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WORKING PAPER

Are road pricing schemes efficient in polycentric cities with endogenous workplace locations?

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This article aims to measure the efficiency of different road pricing schemes (Pigouvian tax, flat tax and cordon toll) to address congestion externalities when the locations of jobs and dwellings within a city are endogenous. The model captures the fact that commuters face a trade-off between taking advantage of the wage premium in the Central Business District (CBD) and being stuck in traffic. I find that the Pigouvian tax strategy is not a social optimum due to the presence of two market failures in the urban economy: congestion and misallocation of jobs within the city. A Pigouvian tax on commuters cannot solve two different problems simultaneously, namely, reducing the congestion level given the locations of jobs and reaching the optimal spatial allocation of firms. Without regulation, the number of jobs in the CBD is too high (and the congestion cost is excessive), while the Pigouvian tax generates a CBD that is too small. In addition, a flat tax is not necessarily worse than a Pigouvian tax, in contrast to the cordon toll.

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KEYWORDS

Polycentric city | Second-best policies | Congestion | Welfare | Urban land use

Executive summary

This article aims to measure the efficiency of different road pricing schemes (Pigouvian tax, flat tax, and cordon toll) to address congestion externality when the locations of jobs and dwellings are endogenous within a polycentric city. The size of the Central Business District (CBD) and the Secondary Business Districts (SBDs) are determined at both the equilibrium ("Laissez-faire") and the optimum (a benevolent planner maximizes the welfare of the inhabitants).
This study analyzes road pricing schemes' effects on congestion level, SBDs' size, and welfare. In addition, the impacts of a second transport mode introduced in the model are discussed. Urban forms and traffic volume in cities with resulting congestion externalities are often treated separately in the economic literature. Adding these features affects firm and household location choices within a municipality.

Since Solow's seminal articles (1972, 1974)1, a body of literature has focused on the optimal supply of road capacity in cities, using the monocentric model including road congestion. More recently, a few authors have considered the polycentric city to analyze the impacts of urban congestion on wages, land rents, social welfare, business and household location choices (Anas and Kim, 1996; Anas, 2013; Zhang and Kockelman, 2016a, 2016b)2.
This literature is scarce and often does not provide complete analytical solutions. Numerical simulations are used to solve complex models, including a labor market, production firms in cities, households, other firms and shopping centers whose location is endogenous with congestion and agglomeration externalities. However, these works do not precisely emphasize how the size of the main or secondary center varies according to road tolls.

The Pigouvian tax strategy does not achieve the optimum. This can be explained by two market failures: congestion and the misallocation of jobs in the city. When firms relocate to an SBD, they do not consider the induced congestion. Without regulation, the number of jobs in the CBD is too high (and the congestion cost is excessive), while the Pigouvian tax generates a CBD that is too small. In addition, a flat tax is not necessarily worse than a Pigouvian tax, in contrast to the cordon toll. When the size of the SBD is too small (too large), a flat-rate tax may be less (more) damaging than a cordon toll. In addition, road pricing scheme efficiency depends primarily on the wage differential between business centers.
Politically, the implementation of congestion charging leads to public outcry. As a result, it is little used worldwide. Public transport utilizing a city's existing road capacity can increase commuters' transport costs but can also alter the wage premium in a major center. There may be a perverse effect such as forcing households to change their place of work, thus improving the attractiveness of the secondary business center where car use predominates, which would also cause urban sprawl. Further research is needed to analyze urban density and congestion policies in polycentric cities, including long-term effects.

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[2] Zhang, W., and Kockelman, K. M. (2016a). Congestion pricing effects on firm and household location choices in monocentric and polycentric cities. Regional Science and Urban Economics. 58: 1–12; Zhang, W., and Kockelman, K. M. (2016b). Optimal policies in cities with congestion and agglomeration externalities: Congestion tolls, labor subsidies, and place-based strategies. Journal of Urban Economics. 95: 64–86; Anas, A. (2013). The Location Effects of Alternative Road-Pricing Policies, Chapter 6 in Infrastructure and Land Policies, eds. Ingram, Gregory K., and Karin L. Brandt. Cambridge, MA: Lincoln Institute of Land Policy; Anas, A., and Kim, I. (1996). General equilibrium models of polycentric urban land use with endogenous congestion and job agglomeration. Journal of Urban Economics. 40:232–256.

1 Introduction

Urban growth leads to serious traffic congestion in cities worldwide. Emerging economies need to set up some efficient urban policies for congestion management. In the United States, Los Angeles was the most congested city in 2016, and commuters lost 104 hours that year due to their home-to-work travel (Cookson and Pishue, 2017). In Thailand, commuters lost 61 hours over a year on average in traffic due to congestion in urban areas. Long travel time delays are also measured in Colombia, Russia and Indonesia according to a recent study by Cookson and Pishue (2017). To improve travel time reliability, road pricing policies, such as the *cordon toll*, have been implemented in a few cities around the world (Small and Gómez-Ibáñez, 1997). An urban toll aims to regulate traffic demand within some urban area or over a portion of a road during certain periods. Since 1975, Singapore has charged commuters entering the city center a *cordon toll* to improve traffic flow. Indeed, traffic volume dropped by 45% between 1975 and 1991 (Santos, 2005).

Including a congestion externality in an urban model may reduce the benefits of agglomeration economies. Firms have incentives to be close to each other, hence generating increasing returns and agglomeration economies (external and internal). However, gathering firms in either a Central Business District (CBD) or Secondary Business Districts (SBDs) leads to high land rents at these locations and high commuting costs for workers (Fujita and Thisse, 2013). Over the long run, high home-to-work costs may induce adjustments of job and household locations, which requires close scrutiny. The literature has studied the impacts of congestion on urban density and land use using standard monocentric city models since the seminal work of Solow (1972). He is one of the few authors that incorporated both land use for road infrastructure and congestion into an urban economic model to determine the optimal allocation of land for road infrastructure[1]. Several studies have used the standard but empirically questionable monocentric model for convenience (Anas and Kim, 1996). Land in the CBD can be exclusively allotted to firms (Wheaton, 1998; Brueckner, 2007; Larson and Yezer, 2015) or to mixed use (Anas and Kim, 1996; Fujita and Ogawa, 1982). Arnott (1979) developed a theoretical model with congestion without internalizing this externality, unlike Solow (1973, 1972). He extended Solow's work on the relationship between private land value and social land value in both residential and road use. The decision to internalize congestion is at issue, as noted by Arnott's works (1979, 2007). As workers respond to a pricing scheme over the long run, complete internalization of the negative externality imposed on other urban dwellers may not be efficient when a second market failure exists (Tikoudis *et al.*, 2015).

The objective of this paper is to understand the evolution of urban congestion and welfare by relocating jobs in SBDs. This article assesses which urban land use regulations and road pricing schemes improve the welfare of the whole city. Three different taxes on commuters are evaluated: a *Pigouvian tax*, a *flat tax* and a *cordon toll*. The urban model is a polycentric city with two externalities: (i) positive agglomeration economies yielding a wage premium in the CBD compared to the SBDs and (ii) a negative congestion externality due to home-to-work commutes. Work and residential places are interdependent in household location choices.Several authors have examined road pricing schemes effects using standard monocentric models (Wheaton, 1998; Brueckner, 2007; Tikoudis *et al.*, 2015), whereas only a few have investigated the same effects in polycentric cities (Zhang and Kockelman, 2016a). This approach enables us to reflect the tendency of developed cities (e.g Los Angeles, Paris, Boston) to evolve toward decentralized and non-monocentric forms.

[1]Strotz (1965) was the first to study the optimal provision of road facilities using a monocentric model with congestion.

Therefore, I can discuss the efficiency effects and other impacts of urban policies (e.g., taxes and redistribution) in relation to previous work on the monocentric case. In addition, the decentralization of jobs within a city may reduce the average commuting distance and thus reduce traffic congestion for each commuter compared to a monocentric city. This study has similarities with Zhang and Kockelman (2016b). They evaluate different urban policies and measure their impacts on job decentralization, population density and firm distribution. However, they do not focus on the optimal allocation of jobs within a polycentric city in relation to road pricing schemes. I provide analytical solutions regarding the no-toll equilibrium, as well as the optimal and second-best sizes of SBDs.

I find that the *Pigouvian tax* on commuters is not optimal. This can be explained by the presence of two market failures in the urban economy: congestion and job misallocation within the city. This tax cannot kill two birds with one stone, namely, reducing the congestion externality for given workplace locations and yielding the optimal spatial allocation of firms. This result highlights firms do not take social costs of congestion into account when they decentralize jobs in the outskirts yielding too large SBDs. In the case of no-toll equilibrium, the CBD (the residential area where workers live) is larger than optimal (i.e., SBDs are too small). A large proportion of workers is eager to commute to the city center due to the CBD's wage premium compared to the SBDs.

For a given city size, the *Pigouvian tax* on commuters makes the CBD too small in a polycentric city. In other words, the SBD expands and increases the congestion externality of each road user around the workplace. This second-best policy overcorrects the congestion externality because firms do not take the effects of jobs decentralization on congestion delay into account. This is due to (i) the free location decisions of workers and (ii) endogenous workplaces location. A *flat tax* and a *cordon toll* do not achieve an optimal location for the SBD. This article recaptures the effects of road pricing schemes that have been demonstrated in the literature on monocentric city. Tikoudis *et al.* (2015) use numerical simulations in a different context and include a labor tax in their model with road pricing schemes to study the tax interaction effects. Their results clarify that a road toll is necessary and welfare improving when no distortions exist in the labor market. Tikoudis *et al.* (2015) and Verhoef (2005) agree that a *flat kilometer tax* is more efficient than a *cordon toll* in a monocentric structure. However, when a polycentric structure emerges, this result does not hold, as the efficiency of the *cordon toll* or the *flat tax* depends mainly on the wage gap between the business centers before the implementation of the road pricing scheme. Based on the initial location of the SBD (a SBD that is too small or too large), a *flat tax* may be less (resp., more) harmful than a *cordon toll.*

Related literature Urban forms and traffic volume in cities with resulting congestion externalities are treated separately. Adding these features affects firm and household location choices within a city. Two bodies of literature address both urban and transport issues. First, the standard urban model (i.e., the monocentric city) has been used since Solow (1972) to address road land use and traffic congestion. One or more urban policies were evaluated in the case of one (Arnott, 1979; Wheaton, 1998) or two (Wheaton, 2004; Tikoudis *et al.*, 2015) externalities in a monocentric city. Wheaton (1998) focused on urban form evolution when the congestion externality is correctly internalized, examining the impacts on resident density and transport capacity in a monocentric circular city. Wheaton (2004) and Arnott (2007) were interested in the interplay between congestion and agglomeration externalities, and Tikoudis *et al.* (2015) questioned the impacts of various second-best road tolls on the labor market and welfare in the long run in order to observe households' decisions to adjust their labor supply and commuting distances.

Second, congestion management has been considered more recently for the polycentric city. Anas and Kim (1996) studied the impacts of congestion on urban structure in a model of a linear, "narrow" city with a link-node road network and households, firms and shopping centers with endogenous locations. In more recent studies, Anas (2013) and Zhang and Kockelman (2016a) analyzed the effects of road pricing schemes on workplace and residential locations as well as on wages, rents, housing prices and land development. Anas (2013) provided some insights for the city of Chicago, whereas Zhang and Kockelman (2016b) considered a general equilibrium model including both agglomeration and congestion externalities with labor market and land use patterns. Extension of the analysis of the impacts of congestion pricing policies on land use, rents and firm locations to a polycentric structure is a recent development in urban economics, as illustrated by Zhang and Kockelman (2016a). However, the existing literature has not fully addressed the impacts of road pricing policies on the size of the CBD when two externalities interplay. This study provides new insights and thus helps enrich knowledge in this stream of literature.

The article is organized as follows. Section 2 describes model's assumptions. Then, I briefly focus on unpriced congestion effects in a polycentric model without wage gap between business centers. I present a configuration in which a polycentric city hosts homogeneous households in the city with a wage gap between business centers in order to find the equilibrium and optimal size of SBDs. Section 4 evaluates tolling schemes efficiency in terms of congestion management and allocation of jobs. In section 5 one simulation is run with a set of fixed parameter values to confirm or reverse our analytical results. Section 6 discuss our results by extending the model to include two transport modes in particular. Section 7 concludes.

2 The model

The urban economy designed here builds on the basic model from Denant-Boèmont *et al.* (2018) and Cavailhès *et al.* (2007). Only one closed-form linear city is built with a fixed population $L > 0$. While Gaigné *et al.* (2012) analyzed an urban system and the carbon footprints of both firms and households in different city structures, this model copes with congestion and a positive agglomeration externality within a single city, especially a polycentric structure. Locations of SBDs are determined endogenously in the polycentric city.

2.1 The city

Consider a city endowed with homogeneous workers who are free to choose their residential location and workplace. The city is star-shaped with m radial travel corridors along which residents live, and transportation takes place. The CBD is a point at $x = 0$. Firms are located either in the CBD or in an SBD and do not occupy physical space. Households and firms do not compete for land. Distances and locations to the CBD are expressed by the same variable x measured from 0. Individuals travel only for commuting purposes. They use a single one-way road that ends at the location of their respective workplace. No wasteful commuting occurs in equilibrium. Each worker decides to locate as close to her workplace as possible because spatial mismatches would not maximize her utility. The location of a SBD z_S along each spoke is determined endogenously (the city is assumed to be symmetric around the CBD). The supply of housing floor space δ is constant per unit of distance from the CBD and normalized to 1. At each location x, a and $(1 - a)$ are the exogenous fractions of land devoted to residential purposes and road infrastructure, respectively. The job allocation within the city in relation to transport-related congestion stemming from the flow of commuters is the primary consideration. Therefore, the parameter a is exogenously given, leading to a particular land use pattern (Solow and Vickrey, 1971)[2]. Accordingly, the total housing space available is equal to yma, with city size y representing the radius limit from the CBD.

