+\tfrac{1-\mu}{2}(\tfrac{1}{2}\tfrac{k}{2}+\tfrac{1}{2}k)+\tfrac{1-\mu}{2}(\tfrac{1}{2}\times 0+\tfrac{1}{2}\tfrac{k}{2})\big)\\ &\quad +\tfrac{(1-\delta)J}{J-1}\big(\mu k+\tfrac{1-\mu}{2}k+\tfrac{1-\mu}{2}\tfrac{k}{2}\big)\Big)\\ &= \tfrac{k}{2J(J-1)}\Big(J\big(3+\mu-\delta(1+\mu)-(1-\mu)\delta^l\big)-2\Big)\end{aligned}$$

If one firm j were to deviate in this equilibrium by setting $\eta_j=\eta\neq\frac{1}{2}$, their deviation profits would be:

$$\begin{aligned}\mathbb{E}(\overline{\Pi}^{u*}_j|\eta^*_j=\eta,\eta^*_{-j}=\tfrac{1}{2}) &= \tfrac{2}{J}\Big(\tfrac{\delta^l J}{J-1}\big(\mu\tfrac{k}{2}+(1-\mu)\eta\times 0+(1-\mu)(1-\eta)\tfrac{k}{2}\big)+\tfrac{\delta^u J-1}{J-1}\big(\mu\tfrac{k}{2}\\ &\quad +(1-\mu)\eta(\tfrac{1}{2}\tfrac{k}{2}+\tfrac{1}{2}k)+(1-\mu)(1-\eta)(\tfrac{1}{2}\times 0+\tfrac{1}{2}\tfrac{k}{2})\big)\Big)\\ &= \tfrac{k}{2J(J-1)}\Big(J\big(2\eta(1-\mu)+2(1+\mu)+\delta^l(1-\mu)(1-4\eta)-\delta(1+\mu)\big)-1-\mu-2\eta(1-\mu)\Big)\end{aligned}$$

The difference between these two expected profits is given by:

$$
\begin{aligned}
\Delta\overline{\Pi}^u_{\eta^*=\frac{1}{2}} &= \tfrac{k}{2J(J-1)}\Big(J\big(2\eta(1-\mu)+2(1+\mu)+\delta^l(1-\mu)(1-4\eta)-\delta(1+\mu)-3-\mu+\delta(1+\mu)+(1-\mu)\delta^l\big) \\
&\quad -1-\mu-2\eta(1-\mu)+2\Big) \\
&= \tfrac{(1-2\eta)(1-\mu)k}{2J(J-1)}\Big(1-(1-2\delta^l)J\Big)
\end{aligned}
$$

As $\delta^l = 1-\delta$ and $\delta^l < \delta$, it is straightforward that $\delta > \frac{1}{2}$ which implies that both δ^l and $1-\delta$ are inferior to $\frac{1}{2}$. It implies that $1-2\delta^l \in]0,1[$.

If $\eta > \frac{1}{2}$, then $\Delta\overline{\Pi}^u_{\eta^*=\frac{1}{2}} > 0 \iff 1-(1-2\delta^l)J < 0$, which is equivalent to $J > \frac{1}{1-2\delta^l}$.

Conversely, if $\eta < \frac{1}{2}$, then $\Delta\overline{\Pi}^u_{\eta^*=\frac{1}{2}} > 0 \iff 1-(1-2\delta^l)J > 0$, meaning $J < \frac{1}{1-2\delta^l}$. In both cases, profitable deviations are possible. The derivative of the deviation profits is given by :

$$
\frac{\partial \mathbb{E}(\overline{\Pi}^{u*}_j | \eta^*_j = \eta, \eta^*_{-j} = \frac{1}{2})}{\partial \eta} = \frac{(1-\mu)k}{J(J-1)}\Big(J(1-2\delta^l)-1\Big)
$$

Hence :
$$
\begin{cases}
\frac{\partial \mathbb{E}(\overline{\Pi}^{u*}_j | \eta^*_j=\eta,\eta^*_{-j}=\frac{1}{2})}{\partial \eta} > 0 & \iff \quad J > \frac{1}{1-2\delta^l} \\
\frac{\partial \mathbb{E}(\overline{\Pi}^{u*}_j | \eta^*_j=\eta,\eta^*_{-j}=\frac{1}{2})}{\partial \eta} < 0 & \iff \quad J < \frac{1}{1-2\delta^l}
\end{cases}
$$