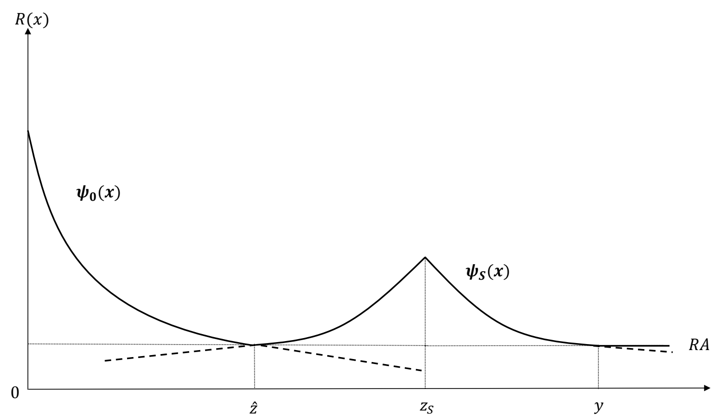

Figure 1: Bids rent near each business district and locations along one spoke of the CBD ($x = 0$) and a SBD ($x = z_S$) within a polycentric city configuration.

[2]The optimal allocation of road facilities has already been discussed in the literature (Strotz, 1965; Solow, 1973, 1972). I do not focus on the optimal land use allocation of residential and road capacities in the model, enabling me to disentangle the different effects at work.

2.2 Households

Only one class of worker is considered in our model. They earn income ω_i with $i = C$ when they work in the CBD or $i = S$ in a SBD. These households have the same preferences and the same utility function, which depends on two consumption goods: land, which is used as a proxy for housing, and the numéraire given by:

$$U(q, h), \tag{1}$$

where q is the consumption of the numéraire, and h is the consumption of housing floor space. Housing demand is assumed to be constant and normalized to 1. As a consequence, the residential density is constant per unit of distance and does not replicate the widely demonstrated fact that population density is decreasing with the distance from the CBD (Brueckner, 1987). Thus, the fixed city size y depends only on exogenous components because our main interest in this study is to observe the rise and fall of the relative share of jobs in the CBD. In addition, the traditional trade-off between low/high land rents and long/short journey to work is respected, as in numerous urban economic models with fixed lot sizes (Lucas and Rossi-Hansberg, 2002; Gaigné *et al.*, 2016)[3]. Each household reaches a common utility level $\bar{U}$ within the city in equilibrium. The time constraint of a worker located at x is given by:

$$1 = T_L(x) + T(x), \tag{2}$$

where $T_L(x)$ is the amount of labor time, and $T(x)$ is the commuting time from her residential location x to the business district's location z_i, with $z_i = 0$ (resp., $z_i = z_S$) if her job is located in the CBD (resp., SBD). Hence, $T(x) > 0$ is the endogenous time spent commuting per unit of distance. It depends on the congestion imposed by other travelers and the transport mode[4]. The budget constraint of a worker located at x can be written as follows:

$$\omega_i(1 - T(x)) + \bar{G} + \bar{R} = q + \frac{R(x)}{a}h + t_0 x + \tau_k(x), \tag{3}$$

where $k =$ *Pigouvian tax*, *flat tax* or *cordon toll*, $t_0 x$ represents the total pecuniary costs of transportation between the workplace and the residence, and $R(x)$ is the land rent at x. $R(x)/a$ is the price paid by a consumer to reside at x. Transport costs contain a fixed component t_0 reflecting fuel, insurance and average maintenance costs for using a car. Thus, pecuniary costs of using a car differ only with the distances traveled by workers. In absence of road pricing schemes, no transfer is received by workers (i.e., it is a competitive market). When a road pricing scheme is implemented, tax revenues and aggregated land rents ALR are returned via a lump sum with $\bar{G}$ and $\bar{R}$:

$$\bar{G} = \frac{G_k}{L}, \tag{4}$$

and:

$$G_k = ma \int_0^y \tau_k(x) \mathrm{d}x,$$

[3]Including an endogenous housing demand would increase mechanically the city size while the share of land use devoted to housing is fixed. Our main objective is to determine the effects of three road pricing schemes disregarding the long-run changes of the size of the city.

[4]In this model, time allocated to sleep and leisure is not taken into account. A model with fixed working times and endogenous leisure time would not qualitatively change our results. A case study with two transport modes is discussed at the end.

where G_k is the total amount of tax collected. They receive the second amount given by:

$$\bar{R} = \frac{ALR}{L},$$

where:

$$ALR = m \int_0^y R(x)\mathrm{d}x \tag{5}$$

is the aggregate land rent at the whole city. This approach is in line with previous work by Zhang and Kockelman (2016a) and Tikoudis *et al.* (2015). I implicitly assume that a benevolent planner levies a tax on off-farm land rents and redistributes it to residents. Otherwise, households incur a welfare loss due to a change in aggregate land rents when policies are implemented in this closed-city model (Solow, 1973; Parry and Bento, 2001).

2.3 Congestion costs and transport infrastructure

A stationary-state congestion model is implemented in the urban economic model (see Small *et al.*, 2007). All workers take a single road to commute. They face no costs to enter the road, which has no distinctive features such as traffic signals or stops. All homogeneous users are assumed to drive a car with the same characteristics. They face a travel time cost that depends on the number of users on the road at any point and the fraction of land devoted to roads. The road's length corresponds to the city size y. Travel time is increasing with the number of commuters on the road. Households commute to the edge of the CBD and/or of the SBDs according to their residential location. Here, $f(x)$ denotes the cumulative number of travelers using the single road who live beyond the distance x. The travel time per unit of distance at the portion x has the following form[5]:

$$\tau_0 + \tau_1 f(x)^{\beta} \quad \text{with} \quad \beta = 1 \tag{6}$$

$$f(x) = \begin{cases} \int_x^{\widehat{z}} \frac{a}{h(1-a)}\mathrm{d}z, & \text{if} \quad x < \widehat{z} \\ \int_x^{y} \frac{a}{h(1-a)}\mathrm{d}z, & \text{if} \quad x > z_S \\ \int_x^{z_S} \frac{a}{h(1-a)}\mathrm{d}z, & \text{if} \quad \widehat{z} < x < z_S \\ 0, & \text{if} \quad x = y. \end{cases} \tag{7}$$

The free-flow travel time is equal to τ_0, and the second term includes the time delay at x induced by the cumulative number of road users living beyond x. This means that the average speed decreases when traffic density increases; namely, there is pure flow congestion. τ_1 is a sensitivity parameter multiplied by the aggregate traffic flow $f(x)$ arriving at a location x along the road. The magnitude of β is widely discussed in the literature (Small, 1992; Arnott *et al.*, 2005) but no consensus has emerged. Arnott (2007) notes that empirical estimates are close to 1.0 when long roads are considered. This travel time function yields a specific traffic congestion at the segment x. The travel demand of drivers living before x is not included in this function. Nevertheless, the traffic slowdown has to be considered when they take the road. Furthermore, each commuter living in x imposes a travel delay on other road users living before and beyond x. In turn, these road users cause congestion externalities incurred by each commuter living in x. Thus, this travel time is integrated from the workplace location (the destination) to the trip

[5]see Small *et al.* (2007), Arnott (2007) and Tikoudis *et al.* (2015)

origin (the residential place) to consider an aggregate congestion externality. Calculations lead to the total commuting time of an individual living at distance x, which is expressed as follows:

$$T(x) = \begin{cases} \displaystyle\int_0^x [\tau_0 + \tau_1 \textstyle\int_x^{\widehat{z}} \frac{a}{h(1-a)} \mathrm{d}z] \mathrm{d}x, & \text{when} \quad 0 < x < \widehat{z} \\ \displaystyle\int_{z_S}^x [\tau_0 + \tau_1 \textstyle\int_x^{y} \frac{a}{h(1-a)} \mathrm{d}z] \mathrm{d}x, & \text{when} \quad z_S < x < y \\ \displaystyle\int_{\widehat{z}}^x [\tau_0 + \tau_1 \textstyle\int_x^{z_S} \frac{a}{h(1-a)} \mathrm{d}z] \mathrm{d}x, & \text{when} \quad \widehat{z} < x < z_S. \end{cases} \tag{8}$$

It allows us to obtain the travel time per unit of distance near the resident's location. Then, the total travel time can be quantified to her workplace. Incurred congestion is measured at x, which leads to a travel delay along a worker's journey to her business district. As there exists spatial symmetry around the SBD, the second term of the equation can be used to measure congestion along the road section from $\widehat{z}$ to z_S. Then, the total commuting time and the time constraint (2) are inserted into the budget constraint (3) to obtain the indirect utility of a worker:

$$V_C(x) = \omega_C - \omega_C \left[x(\tau_0 + t_0) + \frac{\tau_1 a}{h(1-a)} [\widehat{z}x - \frac{x^2}{2}] \right] - \frac{R(x)h}{a} - \tau_k(x) + \bar{G} + \bar{R}, \tag{9}$$

when she lives in and commutes to the CBD and:

$$V_S(x) = \omega_S - \omega_S \left[(x - z_S)(\tau_0 + t_0) + \frac{\tau_1 a}{h(1-a)} [y(x - z_S) + \frac{z_S^2 - x^2}{2}] \right] - \frac{R(x)h}{a} - \tau_k(x) + \bar{G} + \bar{R}, \tag{10}$$

when she lives in and commutes to the SBD. In this competitive market framework, household that places the highest bid obtains housing at x, which is in line with Alonso (1964) and urban economic models.

2.4 Wages

In line with the models of Fujita and Ogawa (1982) and Lucas and Rossi-Hansberg (2002), firms produce a composite good. Their only production factor is labor. Each firm makes a profit denoted Π_i whether it produces in the CBD ($i = C$) or in a SBD ($i = S$):

$$\Pi_i = pq_i - \omega_i T_{Li} \tag{11}$$

where p is the output price, q_i is the output size with $q_i = \mathbf{A}_i T_{Li}^{\gamma}$, $\mathbf{A}_i \geq 1$ and $\gamma \leq 1$, and T_{Li} is the total labor time units[6]. $\mathbf{A}_i$ is a positive agglomeration externality depending on the business centers (CBD and SBDs). This term affects productivity as a positive multiplier. Firms clustered in the CBD take advantage of a more efficient environment that takes concrete form as a productivity drop $\mathbf{A}_C > \mathbf{A}_S = 1$[7]. $\mathbf{A}_C$ is increasing with positive agglomeration economies in the CBD. Wages are fixed by firms. The maximization of (11) with respect to labor time T_{Li} implies the following labor demand:

$$p\mathbf{A}_i \gamma T_{Li}^{\gamma-1} = \omega_i \tag{12}$$

[6] I assume that the marginal productivity of labor is non-increasing in line with Lucas and Rossi-Hansberg (2002).

[7] Agglomeration economies exist in CBDs, but thanks to new information and communication technologies, decentralization of jobs occurs within the city (Baum-Snow and Pavan, 2012).

so that $\Pi_i = (1-\gamma)p\mathbf{A}_i T_{Li}^{\gamma} = (1-\gamma)[p\mathbf{A}_i]^{\frac{1}{1-\gamma}}[\gamma/\omega_C]^{\frac{\gamma}{1-\gamma}}$, where I have inserted (12) into (11). In equilibrium, $\Pi_C = \Pi_S$ leads to:

$$\omega_C = \mathbf{A}_C^{\frac{1}{\gamma}} \omega_S$$

Hence, $\mu = \omega_S/\omega_C = \mathbf{A}_C^{-1/\gamma}$. As a consequence, the wage gap between business centers depends on the magnitude of agglomeration economies $\mathbf{A}_C$ and diseconomies of scale γ.

2.5 Welfare

Finally, the total welfare is calculated. It represents the aggregate indirect utility within the closed city[8]. The sum of the indirect utilities of tenant workers, the land rent incomes of absentee landlords are an integral part of this welfare. Under a no-toll equilibrium, welfare is expressed as follows:

$$\begin{aligned} W^* &= \frac{ma}{h}\left[\int_0^y \omega_i\left(1-T(x)\right)\mathrm{d}x - \int_0^y t_0 x \mathrm{d}x\right] - ALR \\ &+ \left[ALR - m\int_0^y R_A \mathrm{d}x\right] + m\int_0^y R_A \mathrm{d}x \end{aligned}$$

with:

$$ALR = m\int_0^y R(x)\mathrm{d}x$$

Land rent incomes are the aggregate land rents paid by tenant workers minus agricultural land rents R_A redistributed to landlords living outside the city. Finally, agricultural landowners benefit from agricultural land rent incomes. The purpose of this article is to analyze the impacts of the road pricing schemes on welfare and on the city structure. Furthermore, one of the main goals is to determine the conditions under which aggregate welfare is maximized within the closed city.

[8]The first part of the aggregate welfare is the difference between aggregate incomes and aggregate congestion, commuting and housing costs in a city (Wheaton, 2004). The second part is the distribution of rents to absentee landlords.

3 Decentralization of jobs and welfare

Traffic congestion and road pricing schemes are analyzed under a polycentric setting. Jobs locate in a CBD and in SBDs within the city and wages are fixed. Our main objective is to understand the evolution of urban congestion and welfare by relocating jobs and introducing different taxes.

3.1 The polycentric city

A worker deciding to locate close to the CBD earns a gross wage rate ω_C above the wage ω_S offered by firms in the SBDs. No worker has an incentive to move from her workplace or residence in the spatial equilibrium. All households live at each location x such that $\partial V(x)/\partial x = 0$. In this spatial organization, $\widehat{z}$ is the right limit of the area formed by workers commuting to the CBD (i.e., the left limit of the built-up area constituted by individuals working in the SBD). The location of a SBD is determined endogenously through the location of $\widehat{z}$. Individuals locate around each SBD symmetrically. Hence, the endogenous location of a SBD z_S is the midpoint of the area between $\widehat{z}$ and the city limit y. Therefore:

$$z_S = \frac{y + \widehat{z}}{2} \tag{13}$$

Notice that bids rent at y and $\widehat{z}$ are assumed to be equal to the opportunity costs of land R_A. The equilibrium border when the city is polycentric reaches:

$$y = \frac{Lh}{am} \tag{14}$$

Note that the city size y does not change when $\widehat{z} = y$ (i.e., a monocentric configuration). As housing demand remains fixed, the structural density is the same, and y is unchanged. When jobs relocate to an SBD, the size of the CBD decreases from y to $\widehat{z} < y$; hence, workers incur lower transport costs (in terms of both time and money) (see Appendix B).

Congestion delay and traffic flow Regarding traffic flows toward a SBD, the total commuting time of an individual living at location x between a SBD z_S and before a border y is expressed as follows:

$$T(x) = (x - z_S)\tau_0 + \frac{\tau_1 a}{h(1-a)}[y(x - z_S) + \frac{z_S^2 - x^2}{2}] \geq (x - z_S)\tau_0 \tag{15}$$

Notice that when z_S rises, the total commuting time to this location decreases because the CBD limit expands leading to shrinkage of SBD area. Note that our congestion delay cost function is increasing at a decreasing rate. The marginal travel time cost is higher towards the SBD than outwards from the SBD. Accordingly, the opportunity cost of the commuting time is higher near the secondary center than in the outskirts. An individual living near the SBD will have a lower commuting time due to an infinitesimal move $(x - dx)$ than that of an individual living near the city border. Living near the border implies high transportation costs. Congestion delay costs increase with x, but land rents are cheaper in the outskirts. The effect of a on the congestion externality is unclear. Indeed, when a increases, the city limits shrink, which implies a decrease in traveled distances. However, the road's capacity $(1 - a)$ falls, leading to an ambiguous result for a given population density.

As there is symmetry around an SBD, travel distances and congestion delay are the same only if the two locations x (i.e., between $\widehat{z}$ and z_S or between z_S and y) are equidistant from z_S (see equation 15). Therefore, the subsequent analytical properties focus on workers living at the right side of the SBD. Commuters use one single mode of transport to travel, and they obtain the same level of utility within the city. A worker living in the area between the CBD and $\widehat{z}$ has no economic incentives to work in a SBD. No wasteful commuting occurs in equilibrium. Each traffic flow towards the SBD starts from $\widehat{z}$ and y. This means that congestion costs at these points may be lower than in a monocentric configuration (*i.e.*, when $\widehat{z} = y$)[9].

SBD Size As there are two business districts, a wage gap ($0 < \mu < 1$) exists between them in accordance with empirical evidence (see White, 1999; Timothy and Wheaton, 2001). Furthermore, a worker commuting to the SBD may incur a lower transport cost than one commuting to the CBD. This gives the worker an incentive to work in a subcenter of the city. The size of the CBD $\widehat{z}$ is determined by an indifference condition. For a worker living at $\widehat{z}$, she is indifferent between commuting to the CBD and commuting to the SBD due to the same utility level, yielding the following expression:

$$\omega_C \left[1 - (\widehat{z}\tau_0 + \frac{\tau_1 a \widehat{z}^2}{2h(1-a)})\right] - t_0 \widehat{z} = \omega_S \left[1 - (z_S - \widehat{z})\tau_0 - \frac{\tau_1 a (z_S - \widehat{z})^2}{2h(1-a)}\right] - t_0(z_S - \widehat{z}) \qquad (16)$$

The size of the CBD $\widehat{z}$ is smaller than the city limits y in this model. Individuals accept work from firms in SBDs when the gross wage ω_S offered is strictly above the CBD's wage net of total transport costs (i.e., the opportunity cost of commuting, congestion delays and pecuniary costs) in a monocentric city. Employers benefit from this relocation of jobs within a city because they pay lower wages. Workers face shorter commutes and pay lower land rents. Whatever the gross wage ω_C offered by firms in the CBD, the decentralization of jobs holds for a certain level of ω_S. As a consequence, the free market equilibrium yields a non-linear expression of the CBD border $\widehat{z}(\mu)$. For greater clarity, τ_0 is assumed to be equal to zero[10]. Hence:

$$\widehat{z}^*(\mu) = \frac{-3t_0(1-a) - \frac{\mu\omega_C\tau_1 L}{2m} + 2(1-a)\sqrt{\Delta}}{\omega_C(2-\mu/2)\tau_1 a} \qquad (17)$$

with:

$$\Delta \equiv \left[\frac{3t_0}{2} + \frac{\mu\omega_C\tau_1 L}{4m(1-a)}\right]^2 + \frac{\omega_C(4-\mu)\tau_1 L}{2m(1-a)}\left[\frac{\omega_C(1-\mu)ma}{L} + \frac{t_0}{2} + \frac{\mu\omega_C\tau_1 L}{8m(1-a)}\right] > 0$$

in which (13) and (14) have been inserted. $\widehat{z}$ is implicitly defined by the indifference condition (16). The influence area of the SBD (the number of individuals working in the SBD) rises with μ

[9] A commuter incurs a maximum travel time delay $\frac{\tau_1 a y^2}{2h(1-a)}$ at y when one single business center exists. If SBDs exist, each worker enters a road where the maximum time delay falls to $\frac{\tau_1 a \widehat{z}^2}{2h(1-a)}$ ($\widehat{z} < y$). Trivial calculations show that the congestion externality decreases for each commuter (see Appendix B). The maximum traveled distance diminishes, and the traffic flow is split between the business districts. Therefore, for a given density, each worker is better off when firms locate in several business districts. *Ceteris paribus*, congestion costs will rise for inhabitants working in the CBD regardless of whether $\widehat{z}$ or y increases.

[10] Calculations for $\widehat{z}(\mu)$ without this assumption are reported in Appendix $A2$.

and reaches a maximum when $\mu = 1$. The attractiveness of a peripheral business district depends on the average gross wage observed within this SBD. Each worker considers her wage net of total transport and residential costs. For a given city size, a polycentric structure with a prominent business center enables shorter home-to-work distances and reduces pressure on land rents at and close to the CBD. Therefore, transport and rent costs decrease for each commuter. Urban traffic is divided into three flows, and no cross-border commuting between the business districts occurs in equilibrium. As a consequence, all commuters in the city face fewer congestion delays than they would in a monocentric one, but they do not benefit from the most efficient travel time.

The equilibrium land rent at each location is given by: $R(x) = \max\{\Psi_C(x), \Psi_S(x), R_A\}$. $\Psi_C(x)$ (resp., $\Psi_S(x)$) is the bid rent of individuals working in the CBD (resp., SBD). $\partial V(x)/\partial x = 0$ implies:

$$\Psi_C(x) = \frac{a\left[2h(1-a)(\omega_C\tau_0 + t_0)(\widehat{z}^*(\mu) - x) + a\omega_C\tau_1(\widehat{z}^*(\mu) - x)^2\right]}{2h^2(1-a)} + R_A$$

and:

$$\Psi_S(x) = \frac{a\left[2h(1-a)(\omega_C\tau_0 + t_0)(\frac{y-\widehat{z}^*(\mu)}{2} - |z_S - x|) + a\omega_C\tau_1(\frac{y-\widehat{z}^*(\mu)}{2} - |z_S - x|)^2\right]}{2h^2(1-a)} + R_A$$

Bids rent decrease with the distance to the business districts.

3.2 Equilibrium allocation and optimal location of jobs

The no-toll equilibrium is used as a reference to compare optimal and equilibrium locations of the SBD. In equilibrium, the welfare (i.e., the aggregate indirect utilities) of the polycentric city is expressed as follows:

$$W_S(\widehat{z}^*) = \frac{ma}{h}\left[\omega_C\widehat{z}^*(1-\mu) + \mu\omega_C y - \frac{(\mu\omega_C\tau_0 + t_0)\left(y^2 - \widehat{z}^{*2}\right)}{2} - \frac{(\omega_C\tau_0 + t_0)\widehat{z}^{*2}}{2} - \frac{\omega_C\tau_1 a\left[4\widehat{z}^{*3} + \mu(y - \widehat{z}^*)^3\right]}{12h(1-a)}\right]$$

Hence, inserting equation (17) gives us the welfare in equilibrium (Appendix $A3$). Notice that for a value of μ denoted $\underline{\mu}$, the urban economy recaptures a monocentric setting (M), as $\widehat{z}^*(\underline{\mu}) = y$ and $W_S(\underline{\mu}) = W_M$ (see Appendix B). Because the size of a SBD depends on the wage gap between the business districts, it is crucial to analyze the effects of jobs decentralization on welfare according to the level of (μ). Indeed, we have:

$$W_S(\widehat{z}^*) - W_M \quad = \quad \frac{ma}{h}\left[\int_0^{\widehat{z}^*} V_C \quad \mathrm{d}x + 2\int_{z_S}^{y} V_S \quad \mathrm{d}x - \int_0^{y_M} V_M \quad \mathrm{d}x\right] + ALR - ALR_M$$

Job relocation to a SBD has an ambiguous effect on welfare. The effect depends strongly on the magnitude of the wage differential between business districts. Note that in equilibrium without taxation, the welfare of inhabitants working in the CBD improves when the urban economy shifts from a monocentric to a polycentric configuration. The relationship is more complex for inhabitants living near the SBD. On the one hand, they face both shorter travel distances and less congestion. Thus, their opportunity costs of commuting decrease because their income ω_S is smaller than ω_C. On the other hand, they pay the same rent R_A, and the difference in individual utility depends strongly on the wage gap. Indeed, the wage gap is positive for polycentric city's inhabitants. However, the economic shift has no incentive if the wage discrepancy offsets the opportunity costs of commuting and congestion delays. To understand how the wage gap and commuting time costs affect aggregate welfare, the analysis considers the case where the wage rates observed in the CBD and the SBDs are equal. Then, a more realistic case is analyzed where workers earn a higher hourly wage in the CBD than in the SBDs. For each case, a free market equilibrium is solved, and aggregate welfare is maximized by a benevolent planner who sets the optimal size of the SBD.

Specific case with no wage gap between the CBD and the SBDs First, a simple way to disentangle the different effects at work is to equalize the wage rates of the business centers: $\omega_C = \omega_S(\mu = 1)$. Workers living near the CBD (resp., the SBD) face a maximum distance equal to $\widehat{z}$ (resp., $\frac{y-\widehat{z}}{2}$), and traffic is divided into three flows. Hence, there is lower congestion on the roads when jobs relocate to an SBD within a closed city. In equilibrium, inserting $\mu = 1$ into (17) leads to $\widehat{z}^* = \frac{y}{3}$, so the free market size of the residential zone near the SBD is equal to $(y - \widehat{z}^*) = \frac{2y}{3}$. This result is identical to the optimal size of the left endpoint of the SBD ($\widehat{z}^{**}$) when welfare is maximized by a benevolent planner[11]. As a result, it leads to:

$$\widehat{z}^* = \widehat{z}^O = \frac{y}{3} \tag{18}$$

The free market solution is identical to the optimal location of the CBD limit when the congestion externality is not internalized, provided that no wage gap exists between the business districts. Furthermore, it is straightforward to check that welfare in a polycentric city ($W_S(\widehat{z}^O)$) with an equal wage rate is higher than that in a monocentric setting (W_M) (Appendix $A4$). It follows that each parameter that marginally increases the population density (m, a) yields a higher differential in favor of the polycentric city, all things being equal. The free market equilibrium and optimal solution of $\widehat{z}$ yield identical welfare, which is better than that for a monocentric structure. Note that welfare in a polycentric city is decreasing with respect to $\widehat{z}$[12]. In other words, welfare is increasing as the size of the CBD decreases in favor of the SBD until $\widehat{z}$ reaches $\frac{y}{3}$. Assume that there is no wage gap ($A_C = 1$) and that welfare in equilibrium is maximized when $\widehat{z}^* = \frac{y}{3}$ when a share of firms relocates to an SBD. In this case, the free market size is equal to the optimal size (see Appendix $A4$). As a consequence, jobs relocation within a closed city makes workers better off than they would be in a monocentric setting. It is straightforward to check that $\Delta_W > 0$:

$$\Delta_W \equiv W_S(\widehat{z}) - W_M = \frac{m\omega_C \tau_1^2 a^2 (y - \widehat{z})(y + \widehat{z})^2}{4h^2(1-a)} > 0 \tag{19}$$

For all $\widehat{z} < y$, the polycentric city is welfare improving, since the maximum distance for home-to-work commutes is shorter than y[13]. In addition, workers incur a lower level of congestion for each commuting trip when the city size y is unchanged. Thus, the efficient commuting pattern (the

[11] $\frac{\partial W_S(\widehat{z})}{\partial \widehat{z}} = 0$ leads to a single maximum for $W_S(\widehat{z})$ when $\widehat{z} \in [0, y]$, which reaches $\widehat{z}^O = \frac{y}{3}$. The details of these calculations are reported in Appendix $A4$.

[12] We have $W_S(\widehat{z} = 0) > W_S(\widehat{z} = y))$ and $\frac{\partial W_S}{\partial \widehat{z}}\big|_{\widehat{z}=0} > 0 > \frac{\partial W_S}{\partial \widehat{z}}\big|_{\widehat{z}=y}$. In addition, $\frac{\partial W_S(\widehat{z})}{\partial \widehat{z}}$ has a single extremum when $\widehat{z} \in [0, y]$, which is a maximum.

[13] Note that when no CBD exists (i.e., $\widehat{z} = 0$), the SBD is located in the middle of the total residential area (i.e., $\frac{y}{2}$), as our model is symmetric around the CBD located at $x = 0$. This case is a duocentric city (two SBDs per line) where welfare is higher than in a monocentric city, even when the location of the left endpoint of the SBD does not

minimum home-to-work distance) is achieved when $\omega_C = \omega_S$ in comparison with the monocentric pattern[14].