The $(\eta^*, \overline{p}^{u*}, \underline{p}^{u*}) = (\frac{1}{2}, k, k)$ is thus an equilibrium strategy for unlabeled skilled firms if and only if $J = \frac{1}{1-2\delta^l}$, which is possible only if $\delta \geq \frac{1}{4}$ since $J > 2$. The various equilibria depend on the values of δ^l and J:

- If $\delta^l > \frac{1}{2}$, unlabeled skilled firms' equilibrium lying strategy is $\eta^* = 0$
- If $\delta^l < \frac{1}{2}$, unlabeled skilled firms' equilibrium lying strategy is :

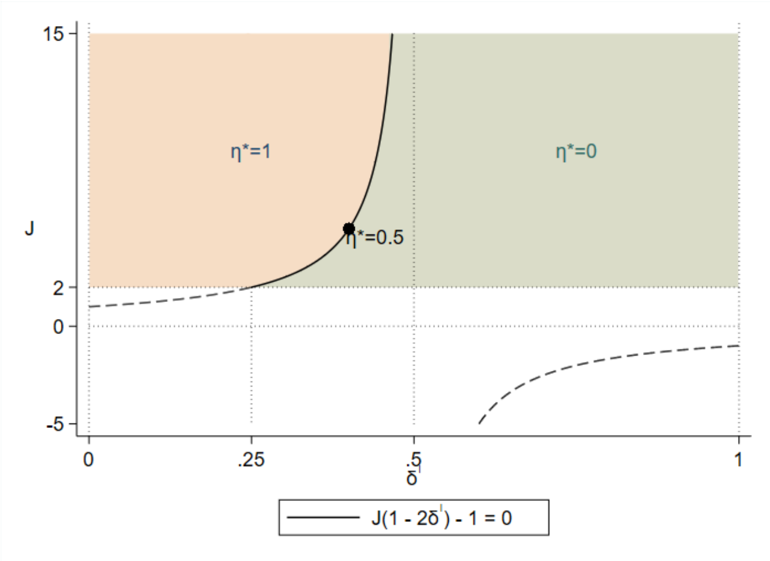
$$
\begin{cases}
\eta^* = 1 & \iff \quad J > \frac{1}{1-2\delta^l} \\
\eta^* = \frac{1}{2} & \iff \quad J = \frac{1}{1-2\delta^l} \quad \text{and} \quad \delta^l \geq \frac{1}{4} \\
\eta^* = 0 & \iff \quad J < \frac{1}{1-2\delta^l}
\end{cases}
$$

□

Labeling can prevent equilibrium overtreatment, but only if the consumer does not always pick labeled firms. Figure 9 sums up how the value of J and δ^l may affect the equilibrium lying strategy of unlabeled firms. If δ is small, lying is always optimal even if a lot of skilled firms are labeled, as the chance of matching with them does not represent enough of a risk to outweigh the certainty of winning over the customer if matched with an unskilled firm. The condition on J can also be rewritten as follows:

$$
\begin{aligned}
J(1-2\delta^l)-1 \quad \lesseqgtr \quad 0 \quad &\iff \quad J(1-\delta^l-(\delta-\delta^u))-1 \quad \lesseqgtr \quad 0 \\
&\iff \quad (1-\delta)J+\delta^u J-1 \quad \lesseqgtr \quad \delta^l J \\
&\iff \quad \tfrac{(1-\delta)J}{J-1}+\tfrac{\delta^u J-1}{J-1} \quad \lesseqgtr \quad \tfrac{\delta^l J}{J-1}
\end{aligned}
$$

The turning point for unlabeled skilled firms is ultimately whether it is more likely to be drawn with a labeled firm or with any other type of firm. If the former is more probable, truth-telling is optimal. Otherwise, either lying yields higher profits or truth-telling is not sustainable in equilibrium. In other words, if labeling is not



Source: Author's computations.