General case with positive agglomeration externalities Now you return to the case where $\omega_S = \mu\omega_C$ with $(0 < \mu < 1)$. The expression of the free market CBD limit $\widehat{z}^*$ is highly non-linear and implicitly defined by the indifference condition for a worker living between the two business centers. $\widehat{z}^*$ is expressed as follows:

$$\omega_C\left[1-\widehat{z}^*\tau_0-\frac{\tau_1 a\widehat{z}^{*2}}{2h(1-a)}\right]-t_0\widehat{z}^*=\omega_S\left[1-(y-z_S)\tau_0-\frac{\tau_1 a(y-z_S)^2}{2h(1-a)}\right]-t_0(y-z_S) \qquad (20)$$

The size of the residential zone from which workers commute to the SBD $(y-\widehat{z}^*)$ depends mainly on the wage gap between the business centers. A wage rate growth in the CBD leads to its expansion at the expense of the size of the SBDs. Standard calculations show that the CBD's size is smaller with free-flow travel time τ_0, under the three congestion pricing schemes and when the border y decreases. Workers favor commuting to the SBD when the free-flow travel speed $1/\tau_0$ diminishes. Similarly, the CBD's size decreases when the slope (τ_1) of the congestion curve increases (see Appendix $A5$ for the details of the calculations). When the available floor space $(\delta > 1)$ per land unit grows, the CBD's size $\widehat{z}^*$ diminishes, and y decreases with the fraction of residential area a. Conversely, city size grows when the lot size h increases. Note that a marginal increase in a leads to a more compact city but lowers the road's capacity, yielding higher congestion delay costs for a given population density.

The determination of an optimal solution by a benevolent planner considers the maximization of aggregate welfare so that $\frac{\partial W_S(\widehat{z})}{\partial \widehat{z}} = 0$ yields a non-linear $\widehat{z}^O$ (see Appendix $A5$). In this case, the optimal solution and the free market equilibrium are not identical. When $\mu = 1$, you revert to the previous case where the optimal and free market sizes are equal $(\widehat{z}^O = \widehat{z}^*)$. Then, you determine whether the free market size of the area where individuals live near the CBD is too small or too large.Welfare in the polycentric city can be written as follows[15]:

$$W_S = \frac{ma}{h}\left[\int_0^{\widehat{z}} V_C \quad \mathrm{d}x + 2\int_{z_S}^{y} V_S \quad \mathrm{d}x\right] + ALR \qquad (21)$$

At the optimum, $\frac{\partial W_S}{\partial \widehat{z}}\big|_{\widehat{z}=\widehat{z}^O} = 0$ because welfare has reached its maximum. Furthermore, welfare W_S is a concave curve on the interval $[0, y]$ since $\frac{\partial^2 W_S}{\partial^2 \widehat{z}} < 0$. Hence, to find the location of the CBD limit in equilibrium relative to the optimum, welfare is derived with respect to $\widehat{z}$ conditional on $\widehat{z} = \widehat{z}^*$:

$$\frac{\partial W_S}{\partial \widehat{z}}\bigg|_{\widehat{z}=\widehat{z}^*} = \left[\frac{ma}{h}\left((y-\widehat{z}^*)\frac{\partial V_S}{\partial \widehat{z}} + \widehat{z}^*\frac{\partial V_C}{\partial \widehat{z}}\right) + \frac{\partial ALR}{\partial \widehat{z}}\bigg|_{\widehat{z}=\widehat{z}^*}\right] \quad < 0 \qquad (22)$$

yield an equilibrium or optimal solution. A particular polycentric pattern leads to the same level of welfare as the duocentric setting when $\widehat{z} = y\frac{\sqrt{5}-1}{2}$ over $[0, y]$. However, traffic is split into two flows, which insufficiently decreases the congestion delay for each commuter. Indeed, only one solution assures that each commuter can minimize her extra travel time (see Proposition 1).

[14] Note that the city is divided into three equal parts when $\omega_C = \omega_S$ with $\widehat{z} = \frac{y}{3}$, and the SBD's size $(y-\widehat{z})$ equals $\frac{2y}{3}$.

[15] When you differentiate W_S with respect to $\widehat{z}$, there are no price effect at the aggregate level since all land rents paid by tenant workers are received by absentee landlords.

As a result, the free market size of the CBD is greater than optimal. In other words, free market SBD area is smaller than the optimum under positive agglomeration and congestion externalities[16]. The intuition is as follows. When the decentralization of jobs occurs, land rents and congestion delays decrease, on average, as a share of workers relocates closer to a new SBD. As a consequence, workers who earned wage rate ω_C have an incentive to remain in the residential zone close to the CBD because their net wage has increased. Then, for a marginal increase of $\widehat{z}$, the total commuting time for a worker living near the limit of the residential zone is identical when she moves from x to $x + dx$[17]. Hence, with a higher wage rate offered in the CBD, most inhabitants commute to the CBD instead of a SBD without taking into account the marginal social costs they impose on other commuters. Accordingly, the size of the residential zone near the CBD is larger than the optimum. Regarding the optimal size, it is straightforward to check that $\frac{y}{3} < \widehat{z}^O < \widehat{z}^*$ as:

$$\left.\frac{\partial W_S}{\partial \widehat{z}}\right|_{\widehat{z}=\frac{y}{3}} > 0 \tag{23}$$

Briefly, the optimal size of the CBD is larger than $\widehat{z} = \frac{y}{3}$ and smaller than $\widehat{z}^*$.

Proposition 1. *A free market equilibrium yields a* ***CBD that is too large*** *when there are two market failures : the congestion and agglomeration externalities.*
Proof *See Appendix* A5.
The next section focuses on the spatial and economic impacts of internalizing the external costs of congestion according to three different road pricing schemes. Their efficiency is evaluated by comparing the tax-induced size of the CBD and the optimal CBD size.

4 Polycentric city and road pricing schemes

The free market equilibrium under a wage gap yields an inefficient outcome for a polycentric city. Here, our objective is to achieve the optimally sized SBD or to bring $\widehat{z}$ closer to that optimal size. Three road pricing schemes are evaluated separately. Each worker pays a tax that affects her income. The tax $\tau_k(x)$ is inserted into the budget constraint of each household, as well as into the revenue return scheme and aggregate land rent redistributions, leading to the following individual welfare:

$$V_S(x) = \omega_i(1 - T(x)) - \tau_k(x) + \bar{G} + \frac{ALR}{L} - \frac{R(x)h}{a} - t_0 x \quad \text{with} \quad i \in [C, S] \tag{24}$$

Land rent is given by $R(x) = \max\{\Psi_C(x), \Psi_S(x), R_A\}$. $\Psi_C(x)$ (resp., $\Psi_S(x)$) is the bid rent of individuals working in the CBD (resp., SBD). $\partial V(x)/\partial x = 0$ implies that:

$$\Psi_C(x) = \frac{a\left[2h(1-a)\left[(\omega_C\tau_0 + t_0)(\widehat{z} - x) + \tau_k(\widehat{z}) - \tau_k(x)\right] + a\omega_C\tau_1(\widehat{z} - x)^2\right]}{2h^2(1-a)} + R_A,$$

[16]Note that you have $\left.\frac{\partial W_S}{\partial \widehat{z}}\right|_{\widehat{z}=\widehat{z}^*} = 0$ when there are no market failures in our urban model. The size of the CBD in equilibrium is identical to the optimum.

[17]In our previous calculations, note that $\widehat{z}^*$ is larger than $(y - \widehat{z}^*)$, and for a longer distance traveled between the CBD and $\widehat{z}^*$, workers experience a marginal negative effect on their indirect utility, which has a larger magnitude than the positive effect experienced by those working in the small SBD. Hence, $\widehat{z}^* \frac{\partial V_C}{\partial \widehat{z}} > (y - \widehat{z}^*)\frac{\partial V_S}{\partial \widehat{z}}$.

and:

$$\Psi_S(x) = \frac{a\left[2h(1-a)\left[(\omega_S\tau_0+t_0)(\frac{y-\widehat{z}}{2}-|z_S-x|)+\tau_k(y)-\tau_k(|z_S-x|)\right]+a\omega_S\tau_1(\frac{y-\widehat{z}}{2}-|z_S-x|)^2\right]}{2h^2(1-a)}+R_A$$

Note that each pricing scheme affects the endogenous CBD size $\widehat{z}$, which is recalculated to fit the model. Welfare in the polycentric city with taxes can now be defined as follows:

$$\begin{aligned} W_S &= \frac{ma}{h}\left[\int_0^{\widehat{z}} V_C \quad \mathrm{d}x + 2\int_{z_S}^{y} V_S \quad \mathrm{d}x\right] + G_k \\ &+ \left[ALR - m\int_0^y R_A \mathrm{d}x\right] + m\int_0^y R_A \mathrm{d}x \end{aligned}$$

Each commuter pays a charge for using her car but benefits from the redistribution of tax revenues and aggregate land rents, since road pricing schemes are capitalized in land rents. Absentee and agricultural landlords continue to receive their revenues. The impacts of these second-best pricing schemes are examined and the results presented.

4.1 Urban toll

Three sets of policies are examined. They aim at tackling the congestion externality caused by each commuter.

The Pigouvian tax The *Pigouvian tax* ($\tau_k(x) = \tau_{PT}(x)$) prices the marginal external cost of congestion. Indeed, a benevolent planner decides to fix the location of a SBD in optimum while I implement this tax when $\widehat{z}$ is determined endogenously. This tax must adjust to correctly internalize the behavior of workers who are free to choose their residential locations. A priori, these free decisions would prevent this road pricing scheme from achieving the optimum. Each road user is priced at the exact marginal social cost she imposes on others for a given location and a given workplace. Commuters use the same road segment towards a common destination. This externality is priced by multiplying the additional travel time per unit of distance by the opportunity cost of working time $\left[\frac{\omega_i}{T_L+T(x)|x-z_i|}\right]$. The labor supply is inelastic within the city, and each worker can place a higher bid to move closer to her workplace with this gross value of time to avoid a travel time delay. The wage rate is divided by the total time endowment normalized to 1 uniformly. Hence, the toll is expressed as follows[18]:

$$\tau_{PT}(x) = \begin{cases} \int_0^x \frac{\omega_C\tau_1 a}{h(1-a)}(\widehat{z}-x), & \text{when} \quad 0 < x \le \widehat{z} \\ \int_{z_S}^x \frac{\omega_S\tau_1 a}{h(1-a)}(y-x), & \text{when} \quad z_S < x \le y \end{cases} \tag{25}$$

This is a location-constrained first-best policy, as workers are free to choose their residential location according to their bid rent[19]. The tax level for a commuter living in x and commuting to the CBD is $\frac{\omega_C\tau_1 a}{h(1-a)}[\widehat{z}x - x^2/2]$. Between $\widehat{z}$ and z_S, urban dwellers pay the same tax as those living

[18]The details of calculations are reported in Appendix C1.

[19]A first-best policy would fix the residential location of each household and the location of the SBD simultaneously.

between z_S and y because jobs located in z_S are in the middle of the residential area. Hence, using the tax level in the indifference condition for the location of the CBD limit $\widehat{z}$ yields[20]:

$$\omega_C\left[1-\widehat{z}\tau_0-\frac{\tau_1 a\widehat{z}^2}{h(1-a)}\right]-t_0\widehat{z}=\omega_S\left[1-(\frac{y-\widehat{z}}{2})\tau_0-\frac{\tau_1 a(\frac{y-\widehat{z}}{2})^2}{h(1-a)}\right]-t_0(\frac{y-\widehat{z}}{2}) \tag{26}$$

Then, you have the following welfare level:

$$W_{PT}(\tau_{PT})=\frac{ma}{h}\left[\int_0^{\widehat{z}}\omega_C\left[1-T(x)\right]-t_0(x)\mathrm{d}x+2\int_{z_S}^{y}\omega_S\left[1-T(x)\right]-t_0(x-z_S)\mathrm{d}x\right]$$

To find the location of the CBD limit $\widehat{z}_{PT}$, you derive (18) with respect to $\widehat{z}$ knowing that $\widehat{z}=\widehat{z}_{PT}$, leading to:

$$\left.\frac{\partial W_{PT}}{\partial\widehat{z}}\right|_{\widehat{z}=\widehat{z}_{PT}}=\frac{ma}{h}\left[(\omega_S\tau_0+t_0)(\frac{3\widehat{z}-y}{2})\right]>0, \tag{27}$$

as $\widehat{z}>\frac{y}{3}$. Hence, the *Pigouvian tax* fails to support the first-best optimum due to the free location decisions of workers[21]. They earn higher wages in the CBD, yielding an equilibrium CBD size that is too large to correctly manage traffic congestion. Including an extra marginal external cost reduces their net wage. They relocate near the SBD, overweighting the marginal loss of remaining near the CBD. As a consequence, the limit of the residential area from which inhabitants commute to the CBD is lower than the optimal location $\widehat{z}^O$. The CBD limit under this kind of tax is lower than both the equilibrium and the optimal location. The hierarchy is as follows:

$$\widehat{z}_{PT}<\widehat{z}^O<\widehat{z}^* \tag{28}$$

As a result, a location-constrained first-best policy excessively internalizes the congestion externality by encouraging SBD overextension within a polycentric city.

Proposition 2. *A Pigouvian tax fails to reach the optimum, yielding a* ***CBD that is too small*** *when there are two market failures : the congestion and agglomeration externalities.*
Proof *See Appendix C.*

The flat tax A *flat kilometer tax* is levied with no differentiation among commuters. This toll is a uniform tax per kilometer traveled. Each worker commutes to her workplace within the city:

$$\tau_F(x)=t_Fx \quad \text{with} \quad t_F>0 \tag{29}$$

The CBD limit $\widehat{z}$ is now implicitly defined by the following expression including the *flat tax* incurred by each individual:

$$\omega_C\left[1-\widehat{z}\tau_0-\frac{\tau_1 a\widehat{z}^2}{2h(1-a)}\right]-(t_0+t_F)\widehat{z}=\omega_S\left[1-(\frac{y-\widehat{z}}{2})\tau_0-\frac{\tau_1 a(\frac{y-\widehat{z}}{2})^2}{2h(1-a)}\right]-(t_0+t_F)(\frac{y-\widehat{z}}{2})$$

[20] see Appendix $C1$ for details of calculations.

[21] There is no reason why $\widehat{z}$ should be lower that or equal to $\frac{y}{3}$. When this *Pigouvian tax* is implemented, there is only one solution yielding $\widehat{z}=\frac{y}{3}$, namely, when $A_C=1$.

All commuters face the uniform tax per unit of distance but they do not pay the same amount of tax. Hence, workers living near the left endpoint of the SBD may be underpriced and those living near the SBD or the CBD may be overpriced. In addition, workers benefit from the revenue return scheme and receive the same lump-sum payment from the collected taxes. At the aggregate level, you have the following welfare:

$$W_F(\tau_F) = \frac{ma}{h}\left[\int_0^{\widehat{z}} \omega_C\left[1 - T(x)\right] - t_0(x)\mathrm{d}x + 2\int_{z_S}^{y} \omega_S\left[1 - T(x)\right] - t_0(x - z_S)\mathrm{d}x\right]$$

Differentiating W_F with respect to $\widehat{z}$ conditional on $\widehat{z} = \widehat{z}_F$ to find the location of $\widehat{z}_F$ yields:

$$\left.\frac{\partial W_F}{\partial \widehat{z}}\right|_{\widehat{z}=\widehat{z}_F} \lesseqgtr 0 \tag{30}$$

Thus, the location of the CBD limit when each commuter pays a *flat tax* could be greater or lower than the optimal location of $\widehat{z}^O$. The location depends mainly on the wage gap between the business centers before the implementation of the *flat tax*. As a consequence, the size of a SBD matters, but the optimum will never be achieved unless the wage rates are equal in both business centers.

The cordon toll A *cordon toll* is then the last road pricing scheme considered. A *cordon toll* is set at location $\alpha\widehat{z}$ within the CBD area where some inhabitants work[22]. Each commuter living between $\alpha\widehat{z}$ and $\widehat{z}$ pays a fixed fee c to pass the cordon. The charging function is given by:

$$\tau_{CT}(x) = \begin{cases} c & \text{if} \quad \alpha\widehat{z} \leq x \leq \widehat{z} \\ 0, & \text{otherwise.} \end{cases} \tag{31}$$

Workers living inside the cordon are not charged. Therefore, they do not pay for the congestion externality they impose on other commuters. The fixed fee to enter the cordon is captured in the rents of those living near the CBD. Hence, there exists a discontinuity in the rent bid at the tollgate location. Moreover, inhabitants living in the SBD area do not pay road taxes and benefit from both tax and aggregate land rent redistribution. The CBD limit is implicitly defined by the following expression:

$$\omega_C\left[1 - \widehat{z}\tau_0 - \frac{\tau_1 a\widehat{z}^2}{2h(1-a)}\right] - t_0\widehat{z} - c = \omega_S\left[1 - (\frac{y-\widehat{z}}{2})\tau_0 - \frac{\tau_1 a(\frac{y-\widehat{z}}{2})^2}{2h(1-a)}\right] - t_0(\frac{y-\widehat{z}}{2})$$

A priori, in this case, the implementation of a *cordon toll* induces a decrease in the CBD size due to the fee incurred by a few workers living between $\alpha\widehat{z_{CT}}$ and $\widehat{z_{CT}}$. Commuters living near the cordon will relocate closer to the SBD until it becomes more expensive than living close to the CBD. The welfare function $W_{CT}(\tau_{CT})$ is expressed as follows[23]:

$$W_{CT}(\tau_{CT}) = \frac{ma}{h}\left[\int_0^{\widehat{z}} \omega_C\left[1 - T(x)\right] - t_0(x)\mathrm{d}x + 2\int_{z_S}^{y} \omega_S\left[1 - T(x)\right] - t_0(x - z_S)\mathrm{d}x\right]$$

[22] α is purely exogenous in this case and does not represent a location of the cordon which would maximize the indirect utility as in Verhoef's (2005) numerical simulations. We have left aside the debate about the optimal location of the cordon discussed in the literature (Mun *et al.*, 2005; Verhoef, 2005). Our main interest is analyzing the pecuniary and job relocation effects due to this pricing scheme. However, I indirectly find an optimal location of the cordon ($\alpha\widehat{z}^O$) through the endogenously determined optimal size of the CBD ($\widehat{z}^O$).

[23] The details of the calculations are reported in Appendix $C3$.

Differentiating W_{CT} with respect to $\widehat{z}$ conditional on $\widehat{z} = \widehat{z}_{CT}$ to find the location of $\widehat{z}_{CT}$ yields:

$$\left.\frac{\partial W_{CT}}{\partial \widehat{z}}\right|_{\widehat{z}=\widehat{z}_{CT}} = \frac{ma}{h}\left[c + (\omega_S \tau_0 + t_0)(\frac{3\widehat{z} - y}{2}) + \frac{\omega_S \tau_1 a(y - \widehat{z})^2}{8h(1-a)}\right] \lesseqgtr 0 \tag{32}$$

As a result, the *cordon toll* does not achieve the optimal location of the SBD. The location of the CBD limit may be lower or greater than the optimal location according to the initial wage gap between the subcenters and the transport costs incurred by individuals (see Appendix $C3$). Since the location of the cordon is exogenous, the fixed fee may be too small or too high as well. Indeed, a fee that overprices commuters yield a small CBD and a fee that underprices them leads to a too large CBD. It is straightforward that an increase in the available land for housing lowers the road capacity leading to a small CBD. Conversely, an increase in the lot size spreads out the CBD for a given road capacity.

Proposition 3. *The flat tax and the cordon toll do not necessarily perform better than the Pigouvian tax. They yield a CBD that is either too low or too large depending on initial wage gap between the business districts and the amount of the respective pricing scheme.*
Proof *See Appendix C.*
Finally, I need to compare the results of the policies with the help of numerical simulations because it is difficult to find convenient analytical solutions.

5 Comparisons between road pricing schemes

This section simulates a closed city with a fixed limit $y = 2$, where the total population is fixed at $L = 2$ and the agricultural land rent is $R_A = \frac{1}{4}$ for a given wage gap $\frac{1}{\mu} = 2$ ($\omega_C = \frac{1}{\mu}\omega_S$). This wage gap is chosen since the polycentric city exists under that condition for all tolling schemes. Table 1 shows the different parameter values used for the scenario[24]. Parameter choice is in accordance with the transport costs hierarchy used in other numerical simulations in the literature (Tikoudis *et al.*, 2015; Zhang and Kockelman, 2016a) (*i.e.*, $\tau_0 > \tau_1$).

Table 1: Calibrated parameters

L	R_A	m	h	δ	a	τ_0	τ_1	t_0	ω_S	$\frac{1}{\mu}$
2	$\frac{1}{4}$	2	1	1	0.5	$\frac{1}{5}$	$\frac{1}{6}$	0	2	2

The following parts present the results of simulations conducted for the polycentric city. The numerical settings here guarantee that all individuals benefit from the same amount of toll and rent revenues and share the same indirect utility $\bar{V}$ at all locations under each road pricing scheme.

CBD limit Analytically, this model has demonstrated that a small increase in the wage gap between the CBD and a SBD expands the CBD's area (measured by $\widehat{z}$) in equilibrium. However, simulations show that CBD limit under each road pricing scheme is always lower than in equilibrium without taxation (Table 2). Therefore, tolling schemes never reach the $\widehat{z}^O$ where welfare reaches an extremum. Land use patterns are affected by second-best policies when each business center offers different wage rates. The endogenous left endpoint of a SBD has changed under a *Pigouvian tax*(PT), a *flat tax*(F) and a *cordon toll*(CT). For any $\frac{1}{\mu} > 1$, a polycentric city becomes monocentric. There exists a limit value of ω_S/ω_C ($\underline{\mu}$) below which the city is always monocentric, as given by $\widehat{z}(\underline{\mu}) = y$. Second-best pricing policies reach the level $\frac{1}{\underline{\mu}}$ except for one. With the *Pigouvian tax* a polycentric city always exists even if the CBD is the most attractive workplace. When $\frac{1}{\mu}$ approaches infinity, the CBD's size reaches a limit lower than y in the simulations. With a *flat tax* and a *cordon toll*, a polycentric city prevails provided that $\frac{1}{\underline{\mu}}$ is lower than approximately 4.31 and 4.03, respectively.

Table 2: Simulated results of CBD size within a polycentric city according to each road pricing scheme. ($1/\mu = 2$).

	No toll	Optimum	τ_{PT}	τ_F	τ_{CT}
CBD's size, $\widehat{z}$	1.577	1.467	1.305	1.489	1.506

In other terms, a subcenter no longer has economic interests, as the gross wage offered ω_S is almost 5 times smaller than ω_C under a *flat tax*. Internalizing the congestion externality at any given location mitigates the desirability of traveling to a unique business center. In a way, these

[24]The numerical simulation with all parameters is consistent with the analytical model such that indirect utilities and aggregate welfare are strictly positive.

findings are in accordance with the non-monocentric model of Zhang and Kockelman (2016b), although our model does not consider endogenous city size. Congestion costs yield higher land rents near the CBD, as each worker wants to reduce her commuting costs (Solow, 1972; Wheaton, 1998). When jobs relocate endogenously, land rents diminish near the CBD. However, marginal congestion pricing for each commuter strengthens their willingness to pay the higher land rents associated with a wage gap between the two business centers. Finally, the optimal size of the residential area for individuals commuting to the CBD is never reached by these road pricing schemes. For $\frac{1}{\mu} = 2$, a *Pigouvian tax* leads to a CBD limit lower than the optimum, and vice versa for the *cordon toll* and *flat tax*. Note that each worker is priced at her exact marginal cost under the *Pigouvian tax*. The *flat tax* and the fixed fee to pass through the cordon are identical ($\tau_F = c = 0.148$). This amount of tax represents the marginal cost that pays a worker living at the location $\frac{y}{3}$. The CBD limit under *Pigouvian taxation* is always lower than the optimal location (**Proposition 2.**).

Efficiency of the three road pricing schemes in the polycentric city This part examines the welfare and land use effects of second-best policies, comparing them to the no-toll equilibrium within a polycentric city. First, the implementation of three different instruments is investigated when there is a wage gap between the CBD and a SBD. Table 3 illustrates relevant characteristics of the no-toll equilibrium, the optimum, the *Pigouvian tax*, *flat tax* and *cordon toll*. τ_F and c are not chosen to maximize welfare. Their level is equal to a marginal external congestion cost at the location $x = y/3$. A welfare improvement at an aggregate level is significant under the second-

Table 3: Simulated results of policy instruments within a polycentric city. ($1/\mu = 2$).

	No toll	Optimum	τ_{PT}	τ_F	τ_{CT}
City limit, y	2.00	2.00	2.00	2.00	2.00
Total travel time costs at $\widehat{z}$	2.091	1.89	1.611	1.930	1.960
Welfare, $W_S(\widehat{z})$	5.27	5.34	5.63	5.33	5.32
Percentage change against no-toll equilibrium					
City limits (%)		0	0	0	0
Total travel time costs at $\widehat{z}$ (%)		-9.6	-22.9	-7.70	-6.26
Welfare (%)		1.33	6.80	1.14	0.95

best instruments mainly due to the redistribution of both tax and land rent revenues. Under a *Pigouvian tax*, the welfare level increases from 5.267 to 5.63, that is, 6.8% higher than the welfare of the no-toll equilibrium. Indeed, the toll and the aggregate land rents are returned as lump sums to each worker who does not own their housing. When the congestion externality is internalized at each given location, the total travel costs (in terms of time and money) falls from 2.091 to 1.611 for a worker living in $\widehat{z}$ and working in the CBD. Total welfare covers all inhabitants, landowners, agricultural landlords and makes them better off with second-best instruments compared to the no-toll equilibrium. Indeed, regarding a *flat tax* and a *cordon toll*, welfare is higher by 1.14% and 0.95%, respectively, for a worker. The optimum size of each working area leads to an increase in the welfare level of 1.33%[25]. The CBD area where workers live is too small under the *Pigouvian tax*. That is why households living in $\widehat{z}$ benefit from the shortest maximum traveled distance between the CBD and their residence.

Travel demand is unaffected, as each worker must commute every day. The rent bids capitalize the tax effects, increasing strongly near each business center. Numerical simulations demonstrate that the optimum size is not achieved with a location-constrained first-best policy where congestion is priced and taxes and land rents are returned as lump sums to workers with a wage gap between the business centers (Table 2). With these different instruments internalizing the congestion externality, welfare in the polycentric city is always above that in the no-toll equilibrium. Workers are better off under each pricing scheme when they live in a polycentric city until $\frac{1}{\mu}$ reaches $\frac{1}{\underline{\mu}}$ according to the simulations. Hence, an increasing pecuniary cost of transport through each road pricing scheme leads to a decrease in the CBD size compared to the equilibrium location, which is in accordance with the results of Zhang and Kockelman (2016b) for a given wage gap. The differentiated analytical solutions are non-linear; hence, numerical simulations help us to confirm that $\widehat{z}^*$ is always superior to $\widehat{z_{toll}}$ when $1 < \frac{1}{\mu} < \frac{1}{\underline{\mu}}$. Note that for two business centers offering the same wage rate, the CBD size $\widehat{z} = y/3$ is identical under the no-toll equilibrium, the optimum, *Pigouvian taxation*, and *flat tax* schemes. Finally, these results are in accordance with the literature, as any taxation leads to an increase in the city's compactness (Zhang and Kockelman, 2016a). Road pricing schemes force workers to relocate closer to the CBD in accordance with the findings of Anas (2013). A *Pigouvian tax* decreases CBD size more than the other pricing schemes. The optimal location of $\widehat{z}$ is higher than that of $\widehat{z}_{PT}$ and lower than $\widehat{z}_F$ and $\widehat{z}_{CT}$ according to our simulations.

[25]Note that welfare in optimum is lower than that with the *Pigouvian tax* due to the revenue return scheme for a given city limit.

6 Discussion

This section addresses the implementation of a mass transit service close to the CBD and considers the possible impacts on congestion.