Figure 9: Unlabeled skilled firms' optimal lying strategy depending on the value of J and δ^l

sufficiently widespread and if there are not enough skilled firms overall, overtreatment will prevail. It is clearly not yet achieved in the European case, yet there have been efforts to generalize certifications and labels. The consumer's utility remains unchanged, since overtreatment does not lower their equilibrium payoffs. If the general goal is to ensure more efficient renovations, labels seem to miss the mark. Regarding the effects of unlabeled skilled firms' reporting strategy on other firms, $\eta = 0$ even raises unskilled firms' expected profits:

$$\begin{aligned}
\mathbb{E}(\underline{\Pi}^*|\eta = 1) &= \frac{2}{J}\Big(\frac{(1-\delta)J-1}{J-1}\frac{k}{2} + \frac{\delta^u J}{J-1}\times 0 + \frac{\delta^l J}{J-1}\big(\mu\times 0 + (1-\mu)\frac{k}{2}\big)\Big) \\
&= \frac{k}{J(J-1)}\Big(J(1-\delta+(1-\mu)\delta^l) - 1\Big) \\
\mathbb{E}(\underline{\Pi}^*|\eta = 0) &= \frac{2}{J}\Big(\frac{(1-\delta)J-1}{J-1}\frac{k}{2} + \frac{\delta J}{J-1}\big(\mu\times 0 + (1-\mu)\frac{k}{2}\big)\Big) \\
&= \frac{k}{J(J-1)}\Big(J(1-\delta\mu) - 1\Big)
\end{aligned}$$

As $\mathbb{E}(\underline{\Pi}^*|\eta = 1) - \mathbb{E}(\underline{\Pi}^*|\eta = 0) = \frac{(1-\mu)k}{J-1}(\delta^l - \delta)$ and $\delta^l \leq \delta$, it is always better for them when unlabeled skilled firms are truthful. Their equilibrium profits are also strictly higher than in the previous label setup in both cases. Conversely, $\eta = 0$ decreases the equilibrium profits of labeled firms, making the certification less attractive:

$$\begin{aligned}
\mathbb{E}(\overline{\Pi}^{l*}|\eta = 1) &= \frac{2}{J}\Big(\frac{\delta^l J-1}{J-1}\frac{k}{2} + \frac{\delta^u J}{J-1}\big(\mu\frac{k}{2} + (1-\mu)k\big) + \frac{(1-\delta)J}{J-1}\big(\mu k + (1-\mu)\frac{k}{2}\big)\Big) \\
&= \frac{k}{J(J-1)}\Big(J\big(1+\mu+\delta(1-2\mu) - \delta^l(1-\mu)\big) - 1\Big) \\
\mathbb{E}(\overline{\Pi}^{l*}|\eta = 0) &= \frac{2}{J}\Big(\frac{\delta J-1}{J-1}\frac{k}{2} + \frac{(1-\delta)J}{J-1}\big(\mu k + (1-\mu)\frac{k}{2}\big)\Big) \\
&= \frac{k}{J(J-1)}\Big(J\big(1+\mu(1-\delta)\big) - 1\Big)
\end{aligned}$$

As $\mathbb{E}(\overline{\Pi}^{l*}|\eta = 1) - \mathbb{E}(\overline{\Pi}^{l*}|\eta = 0) = \frac{(1-\mu)k}{J-1}(\delta - \delta^l)$ and $\delta^l \leq \delta$, it is always better for them when unlabeled skilled firms choose to be dishonest. These results establish that (1) decreasing the value of the label from the consumer's perspective is necessary in order to prevent equilibrium overtreatment and (2) that overtreatment is more efficient than labels to deter the entry of unskilled firms. When overtreatment is not an equilibrium behavior, the profits of labeled firms are lower and those of unskilled firms are higher. These results provide some insights as to why why fraud persists despite the existence of numerous certifications on the market. They have also not yet been largely adopted, which could reflect either that skilled firms do not see what could be gain from getting them, or that there is not a large share of skilled firms on the market.

6. Conclusion

The equilibria described in this paper successfully replicate key features of many European countries' construction sectors, that is: low and non-discriminating resale prices and the persistence of fraud in equilibrium despite intense competitive pressure. In particular, unskilled firms can make positive profits in most cases, and this would be especially true if considering a larger number of consumers - which would consist in independent repetitions of the described games. Labels are not an appropriate policy tool to push them out of the market, and may even increase their profits if they successfully prevent overtreatment.