Incidence of modal choice on congestion and urban structure Over 60 million passenger-kms over the year were covered by mass transit in the Ile-de-France region of France in 2014. This transit activity has grown by more than 30% since 2000 (CGDD, 2016). Therefore, to remain realistic, assume that another mode of transit is introduced in the city, public transport (bus). This mode uses a transportation infrastructure assumed to be provided by a public planner (municipal government) at no cost. The market area of this mass transit mode is between the CBD ($x = 0$) and x_B. As there exists mixed land use for transport modes, workers prefer using mass transit over their car when they live close to the CBD according to Limtanakool *et al.*, (2006). Travelers use only one mode, there is no park and ride. Furthermore, car ownership levels increase with the distance from the city center (Dasgupta *et al.*, 1985). Assume that each worker in this area lives close to a bus stop, so they face no costs of access. Between the CBD and the outer border of the public transport area x_B, the road's capacity $(1-a)$ is equally divided between car and bus lanes on the single road. Beyond x_B to y, each worker uses exclusively her car. A mass transit user faces only a commuting time cost that is higher than the free-flow travel time of car users. This cost depends on the waiting times at bus stops and the travel time to the destination[26]. In other words, $T_B(x) > T_A(x)$ without a congestion externality, but passengers pay a fare t_B, which is lower than average capital cost t_0 of car ownership. To sum up, transport pecuniary costs are given as follows:

$$t_A(x) = t_0 x \quad \text{and} \quad t_B(x) = t_B \tag{33}$$

A share of the population uses now public transport[27], and each car user incurs congestion linked to the number of commuter on the road between x_B and $\widehat{z}$, as well as reduced road capacity between the CBD's edge and x_B. Each car user incurs a total commuting time expressed as follows:

$$T_A(x) = \begin{cases} \displaystyle\int_0^x [\tau_0 + 3\tau_1 \textstyle\int_{x_B}^x \int_x^{\widehat{z}} \frac{l}{(1-a)} \mathrm{d}z] \mathrm{d}x, & \text{when} \quad 0 < x_B < x < \widehat{z} \\ \displaystyle\int_{z_S}^x [\tau_0 + \tau_1 \textstyle\int_x^y \frac{l}{(1-a)} \mathrm{d}z] \mathrm{d}x, & \text{when} \quad z_S < x < y \end{cases} \tag{34}$$

Note that the congestion parameter increases due to the decrease in road capacity near the business center of each worker. The total congestion delay now depends on the size of the residential area where car users live (i.e., $\widehat{z} - x_B$).

The polycentric city In a polycentric city in equilibrium, the individual welfare of a worker is identical at all locations. An individual living between 0 and x_B has the following indirect utility:

$$V(x)_B = \omega_C(1 - T_B(x)) - \frac{R(x)h}{a} - t_B \tag{35}$$

[26] Each user faces an exogenous commuting time cost that does not depend on traveled distances. Waiting times at bus stops depend on the bus fleet size and on the frequency at which the buses run (Small, 2004).

[27] Creutzig (2014) sets up a model including public transport close to the CBD and imposes this mode of travel on the residents living in its market area. A simplification occurs by abstracting from providing public transit and public transit infrastructure at a cost.

Between x_B and $\widehat{z}$:

$$V(x)_A = \omega_C(1 - T_A(x)) - \frac{R_A h}{a} - t_0 x \tag{36}$$

The market size for mass transit is specified by the following indifference condition linked to the time and pecuniary costs of each mode:

$$T_B(x_B) + t_B = T(x_B) + t_0 x_B \tag{37}$$

Initially, the residential area where workers live and from which they commute to the CBD is larger than that of the SBD. The introduction of a bus lane on the existing road infrastructure provides a disincentive for residents living close to the CBD to use their car. Workers living near the border $\widehat{z}$ will see a rise in their transportation costs (both pecuniary and time costs). Therefore, the equilibrium size of the CBD will decrease until the reduction in car users offsets the higher congestion delay in the zone shared with the bus lane. The size of the SBD will increase; hence, congestion near this business district will increase. From an economic point of view, the introduction of two transport modes may reduce the size of the CBD to achieve the optimal size, which has the same effect as a road pricing scheme[28]. In the case of a road pricing scheme targeting drivers, $(t_A(x) = t_0 x + \tau_k(x))$, the effect would be similar to our results in section 4, namely, a decrease in the equilibrium size of the CBD and an increase in the number of workers commuting to a SBD. A second transport mode would reinforce the effects of the different taxes as second-best policies. The intuition is as follows: when a second mode of transport is used without improving the road infrastructure for motorists, they face (i) lower transport costs (user effect) and (ii) lower road capacity (time cost effect). When the *user effect* is marginally predominant, the CBD grows, attracting workers from the SBD until there is no incentive to relocate. When the *time cost effect* dominates, CBD size decreases.

[28]However, from an environmental point of view, the size of the road network used by drivers does not necessarily decrease. This is why CO_2 emissions may remain equivalent for cities of a given size.

7 Conclusion

This article develops and explores economic tools in order to tackle the congestion externality in a polycentric setting. For a given city size, the optimal response is consistent with the development of SBDs, clustering firms that can offer the same net wage rate as in the CBD. With unpriced congestion, this urban structure reduces the negative externality incurred by each road user and raises welfare within the city compared to a monocentric setting. When the economy in the polycentric city is consistent with empirical findings, the equilibrium size of the CBD is larger than optimum one. The CBD is more attractive due to a higher wage rate and a higher wage net of transport costs in comparison with a monocentric structure for a given city size. Three main results are noteworthy.

First, the implementation of a *Pigouvian tax* on commuters is only a location-constrained first-best policy because the optimal location of a SBD is not achieved. This tax yields a residential area that is too small and close to a CBD. Welfare is definitely greatest compared with the other road pricing schemes regarding the revenue return schemes in our numerical example. However, at the aggregate level, this tax does not internalize the decentralization of jobs in SBDs, preventing it from achieving the optimum. This *Pigouvian tax* reminds us to internalize the congestion externality that road users impose to each other not only at the starting point of their trip but also during their daily commute to their workplace. Firms that do not internalize effects on congestion when jobs are decentralized must be taxed.

Second, a *flat tax* and a *cordon toll* are second-best policies that do not yield an optimal location of the SBDs. On the one hand, a *flat tax* is homogeneous across road users; hence, they are not distinguished according to the marginal damage they impose on other commuters. Indeed, workers living near a business center (CBD or SBDs) are underpriced in relation to the congestion delay they impose on other road users and the proportionality of the tax with respect to the traveled distances. On the other hand, a *cordon toll* clearly differentiates among commuters, similar to the *Pigouvian tax*, but the tax burden is asymmetric. The residential area where individuals live and from which they commute to the CBD is larger than that near the SBDs; hence, the *cordon toll* is imposed in the former area. Only workers living between the *cordon toll* location and the CBD limit pay a fixed fee to commute to the CBD. Simultaneously, individuals living in other residential areas near SBDs do not incur higher transport costs because the marginal congestion delays they impose on other users may be much lower than those imposed by individuals living close to the CBD. To sum up, the efficiency of these road pricing schemes depends mainly on the wage gap between business centers within the polycentric city before the implementation of this road pricing scheme[29]. According to the initial location of the SBDs (too small or too large), a *flat tax* may be less (resp., more) harmful than a *cordon toll*.

Finally, I demonstrate that a polycentric city in an unpriced congestion context yields a suboptimal location of the CBD limit, which is too large compared to the optimal size. According to the magnitude of the transport costs, the CBD may be too small or too large. The location is suboptimal since the indifferent worker living at the CBD limit (the left endpoint of the SBD area) decides to commute to the CBD without taking into account into account the marginal damage costs incurred by road users commuting to the CBD as well. Conversely, this indifferent worker does not pay attention to the marginal damage costs incurred by workers living near a SBD when

[29]Accordingly, the travel time and pecuniary costs incurred by each worker depend on the wage gap, which determines the size of the influence area of each business center.

she decides to drive towards it. In addition, an absence of a marginal damage obtained by individuals working in the CBD are not taken into account as well. Her marginal move due to a change in the wage gap will not induce large differences in her transport costs (time and money) compared to her previous location under the suboptimal equilibrium.

Developing subcenters may alleviate the congestion cost for each road user provided that no distortions exist for the labor or housing markets when congestion is unpriced. However, a transport improvement yields direct benefits and indirect costs, as noted by Arnott (1979) in the same context. As long as growth of traffic flows and urban sprawl costs do not substantially decrease the direct benefits from jobs relocations, urban planning remains a key policy for addressing urban transport issues compared to pricing schemes. Indeed, road pricing schemes benefit all landlords regardless of the closed-city structure (mono- or polycentric). Hence, pricing congestion implies political concerns regarding the redistribution of tax revenues when negative externality is internalized. In addition, the desirability of multiple business centers may decrease due to an increase in the average traveled distance; hence, the congestion externality may worsen when housing size adjusts in the long run.

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Appendix

A. The polycentric city

A1. Equilibrium border

The population constraint is given by:

$$\int_0^{\widehat{z}} \frac{a}{h}\mathrm{d}x + \int_{\widehat{z}}^{y} \frac{a}{h}\mathrm{d}x = \frac{L}{m}$$

so that:

$$\begin{aligned}
\int_0^{\widehat{z}} \frac{a}{h}\mathrm{d}x + \int_{\widehat{z}}^{y} \frac{a}{h}\mathrm{d}x &= \frac{L}{m} \\
\int_0^{\widehat{z}} \frac{a}{h}\mathrm{d}x + 2\int_{z_p}^{y} \frac{a}{h}\mathrm{d}x &= \frac{L}{m} \\
\frac{a}{h}\widehat{z} + 2\frac{a}{h}\frac{y-\widehat{z}}{2} &= \frac{L}{m}
\end{aligned}$$

leading to:

$$y = \frac{Lh}{ma}$$

A2. CBD limit in equilibrium with a wage gap

A worker living at $\widehat{z}$ is indifferent between traveling to the CBD or the SBD. Therefore, it leads to:

$$\omega_C\left[1 - \widehat{z}\tau_0 - \frac{\tau_1 a\widehat{z}^2}{2h(1-a)}\right] - t_0\widehat{z} = \omega_S\left[1 - \tau_0(\frac{y-\widehat{z}}{2}) - \frac{\tau_1 a(y-\widehat{z})^2}{8h(1-a)}\right] - t_0(\frac{y-\widehat{z}}{2})$$

Inserting $\mu\omega_C = \omega_S$ due to the wage gap between the business districts yields:

$$\widehat{z}^*(\mu) = \frac{-2h(1-a)\left[\omega_C(2+\mu)\tau_0 + 3t_0\right] - \mu\omega_C\tau_1 ay + 2\sqrt{\mu\omega_C^2\tau_1^2a^2y^2 + \Delta^*}}{\omega_C(4-\mu)\tau_1 a}$$

with:

$$\Delta^* = h^2(-1+a)^2\left[\omega_C(2+\mu)\tau_0 + 3t_0\right]^2 + 2h(1-a)\left[(4+\mu^2)\omega_C^2 - 5\mu\omega_C^2 + \omega_C(2+\mu)\tau_0 y + 3\mu\omega_C^2\tau_0 y\right]\tau_1 a$$

leading to:

$$\Gamma = \omega_C\left[1 - \mu + \tau_0\left(\frac{\widehat{z}(-2-\mu) + \mu y}{2}\right) + \frac{\tau_1 a\left[\mu(y-\widehat{z})^2 - 4\widehat{z}^2\right]}{8h(1-a)}\right] + t_0\left(\frac{y-3\widehat{z}}{2}\right),$$

with $\Gamma \equiv 0$.

According to the implicit function theorem, it gives:

$$\frac{\partial \Gamma}{\partial k}+\frac{\partial \Gamma}{\partial \widehat{z}}\frac{\partial \widehat{z}}{\partial k}=0$$

where k is an exogenous variable from the model. Equivalently, it leads to:

$$\frac{\partial \widehat{z}}{\partial k}=-\frac{\partial \Gamma/\partial k}{\partial \Gamma/\partial \widehat{z}}$$

Trivial calculations yield:

$$\text{sign}\frac{\partial \widehat{z}}{\partial \tau_0}=\text{sign}\frac{\partial \widehat{z}}{\partial \mu}<0, \quad \text{sign}\frac{\partial \widehat{z}}{\partial y}>0$$

$$\text{sign}\frac{\partial \Gamma}{\partial t_0}=\text{sign}\left[\frac{y-3\widehat{z}}{2}\right]<0$$

where the term in brackets is equal to $\frac{y}{2}$ when $\widehat{z}=0$ and decreases when $\widehat{z}$ increases. In addition it gives:

$$\text{sign}\frac{\partial \Gamma}{\partial \tau_1}=\text{sign}\frac{\partial \Gamma}{\partial h}=\text{sign}\frac{\partial \Gamma}{\partial a}=\text{sign}\left[\mu(y-\widehat{z})^2-4\widehat{z}^2\right]<0 \qquad (0<\mu<1)$$

where the term in brackets is equal to μy^2 when $\widehat{z}=0$ and decreases when $\widehat{z}$ increases.

A3. Welfare Welfare within the polycentric city is now defined as follows:

$$\begin{aligned}W_S(\widehat{z}^*(\mu)) &= \frac{ma}{h}\left[\int_0^{\widehat{z}}\omega_C\left[1-T(x)\right]-t_0x\mathrm{d}x+2\int_{z_p}^{y}\omega_S\left[1-T(x)\right]-t_0(x-z_S)\mathrm{d}x\right]-ALR \\ &+ \left[ALR-m\int_0^y R_A\mathrm{d}x\right]+m\int_0^y R_A\mathrm{d}x\end{aligned}$$

$$W_S(\widehat{z}^*(\mu))=\frac{ma}{h}\left[\omega_C\widehat{z}^*(1-\mu)+\mu\omega_C y-\frac{(\mu\omega_C\tau_0+t_0)\left(y^2-\widehat{z}^{*2}\right)}{2}-\frac{(\omega_C\tau_0+t_0)\widehat{z}^{*2}}{2}-\frac{\omega_C\tau_1 a\left[4\widehat{z}^{*3}+\mu(y-\widehat{z})^{*3}\right]}{12h(1-a)}\right]$$

A4. Case with no wage gap between business districts

The aggregate welfare in the no-toll equilibrium can be defined as follows:

$$\begin{aligned}W_S^* &= \frac{ma}{h}\left[\int_0^{\widehat{z}}\omega_C\left[1-T(x)\right]-t_0x\mathrm{d}x+2\int_{z_p}^{y}\omega_C\left[1-T(x)\right]-t_0(x-z_S)\mathrm{d}x\right]-ALR \\ &+ \left[ALR-m\int_0^y R_A\mathrm{d}x\right]+m\int_0^y R_A\mathrm{d}x\end{aligned}$$

Free-market and optimal sizes of the CBD limit when $\mu = 1$
The indifference function of $\widehat{z}$ is expressed as follows:

$$\omega_C\left[1-\widehat{z}\tau_0-\frac{\tau_1 a\widehat{z}^2}{2h(1-a)}\right]-t_0\widehat{z}=\omega_C\left[1-(y-z_S)\tau_0-\frac{\tau_1 a(y-z_S)^2}{2h(1-a)}\right]-t_0(y-z_S)$$

yielding:

$$\widehat{z}^2\left[\frac{3\omega_C\tau_1 a}{8h(1-a)}\right]+\widehat{z}\left[\frac{3\omega_C\tau_0}{2}+\frac{3t_0}{2}+\frac{\omega_C\tau_1 a}{4h(1-a)}\right]-\left[\frac{\omega_C\tau_0 y}{2}+\frac{t_0 y}{2}+\frac{\omega_C\tau_1 a y^2}{8h(1-a)}\right]=0$$

leading to:

$$\widehat{z}^*=\frac{y}{3}$$

Then, the size of the CBD in which workers live is determined by a benevolent planner in order to maximize welfare $W_S(\widehat{z})$ subject to:

$$\begin{cases} V_C(\widehat{z})=\omega_C(1-x\tau_0-\frac{\tau_1 a}{h(1-a)}[\widehat{z}x-x^2/2])-\frac{\Psi_S(x)h}{a}-t_0x & > \quad 0 \\ V_S(\widehat{z})=\omega_C(1-(x-z_S)\tau_0-\frac{\tau_1 a}{h(1-a)}[y(x-z_S)+\frac{z_S^2-x^2}{2}])-\frac{\Psi_S(x)h}{a}-t_0(x-z_S) & > \quad 0 \end{cases}$$

You have:

$$\frac{\partial W_S(\widehat{z})}{\partial\widehat{z}}=0$$

Hence:

$$\frac{m\omega_C\tau_1^2a^2(y+\widehat{z})(y-3\widehat{z})}{4h^2(1-a)}=0$$

leading to:

$$\widehat{z}^O=\frac{y}{3}$$

Hence, $\widehat{z}^*=\widehat{z}^O$ and their expression has been inserted in the aggregate welfare function (**??**) yielding:

$$W_S(\widehat{z}^O)=L\left[\omega_C-\frac{(\omega_C\tau_0+t_0)y}{2}-\frac{\omega_C\tau_1 a y^2}{27h(1-a)}\right]$$

As a result, it leads to:

$$W_S(\widehat{z}^O)-W_M^*=\frac{m\omega_C\tau_1 a^2(y-\widehat{z})(y+\widehat{z})^2}{4h^2(1-a)} \quad > \quad 0 \tag{38}$$

A5. Proof of proposition 1

Aggregate welfare, including a wage gap within the city, can be defined as follows:

$$\begin{aligned} W_S &= \frac{ma}{h}\left[\int_0^{\widehat{z}}[1-T(x)]-t_0x\mathrm{d}x+2\int_{\widehat{z_S}}^{y}\omega_S[1-T(x)]-t_0(x-z_S)\mathrm{d}x\right]-ALR \\ &+ [ALR-m\int_0^y R_A\mathrm{d}x]+m\int_0^y R_A\mathrm{d}x \end{aligned}$$

with:

$$ALR = m\int_0^{\widehat{z}} \Psi_C(x)\mathrm{d}x + 2m\int_{z_S}^{y} \Psi_S(x)\mathrm{d}x$$

yielding:

$$W_S \;=\; \omega_C\widehat{z} + \omega_S(y-\widehat{z}) - \omega_C\tau_0\widehat{z} - \omega_S\tau_0(y-\widehat{z}) - \frac{\omega_C\tau_1 a\widehat{z}^3}{3h(1-a)} - \frac{2\omega_S\tau_1 a(\frac{y-\widehat{z}}{2})^3}{3h(1-a)} - t_0\frac{y^2}{2}$$

Then, the left endpoint of the SBD $\widehat{z}^O$ is determined when there exists a wage gap between the business centers. The maximization of aggregate welfare $W_S(\widehat{z})$ is subject to:

$$V_S(\widehat{z})\begin{cases} \omega_C(1-x\tau_0-\frac{\tau_1 a}{h(1-a)}[\widehat{z}x-x^2/2]) - \frac{\Psi_C(x)h}{a} - t_0x \quad > \quad 0 \\ \omega_P(1-(x-z_S)\tau_0-\frac{\tau_1 a}{h(1-a)}[y(x-z_S)+\frac{z_S^2-x^2}{2}]) - \frac{\Psi_S(x)h}{a} - (x-z_S)t_0 \quad > \quad 0 \\ \widehat{z} < y \end{cases}$$

Then, it leads to:

$$\frac{\partial W_S(\widehat{z})}{\partial \widehat{z}} = \omega_C - \omega_S + \omega_S\tau_0 - \omega_C\tau_0 - \frac{\omega_C\tau_1 a\widehat{z}^2}{h(1-a)} + \frac{\omega_S\tau_1 a(y-\widehat{z})^2}{4h(1-a)}$$

Thus, $\frac{\partial W_S(\widehat{z})}{\partial \widehat{z}} = 0$ leads to a single maximum for $W_S(\widehat{z})$ when $\widehat{z} \in [0,y]$, which reaches:

$$\widehat{z}^O(\mu) = \frac{-2h(1-a)\omega_C(1-\mu)\tau_0 - \mu\omega_C\tau_1 ay + 2\sqrt{\mu\omega_C^2\tau_1^2a^2y^2 + \Delta^O}}{\omega_C(4-\mu)\tau_1 a}$$

with:

$$\Delta^O = h^2(-1+a)^2\left[\omega_C(1-\mu)\right]^2\tau_0^2 + h(1-a)\left[(4+\mu^2)\omega_C^2 + \mu\omega_C^2\tau_0 y - \mu^2\omega_C^2\tau_0 y\right]\tau_1 a$$

Furthermore, $W_S(\widehat{z})$ is a concave curve since $\frac{\partial^2 W_S}{\partial^2 \widehat{z}}$ yields the following result:

$$\frac{\partial^2 W_S}{\partial^2 \widehat{z}} = \frac{ma}{h}\left[(\omega_S-\omega_C)\tau_0 - \frac{\left[24\omega_C\widehat{z} + 6\omega_S(y-\widehat{z})\right]\tau_1 a}{12h(1-a)}\right] < 0 \tag{39}$$

This optimal CBD size varies positively with the wage rate discrepancy between the business centers within the polycentric setting:

$$\frac{\partial W_S}{\partial \widehat{z}^O} > 0, \quad \text{when} \quad 1 < \frac{1}{\mu} < \frac{1}{\underline{\mu}}$$

B. Monocentric city versus Polycentric city

Congestion level
Note that the differential in the total commuting time cost between the two cities is given by $\Delta T(y)$, with:

$$\begin{aligned}\Delta T(y) &= T_M(y_M) - T(y) \\ &= \tau_0 z_S + \frac{\tau_1 a[y^2 - (\frac{y-\widehat{z}}{2})^2]}{2h(1-a)}\end{aligned}$$

In addition, you have $\Delta T(\widehat{z})$, with:

$$\begin{aligned}\Delta T(\widehat{z}) &= T_M(y_M) - T(\widehat{z}_P) \\ &= \tau_0(y-\widehat{z}) + \frac{\tau_1 a(y^2 - \widehat{z}^2)}{2h(1-a)}\end{aligned}$$

Thus, $\Delta T(y) > 0$ and $\Delta T(\widehat{z}) > 0$, with an average congestion delay that is higher in the monocentric setting.

Welfare comparison When there is no wage gap within the polycentric city, the welfare is given by:

$$W_S(\widehat{z}^*) = \frac{ma}{h}\left[\omega_C y - \frac{(\omega_C\tau_0 + t_0)y^2}{2} - \frac{\omega_C\tau_1 a(4\widehat{z}^{3*} + (y-\widehat{z}^*)^3)}{12h(1-a)}\right]$$

and in a monocentric setting:

$$W_M = L\left[\omega_C - \frac{y_M(\omega_C\tau_0 + t_0)}{2} - \frac{\omega_C\tau_1 a y^2}{3h(1-a)}\right]$$

Therefore, the comparison with a monocentric setting yields $\Delta_V = W_M - W_S$, with:

$$\begin{aligned}\Delta_V &= \omega_C(1-\mu)(y - \widehat{z_{toll}}) + \omega_C\tau_0(1-\mu)(\widehat{z_{toll}} - y) \\ &+ \frac{\omega_C\tau_1 a(\widehat{z_{toll}}^3 - y^3)}{3h(1-a)} + \frac{\omega_S\tau_1 a(\frac{y-\widehat{z_{toll}}}{2})^3}{3h(1-a)}\end{aligned}$$

Regarding the evolution of the welfare between a monocentric and a polycentric setting, it leads to the following expression:

$$\Delta_W(\mu) = \frac{ma(y-\widehat{z})\left[h(1-a)\left(12(\omega_S - \omega_C) + 6\omega_C\tau_0(y+\widehat{z}) - 3\omega_S\tau_0(y-\widehat{z}) + 3t_0(y+3\widehat{z})\right) + 4\omega_C\tau_1 a(y^2 + y\widehat{z} + \widehat{z}^2)\right]}{12h^2(1-a)} \tag{40}$$

It is straightforward to check that the aggregate welfare levels are equal when $\widehat{z}(\underline{\mu}) = y$. In other words, when $\underline{\mu} = 1 - (\tau_0 + \frac{t_0}{\omega_C})y - \frac{\tau_1 a y^2}{2h(1-a)}$. Here, the difference in welfare depends heavily on the magnitude of the wage gap between the business centers within the polycentric city.

C. Road pricing schemes in the polycentric city

C1. Quasi-first-best vs. equilibrium

The city limits remain identical to the benchmark equilibrium, as housing size is exogenous. Thus, the bid rent in a suboptimal equilibrium is given by:

$$\Psi_C(x) = \frac{a\left[h(1-a)(\omega_C\tau_0 + t_0)(\widehat{z_S} - x) + \omega_C\tau_1 a(\widehat{z_S} - x)^2\right]}{h^2(1-a)} + R_A$$

and when $z_S < x < y$:

$$\Psi_S(x) = \frac{a\left[h(1-a)(\omega_S\tau_0 + t_0)(y - x) + \omega_S\tau_1 a(y - x)^2\right]}{h^2(1-a)} + R_A$$

The *Pigouvian tax* is determined as follows:

$$\tau_{PT}(x) = \int_0^x \left[\frac{\tau_1 a}{h(1-a)} \int_x^{\widehat{z}} \omega_C \quad \mathrm{d}z\right]\mathrm{d}x \quad \text{and} \quad \tau_{PT}(x) = \int_{\widehat{z}}^x \left[\frac{\tau_1 a}{h(1-a)} \int_x^{y} \omega_S \quad \mathrm{d}z\right]\mathrm{d}x,$$

with:

$$\begin{aligned}
\tau_{PT}(x) &= \int_0^x \left[\frac{\tau_1 a}{h(1-a)} \int_x^{\widehat{z}} \omega_C \quad \mathrm{d}z\right]\mathrm{d}x \\
&= \frac{\tau_1 a}{h(1-a)} \int_0^x \omega_C(\widehat{z} - x) \quad \mathrm{d}x \\
&= \frac{\omega_C\tau_1 a(\widehat{z}x - x^2/2)}{h(1-a)}
\end{aligned}$$

when $0 < x < \widehat{z}$. Because of symmetry around a SBD, it leads to:

$$\begin{aligned}
\int_{\widehat{z}}^x \left[\frac{\tau_1 a}{h(1-a)} \int_x^y \omega_S \mathrm{d}z\right]\mathrm{d}x &= \int_{z_S}^x \left[\frac{\tau_1 a}{h(1-a)} \int_x^y \omega_S \mathrm{d}z\right]\mathrm{d}x \\
&= \frac{\tau_1 a}{h(1-a)} \int_{z_S}^x \omega_S(y - x)\mathrm{d}x \\
&= \frac{\omega_S\tau_1 a\left[y(x - z_S) - \frac{x^2 - z_S^2}{2}\right]}{h(1-a)}
\end{aligned}$$

when $\widehat{z} < x < y$. Using the *Pigouvian tax* and the indifference condition of a worker living in $\widehat{z}$ yields:

$$\omega_C\left[1 - \widehat{z}\tau_0 - \frac{\tau_1 a\widehat{z}^2}{h(1-a)}\right] - t_0\widehat{z} = \omega_S\left[1 - (\frac{y - \widehat{z}}{2})\tau_0 - \frac{\tau_1 a(\frac{y-\widehat{z}}{2})^2}{h(1-a)}\right] - t_0(\frac{y - \widehat{z}}{2})$$

in which $\omega_S = \mu\omega_C$ and (13) have been inserted, leading to:

$$\widehat{z}_{PT}(\mu) = \frac{h(-1+a)\left[\omega_C(2+\mu)\tau_0 + 3t_0\right] - \omega_S\tau_1 ay + \sqrt{4\mu\omega_C^2\tau_1^2a^2y^2 + \Delta_P}}{\omega_C(4-\mu)\tau_1 a}$$

with:

$$\begin{aligned} \Delta_P &= h^2(-1+a)^2\left[\omega_C(2+\mu)\tau_0 + 3t_0\right]^2 \\ &+ 4h(1-a)\left[(4+\mu^2)\omega_C^2 - 5\mu\omega_C^2 + \omega_C(2+\mu)\tau_0 y + 3\mu\omega_C^2\tau_0 y\right]\tau_1 a \end{aligned}$$