These theoretical results can provide some explanations as to why such public policies have not been successful in undermining fraud and boost the energy gains that are supposedly achievable in the residential sector. They also provide arguments against current policies like direct funding for energy retrofits - the French *MaPrimeRénov'* for instance. As they lower the actual cost of these renovations, they may make household less mindful when choosing contractors. Finally, the variable that seems to play the most important role in deterring fraud is the share of skilled firms, which gives ground to policies aiming to enhance expertise in the sector. Their impact may not be as immediate as labels', but they would have a more decisive impact on undertreatment. Using French insurance data, the NGO *Agence Qualité Bâtiment*[5] found that the total compensation paid to households amounted to approximately 847 million euros in 2020 (AQC 2022). It has increased every year by 5.9% on average since 2011, which was when the RGE label was introduced. The same year, the state spent 5.4 billion euros to found 629 635 apprenticeship contracts, 11% of which were in the construction sector (CDC 2022). Using the mean cost, it adds up to 594 million euros spent on apprenticeships in the sector - 70% of the cost of defects and malfunctions. From a social welfare perspective, improving professional training for contractors could be a better allocation of resources.

This model is yet limited in some dimensions. It would be interesting to further develop a dynamic setup to see how an increase of skills would actually impact equilibrium payoffs. Overtreatment is also a more serious issue if p and p are not equal, as the direct funding of energy retrofit has become a widespread policy in Europe. For instance, the French state has a projected budget of 368.9 million euros dedicated to energy retrofits for 2023 (PLF 2023), hence generalized overtreatment could lead to a drastic misallocation of public resources.

[5] It can be translated to "Construction Quality Agency".

Appendices

A. Simple setup : customer utility

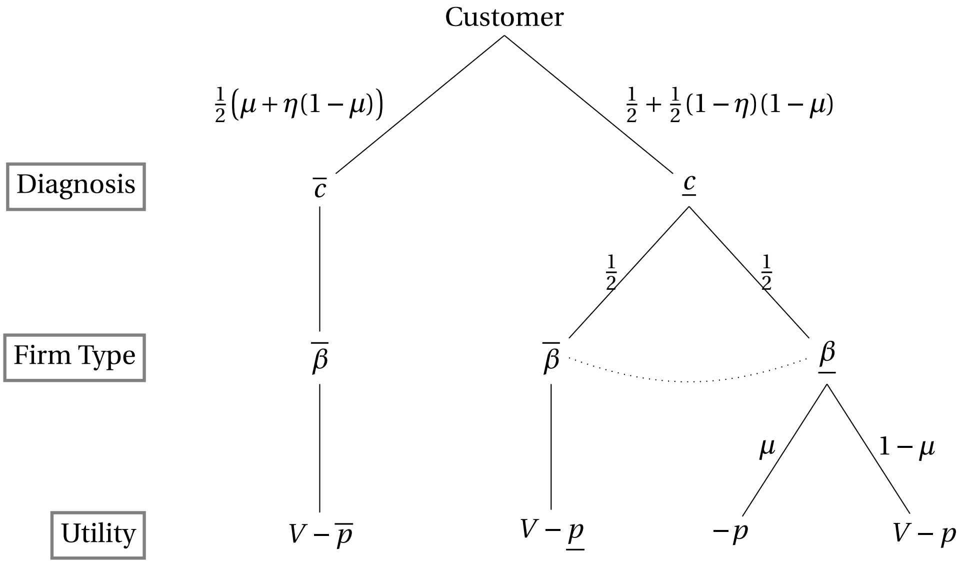


Figure 10: Customer's expected utility depending on the diagnosis received if $\eta < 1$