Welfare is now given by:

$$\begin{aligned} W_{PT} &= \frac{ma}{h}\left[\int_0^{\widehat{z}} V_C \mathrm{d}x + 2\int_{z_S}^{y} V_S\right] + G \\ &+ \left[ALR - \int_0^y R_A \mathrm{d}x\right] + \int_0^y R_A \mathrm{d}x \end{aligned}$$

where aggregate land rents and toll revenues returned as lump sums:

$$ALR = \frac{ma}{h}\left[\int_0^{\widehat{z}} \frac{\Psi_C h}{a}\mathrm{d}x + 2\int_{z_S}^{y}\frac{\Psi_S h}{a}\mathrm{d}x\right] \quad \text{and} \quad G = \frac{ma}{h}\int_0^y \tau_k(x)\mathrm{d}x$$

Thus, the following welfare is:

$$\begin{aligned} W_{PT} &= \frac{ma}{h}\left[\omega_C\widehat{z} - \frac{(\omega_C\tau_0 + t_0)\widehat{z}^2}{2} - \frac{\omega_C\tau_1 a\widehat{z}^3}{3h(1-a)} + \omega_S(y-\widehat{z}) - \frac{(\omega_S\tau_0 + t_0)(y-\widehat{z})^2}{4} - \frac{(\omega_S\tau_1 a(y-\widehat{z})^3}{12h(1-a)}\right] \\ &+ \frac{ma}{h}\left[\frac{(\omega_C\tau_0 + t_0)\widehat{z}^2}{2} + \frac{\omega_C\tau_1 a\widehat{z}^3}{3h(1-a)} + \frac{(\omega_S\tau_0 + t_0)(y-\widehat{z})^2}{4} + \frac{(\omega_S\tau_1 a(y-\widehat{z})^3}{12h(1-a)}\right] + mR_A y \end{aligned}$$

Then, you differentiate W_S with respect to $\widehat{z}$ leading to:

$$\begin{aligned} \frac{\partial W_S}{\partial\widehat{z}} &= \frac{ma}{h}\left[\omega_C - (\omega_C\tau_0 + t_0)\widehat{z} - \frac{\omega_C\tau_1 a\widehat{z}^2}{h(1-a)} - \omega_S + \frac{(\omega_S\tau_0 + t_0)(y-\widehat{z})}{2} + \frac{(\omega_S\tau_1 a(y-\widehat{z})^2}{4h(1-a)}\right] \\ &+ \frac{ma}{h}\left[(\omega_C\tau_0 + t_0)\widehat{z} + \frac{\omega_C\tau_1 a\widehat{z}^2}{h(1-a)} - \frac{(\omega_S\tau_0 + t_0)(y-\widehat{z})}{2} - \frac{(\omega_S\tau_1 a(y-\widehat{z})^2}{4h(1-a)}\right] \end{aligned}$$

When $\widehat{z} = \widehat{z}_{PT}$:

$$\left.\frac{\partial W_P}{\partial\widehat{z}}\right|_{\widehat{z}=\widehat{z}_P} = \frac{ma}{h}\left[\omega_C - \omega_S\right] > 0$$

C2. The polycentric city: Second-best vs. equilibrium

The flat tax The bids rent including the *flat tax* are given by:

$$\Psi_C(x) = \frac{a\left[2h(1-a)(\omega_C\tau_0 + t_0 + \tau_k)(\widehat{z}_F - x) + \omega_C\tau_1 a(\widehat{z}_F - x)^2\right]}{2h^2(1-a)} + R_A$$

and:

$$\Psi_S(x) = \frac{a\left[2h(1-a)(\omega_S\tau_0 + t_0 + t_F)(y-x) + \omega_S\tau_1 a(y-x)^2\right]}{2h^2(1-a)} + R_A$$

when $z_S < x < y$. Using the indirect utility formula with the *flat tax* and (29) yields the indirect utility given by:

$$V_S = \omega_S\left[1 - (\frac{y-\widehat{z}}{2})\tau_0 - \frac{a\tau_1(\frac{y-\widehat{z}}{2})^2}{2h(1-a)}\right] + G + \frac{ALR}{L} - (t_0 + t_F)(\frac{y-\widehat{z}}{2}) - \frac{R_A h}{a}$$

The indifference condition of a worker living in $\widehat{z}$ yields:

$$\omega_C\left[1 - \widehat{z}\tau_0 - \frac{\tau_1 a\widehat{z}^2}{2h(1-a)}\right] - (t_0 + t_F)\widehat{z} = \omega_S\left[1 - (\frac{y-\widehat{z}}{2})\tau_0 - \frac{\tau_1 a(\frac{y-\widehat{z}}{2})^2}{2h(1-a)}\right] - (t_0 + t_F)(\frac{y-\widehat{z}}{2})$$

yielding:

$$\widehat{z}_F = \frac{-2h(1-a)\left[\tau_0(2\omega_C + \omega_S) + 3(t_0 + t_F)\right] - \omega_S\tau_1 a y + 2\sqrt{\Delta_F}}{(4\omega_C - \omega_S)\tau_1 a}$$

with:

$$\begin{aligned} \Delta_F &= h^2(-1+a)^2\left[\tau_0(2\omega_C + \omega_S) + 3(t_0 + t_F)\right]^2 \\ &+ 2h(1-a)\tau_1 a\left[4\omega_C^2 + \omega_S^2 + 3\omega_C\omega_S\tau_0 y - 5\omega_C\omega_S + (2\omega_C + \omega_S)(t_0 + t_F)y\right] + \omega_C\omega_S\tau_1^2 a^2 \end{aligned}$$

Welfare is given by:

$$W_F(t_F) = \frac{ma}{h}\left[\int_0^{\widehat{z}} \omega_C\left[1 - T(x)\right] - t_0(x)\mathrm{d}x + 2\int_{z_S}^{y} \omega_S\left[1 - T(x)\right] - t_0(x - z_S)\mathrm{d}x\right]$$

with:

$$ALR = \frac{ma}{h}\left[\int_0^{\widehat{z}} \frac{\Psi_C h}{a}\mathrm{d}x + 2\int_{z_S}^{y} \frac{\Psi_S h}{a}\mathrm{d}x\right]$$

Hence, the following welfare is:

$$W_F(t_F) = \frac{ma}{h}\left[\omega_C\widehat{z} - \frac{(\omega_C\tau_0 + t_0)\widehat{z}^2}{2} - \frac{\omega_C\tau_1 a\widehat{z}^3}{3h(1-a)} + \omega_S(y-\widehat{z}) - (\omega_S\tau_0 + t_0)\left(\frac{y-\widehat{z}}{2}\right)^2 - \frac{\omega_S\tau_1 a(y-\widehat{z})^3}{12h(1-a)}\right]$$

Then, we differentiate W_S with respect to $\widehat{z}$ yielding:

$$\frac{\partial W_S}{\partial\widehat{z}} = \frac{ma}{h}\left[\omega_C - (\omega_C\tau_0 + t_0)\widehat{z} - \frac{\omega_C\tau_1 a\widehat{z}^2}{h(1-a)} - \omega_S + (\omega_S\tau_0 + t_0)\left(\frac{y-\widehat{z}}{2}\right) - \frac{\omega_S\tau_1 a(y-\widehat{z})^2}{4h(1-a)}\right]$$

When $\widehat{z} = \widehat{z}_F$:

$$\left.\frac{\partial W_S}{\partial\widehat{z}}\right|_{\widehat{z}=\widehat{z}_F} = \frac{ma}{h}\left[\omega_C - \omega_S - \frac{\omega_C\tau_1 a\widehat{z}^2}{2h(1-a)} + \frac{\omega_S\tau_1 a(y-\widehat{z})^2}{8h(1-a)} - t_F(\frac{y-3\widehat{z}}{2})\right] \lesseqgtr 0$$

C3. The polycentric city: Second-best vs. equilibrium

Cordon toll The amount of collected tax is redistributed to each household and expressed as follows:

$$G = \int_{\beta\widehat{z_{CT}}}^{\widehat{z_{CT}}} \tau_k(x)\mathrm{d}x$$

leading to:

$$G = \frac{ma(\widehat{z_{CT}} - \beta\widehat{z_{CT}})^2 c}{2hL}$$

The bid rent including the *cordon toll* is given by:

$$\Psi_C(x) = \frac{a\left[2h(1-a)(\omega_C\tau_0 + t_0)(\widehat{z_{CT}} - x) + \omega_C\tau_1 a(\widehat{z_{CT}} - x)^2\right]}{2h^2(1-a)} + R_A + \frac{ca}{h}$$

When $\alpha\widehat{z_{CT}} < x < \widehat{z_{CT}}$:

$$\Psi_C(x) = \frac{a\left[2h(1-a)(\omega_C\tau_0 + t_0)(\widehat{z_{CT}} - x) + a\omega_C\tau_1(\widehat{z_{CT}} - x)^2\right]}{2h^2(1-a)} + R_A$$

The indirect utility of the inhabitant who lives at $\widehat{z_{CT}}$ and works in the CBD is now given by:

$$V_{CT}(\widehat{z_{CT}}) = \omega_C\left[1 - \widehat{z_{CT}}\tau_0 - \frac{a\tau_1\widehat{z_{CT}}^2}{2h(1-a)}\right] + G + \frac{ALR}{L} - t_0\widehat{z_{CT}} - c - \frac{R_A h}{a}$$

When $z_S < x < y$:

$$V_{CT}(y) = \omega_S\left[1 - (\frac{y - \widehat{z}_{CT}}{2})\tau_0 - \frac{a\omega_S\tau_1(\frac{y-\widehat{z}_{CT}}{2})^2}{2h(1-a)})\right] + G + \frac{ALR}{L} - t_0(\frac{y - \widehat{z}_{CT}}{2}) - \frac{RAh}{a}$$

The limit of the CBD $\widehat{z}$ is implicitly defined by:

$$\omega_C\left[1 - \widehat{z}\tau_0 - \frac{\tau_1 a\widehat{z}^2}{2h(1-a)}\right] - t_0\widehat{z} - c = \omega_S\left[1 - (\frac{y-\widehat{z}}{2})\tau_0 - \frac{\tau_1 a(\frac{y-\widehat{z}}{2})^2}{2h(1-a)}\right] - t_0(\frac{y-\widehat{z}}{2})$$

yielding:

$$z_{CT} = \frac{-2h(1-a)\left[\tau_0(2\omega_C + \omega_S) + 3t_0\right] - \omega_S\tau_1 ay + 2\sqrt{\Delta_C}}{(4\omega_C - \omega_S)\tau_1 a}$$

with:

$$\begin{aligned}\Delta_C &= h^2(-1+a)^2\left[\tau_0(2\omega_C + \omega_S) + 3t_0\right]^2 \\ &+ 2h(1-a)\tau_1 a\left[4\omega_C^2 + \omega_S^2 + 3\omega_C\omega_S\tau_0 y - 5\omega_C\omega_S + (2\omega_C + \omega_S)t_0 y - (4\omega_C - \omega_S)c\right] + \omega_C\omega_S\tau_1^2 a^2 y^2\end{aligned}$$

Welfare is given by:

$$W_{CT} = \frac{ma}{h}\left[\int_0^{\alpha\widehat{z}} \omega_C\left[1 - T(x)\right] - t_0(x)\mathrm{d}x + \int_{\alpha\widehat{z}}^{\widehat{z}} \omega_C\left[1 - T(x)\right] - t_0(x)\mathrm{d}x + 2\int_{z_S}^{y} \omega_S\left[1 - T(x)\right] - t_0(x - z_S)\mathrm{d}x\right]$$

with:

$$ALR = \frac{ma}{h}\left[\int_0^{\widehat{z}} \frac{\Psi_C h}{a}\mathrm{d}x + 2\int_{z_S}^{y} \frac{\Psi_S h}{a}\mathrm{d}x\right]$$

Hence, the following welfare is:

$$W_{CT} = \frac{ma}{h}\left[\omega_C\widehat{z} - \frac{(\omega_C\tau_0 + t_0)\widehat{z}^2}{2} - \frac{\omega_C\tau_1 a\widehat{z}^3}{3h(1-a)} + \omega_S(y-\widehat{z}) - (\omega_S\tau_0 + t_0)\left(\frac{y-\widehat{z}}{2}\right)^2 - \frac{\omega_S\tau_1 a(y-\widehat{z})^3}{12h(1-a)}\right]$$

Then, we differentiate W_{CT} with respect to $\widehat{z}$ yielding:

$$\frac{\partial W_{CT}}{\partial\widehat{z}} = \frac{ma}{h}\left[\omega_C - (\omega_C\tau_0 + t_0)\widehat{z} - \frac{\omega_C\tau_1 a\widehat{z}^2}{h(1-a)} - \omega_S + (\omega_S\tau_0 + t_0)\left(\frac{y-\widehat{z}}{2}\right) + \frac{\omega_S\tau_1 a(y-\widehat{z})^2}{4h(1-a)}\right]$$

When we differentiate W_S with respect to $\widehat{z}$ conditional on $\widehat{z} = \widehat{z}_C$, it yields:

$$\left.\frac{\partial W_S}{\partial\widehat{z}}\right|_{\widehat{z}=\widehat{z}_C} = \frac{ma}{h}\left[c + \frac{\omega_S\tau_1 a(y-\widehat{z})^2}{8h(1-a)} + \frac{(\omega_S\tau_0 + t_0)(3\widehat{z} - y)}{2}\right] \lesseqgtr 0$$

WORKING PAPER

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