B. Obtaining total probabilities in the one-shot game with J firms and 2 firms drawn

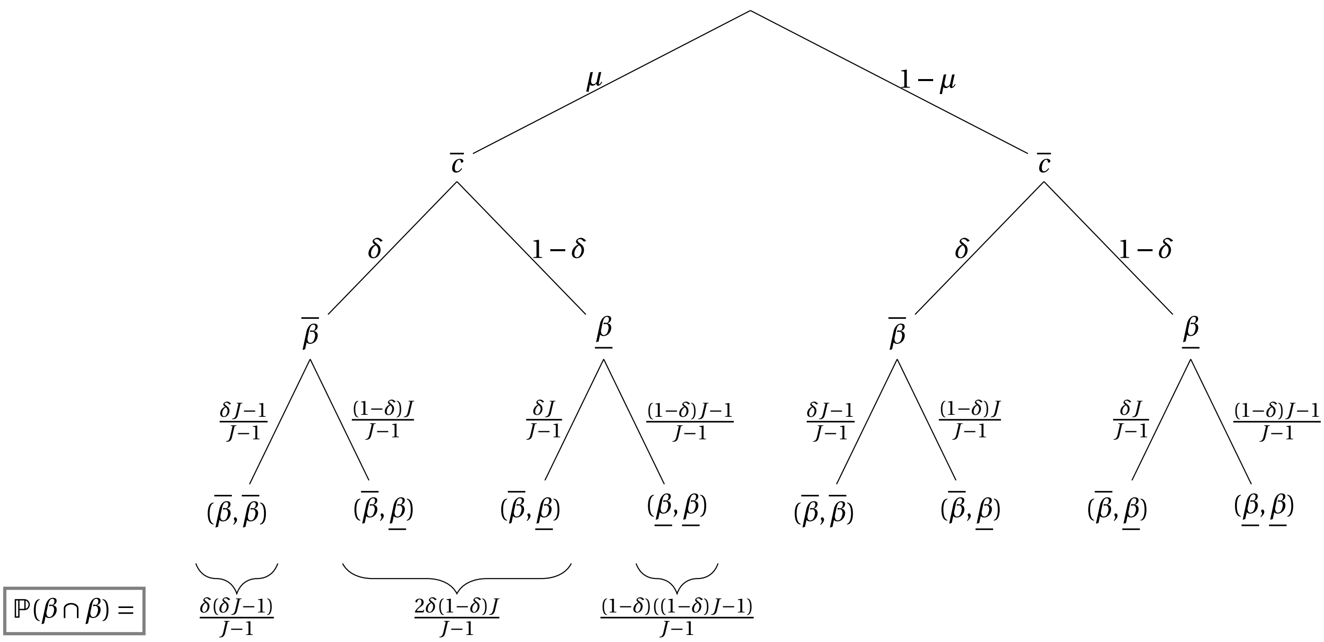


Figure 11: Probability of drawing each pair of firms

C. Introducing heterogeneous reserve prices in the general model

Given the same setup as in section 4, assume firms now have individual reserve prices. Formally, each firm j is randomly assigned a reserve price $k_j \in \mathbb{R}_+^*$ following a continuous cumulative distribution function F that is common knowledge. The main results from propositions 3 and 4 still hold, meaning in equilibrium $\eta_j^* = 1$, $\overline{p}^* = k_j$ and $p* = k_j$ for all firms j depending on their types.

Regarding skilled firms the potential competition still pushes them to post their minimal price k_j in all cases. As they do not know their competitor's type, if they offer a higher resale price there will always be a profitable deviation $\overline{p}_j - \epsilon$ or $\underline{p}_j - \epsilon$, where $\epsilon \in \mathbb{R}_+^*$. As both prices are equal, $\eta_j = 1$ is optimal to maximize their chances of winning over the customer. Their expected equilibrium payoffs are hence:

$$\begin{aligned}
\mathbb{E}(\overline{\Pi}_j^*) &= \tfrac{2}{J}\Big(\tfrac{\delta J-1}{J-1}\big(1-F(k_j)\big)k_j + \tfrac{(1-\delta)J}{J-1}k_j\Big) \\
&= \tfrac{2k_j}{J}\Big(1-\tfrac{\delta J-1}{J-1}F(k_j)\Big)
\end{aligned}$$

Unskilled firms will also post their minimal price, following the reasoning of Proposition 4. Their equilibrium expected profits are hence:

$$\mathbb{E}(\underline{\Pi}_j^*) = k_j\Big(1-F(k_j)\Big) \times \frac{2}{J} \times \frac{(1-\delta)J-1}{J-1}$$

The sign of the derivatives of expected profits with respect to J and δ remain unchanged. In particular, Proposition 5 still holds as $\frac{\partial \mathbb{E}(\underline{\Pi}_j^*)}{\partial J} = -\frac{k_j\left(1-F(k_j)\right)}{J^2(1-J)^2}\Big(2(1-\delta)J^2-4J+2\Big)$. This derivative is positive if and only if $-(1-\delta)J^2+4J-2 \geq 0$. The derivatives of expected profits with respect tot k_j are given by:

$$\begin{aligned}
\frac{\partial \mathbb{E}(\overline{\Pi}_j^*)}{\partial k_j} &= \tfrac{2}{J}\Big(1-\tfrac{\delta J-1}{J-1}\big(F(k_j)+k_j f(k_j)\big)\Big) \\
\frac{\partial \mathbb{E}(\underline{\Pi}_j^*)}{\partial k_j} &= \tfrac{2(1-\delta)J-2}{J(J-1)}\Big(1-F(k_j)-k_j f(k_j)\Big)
\end{aligned}$$

The expected profits of a skilled firm are an increasing function of its reserve price k_j if and only if $\frac{J-1}{\delta J-1} \geq F(k_j)+k_j f(k_j)$. Using partial integration, we find the following:

$$\begin{aligned}
& \int_0^{k_j} \tfrac{J-1}{\delta J-1}\partial t \geq \int_0^{k_j} F(t)\partial t + \int_0^{k_j} t f(t)\partial t \\
\iff\ & \tfrac{J-1}{\delta J-1}k_j \geq \big[tF(t)\big]_0^{k_j} - \int_0^{k_j} t f(t)\partial t + \int_0^{k_j} t f(t)\partial t \\
\iff\ & \tfrac{J-1}{\delta J-1}k_j \geq k_j F(k_j) \\
\iff\ & \tfrac{J-1}{\delta J-1} \geq F(k_j)
\end{aligned}$$

Since $\frac{J-1}{\delta J-1} \geq 1 \forall \delta \in [0,1]$, we find that $\frac{\partial \mathbb{E}(\overline{\Pi}_j^*)}{\partial k_j} \geq 0 \forall k_j$. It is easy to show that $\frac{\partial \mathbb{E}(\underline{\Pi}_j^*)}{\partial k_j} \geq 0 \forall k_j$ by following the same steps. This result implies that the negative effect of a marginally higher k_j on the probability that firm j sets a lower price than its competitor is completely offset by the direct gain if it wins over the customer.

The analysis of the customer's expected utility is identical to the one carried out in sections 4.3 and 4.4, assuming k is now defined by:

$$k = \begin{cases} \min(k_j, k_{-j}) & \text{if } \beta_j = \beta_{-j} \\ k_j & \text{if } (\beta_j, \beta_{-j}) = (\overline{\beta}, \underline{\beta}) \end{cases}$$

D. Obtaining total probabilities in the game with labeled firms

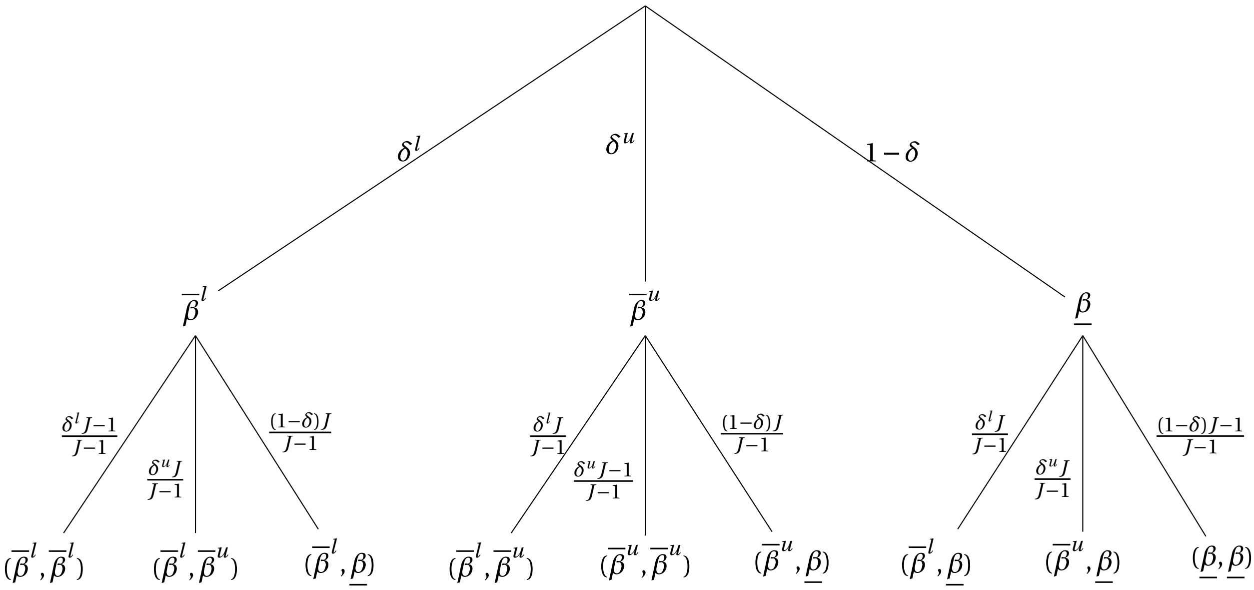


Figure 12: Probability of drawing each pair of firms in the setup with labeled firms

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